Math 20300 Spring 2006 Final Exam Answers:

1a) $4(x-1)+7(y-1)+5z=0$ b) $\frac{x-1}{4}=\frac{y-1}{7}=\frac{z}{5}$ c) $\frac{\sqrt{90}}{2}$

2a) $\langle 3x^2y,\ \ x^3+2ye^z,\ \ y^2e^z\rangle$ b) $\frac{3}{7}$ c) $3(x-1)+3(y-1)+z=0$

3a) Absolutely convergent – use the limit comparison test with $\sum\frac{1}{n^{3/2}}$.

b) Conditionally convergent. Use the integral test to show that the series is not absolutely convergent and the alternating series test to then show it is conditionally convergent.

c) Divergent – the terms of the series do not have a limit. ($\frac{n}{\sqrt{4n^2+1}}\to\frac{1}{2}$ as $n\to\infty$, so in the limit the terms of the alternating series oscillate between $\frac{1}{2}$ and $-\frac{1}{2}$.)

4. The volume is 8 times the volume of the portion in the first octant. This gives

$$V=8\int_0^{\frac{\pi}{2}}\int_0^2\int_0^{\frac{3}{2}\sqrt{4-r^2}} r\,dz\,dr\,d\theta=16\pi$$

(The upper z-limit is found by expressing the boundary condition $z=3\sqrt{1-\frac{x^2}{4}-\frac{y^2}{4}}$ in cylindrical coordinates.)

5. The series is absolutely convergent in the interval $\left[\frac{11}{4},\frac{13}{4}\right]$, i.e. $|x-3|\le\frac{1}{4}$.

6. The critical points are $(0,0)$, $(3,6)$, and $(3,-6)$. Local maximum: $(0,0)$ Saddle points: $(3,6)$ and $(3,-6)$.

7. $M=\frac{\pi}{2}\int_0^1\int_0^{\frac{1}{1+x^2}} dy\,dx=\frac{\pi^2}{8}$, $M_y=\frac{\pi}{2}\int_0^1\int_0^{\frac{1}{1+x^2}} x\,dy\,dx=\frac{\pi}{4}\ln 2$, $\bar{x}=\frac{2\ln 2}{\pi}$.

8a). Use the expansion $e^{-t^2}=1-t^2+\frac{t^4}{2!}-\frac{t^6}{3!}+\cdots+\frac{t^{2k}}{k!}+\cdots$ and integrate term-by-term from 0 to 0.1. This gives

$$\int_0^{0.1} e^{-t^2}dt=0.1-\frac{(0.1)^3}{3}+\frac{(0.1)^5}{5(2!)}+\cdots$$

where the terms alternate in sign. By the alternating series test, the sum of the first two terms differs from the limit by at most the next term, $\frac{(0.1)^5}{5(2!)}$. Since the latter is less than 10^{-5}, we can take as our approximation the sum of first two terms, $0.1-\frac{(0.1)^3}{3}\approx 0.09967$.

8b) $x=\frac{\sqrt{3}}{2}-\frac{t}{2}$, $y=\frac{1}{2}+\frac{\sqrt{3}\,t}{2}$, $z=-\ln 2+t\sqrt{3}$

9. a) $f(x)=8+5(x-1)+(x-1)^2$ b) $SA=\iint_R \sqrt{1+z_x^2+z_y^2}\,dA=\frac{\sqrt{5}}{2}\iint_R dA=8\pi\sqrt{5}$, where R is the disc bounded by the circle $x^2+y^2=16$.

10a) $\frac{\partial u}{\partial t}=-\sin t\frac{2xy^2}{(x^2+y^2)^2}-e^s\frac{2x^2y}{(x^2+y^2)^2}$, where x and y are replaced by $x=\cos t$ and $y=te^s$.

b) Restricting the analysis to the 1st octant, the volume is given by $V=\int_0^1\int_0^x (1-x^2)\,dy\,dx=\frac{1}{4}$. (Sketch omitted)

11a) $\int_0^1\int_0^{x^2}\sqrt{x^3+1}\,dy\,dx=\frac{2}{9}(2^{\frac{3}{2}}-1)$ (Sketch omitted)

b) The limit does not exist. Along the x axis the expression is equal to one, but on the line $y=x$ the expression equals two.

12a) $V=\int_0^{2\pi}\int_0^{\pi/3}\int_0^2 \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta=\frac{8\pi}{3}$

b) The series converges absolutely, since for all n, $\frac{|\sin(n^2)|}{n^2+1}<\frac{1}{n^2}$, and the series $\sum_n \frac{1}{n^2}$ is convergent. Since the series is absolutely convergent, it is also convergent.