1. (a) An ellipsoid with vertices $(\pm 2, 0, 0)$, $(0, \pm 2, 0)$ and $(0, 0, \pm 8)$. The traces are a circle of radius 2 and an ellipse with semimajor axis 8 on the z-axis and semiminor axis 2.

(b) Let $F = \frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{64}$; $\nabla F = \langle \frac{x}{2}, \frac{y}{2}, \frac{z}{32} \rangle$; $\nabla F(1, -1, 4\sqrt{2}) = \langle \frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{8} \rangle$.
The plane has equation $\frac{1}{2}(x-1) - \frac{1}{2}(y+1) + \frac{\sqrt{2}}{8}(z - 4\sqrt{2}) = 0$ or $x - y - \frac{\sqrt{2}}{4}z = 4$

2. (a) Proof: $\langle 1, 2, -3 \rangle \neq c\langle 1, 2, -1 \rangle$ for any nonzero real number c. Note: Verifying that the dot product of the two vectors above is not zero, shows the line is not perpendicular to the **normal** to the plane, i.e, that the line and the plane are not parallel.

(b) $-1 - 3t = 3 + t + 2(2 + 2t)$; $t = -1$; substitute into equations of line: $(2, 0, 2)$

(c) The normal $\langle 1, 2, -1 \rangle$ to the plane is the direction of the line; a point on the line is $(0, 0, 4)$; the symmetric form is $\frac{x}{1} = \frac{y}{2} = \frac{z-4}{-1}$.

3. $2\int_0^2 \int_{x^2}^4 x^2 y \, dy \, dx = \int_0^2 x^2 y^2 |_{y=x^2}^4 \, dx = \int_0^2 16x^2 - x^6 \, dx = (\frac{16x^3}{3} - \frac{x^7}{7})|_0^2 = 2^7(\frac{1}{3} - \frac{1}{7}) = \frac{512}{21}$; or
$2\int_0^4 \int_0^{\sqrt{y}} x^2 y \, dx \, dy = 2\int_0^4 \frac{x^3 y}{3} |_{x=0}^{y^{1/2}} \, dy = \frac{2}{3}\int_0^4 y^{5/2} \, dy = \frac{2}{3} \cdot \frac{2}{7} y^{7/2} |_0^4 = \frac{512}{21}$.

4. (a) $\nabla f = -e^{-xyz}\langle yz, xz, xy \rangle + \langle 1, 0, 0 \rangle$; $\nabla f(2, 0, 1) = -\langle 0, 2, 0 \rangle + \langle 1, 0, 0 \rangle = \langle 1, -2, 0 \rangle$.
$D_u f = \langle 1, -2, 0 \rangle \cdot \langle 3, 1, 1 \rangle / \sqrt{11} = \frac{1}{\sqrt{11}}$. CAUTION: $(3\mathbf{i} - 2\mathbf{j})/\sqrt{11}$ is wrong!

(b) (i) $\nabla f(2, 0, 1) = \langle 1, -2, 0 \rangle$; (ii) $\| \nabla f(2, 0, 1) \| = \sqrt{5}$.

5. The boundary surfaces define two bounded regions of equal volume: one region in the first octant and one region for which all x- and y-coordinates are negative. For the region in the first octant, $z = 4 - x^2$ is the top surface and the triangle in the xy-plane with sides $x = 2$ (which is the intersection of $z = 4 - x^2$, and $z = 0$), $y = 0$, and $x = y$. The volume is
$\int_0^2 \int_0^x 4 - x^2 \, dy \, dx = \int_0^2 4x - x^3 \, dx = (2x^2 - \frac{x^4}{4})|_0^2 = 4$.

6. Apply the ratio test to obtain convergence if

$$\lim_{n\to\infty} \left| \frac{2^{n+1}(x-1)^{n+1}}{n+2} \cdot \frac{n+1}{2^n(x-1)^n} \right| = \lim_{n\to\infty} \frac{2(n+1)}{n+2}|x-1| = 2|x-1| < 1, \text{ or } |x-1| < \frac{1}{2}.$$

Thus, the series converges in the interval $(\frac{1}{2}, \frac{3}{2})$, and the endpoints must be tested separately: Substituting $x = \frac{1}{2}$ yields the series $\sum_0^\infty \frac{(-1)^n}{n+1}$, the alternating harmonic series, which is convergent. Substituting $x = \frac{3}{2}$ yields the harmonic series, which is divergent, and the interval of convergence is $[\frac{1}{2}, \frac{3}{2})$.

The radius of convergence can also be calculated as $1/\lim \left| \frac{2^{n+1}}{n+2} \cdot \frac{n+1}{2^n} \right| = \frac{1}{2}$.

7. (a) Use the comparison and integral tests to test the series for absolute convergence: $\left| \frac{\cos n}{n(\ln n)^2} \right| \leq \frac{1}{n(\ln n)^2}$ and $\int_2^\infty \frac{1}{x(\ln x)^2} = \int_{\ln 2}^\infty \frac{1}{u^2} \, du < \infty$, so the series of absolute values of the terms of the given series is term by term smaller than a convergent series, and the answer is that the given series is absolutely convergent.

(b) For each n, $a_{n+1}/a_n = -2/9$, so this is a geometric series with common ratio having absolute value less than 1, and the series is absolutely convergent. The sum is $\frac{-2/9}{1-(-2/9)} = -\frac{2}{11}$.

(c) Divide the numerator and denominator of the nth term by 2^n to obtain $\frac{(-2)^n}{2^n+n} = \frac{(-1)^n}{1+n \cdot 2^{-n}}$, so that, for large n, the terms alternate between size approximately 1 and -1. Thus, the terms of series do not converge to zero (or, in fact, any number), and the series is divergent.

8. (a) (i) Along a path with $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{x^2} = 1$. Similarly, along a path $x = -y$, the limit is 0. Since the limits along at least two different paths are distinct, the function does not have a limit at $(0,0)$.

(ii) Evaluate continuous functions to find their limit: $\frac{0+0-1}{e^{0^2+0^2}} = -1$

(b) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = (2u)(3) + (4v)(1) = 6(3x+y) + 4(x+4y) = 22x + 22y$. At $(x,y) = (1,-1)$, $\frac{\partial f}{\partial x} = 22 - 22 = 0$; or $f(u(x,y), v(x,y)) = (3x+y)^2 + 2(x+4y)^2$, and one can simplify and differentiate or differentiate and simplify to obtain the same result.

9. There are no singular or boundary points, so all extrema occur at stationary points for $f(x,y) = x^2y + x^2 + 8y^2$:

$$f_x = 2xy + 2x = 2x(y+1) = 0; \qquad f_y = x^2 + 16y = 0$$

From the first equation, $x = 0$ or $y = -1$. Substituting these values into the second equation yields the three critical points $(0,0)$ and $(\pm 4, -1)$. We calculate the discriminant $D(x,y)$:

$$\begin{aligned} f_{xx} &= 2y+2; \qquad f_{xy} = 2x; \qquad f_{yy} = 16; \\ D(x,y) &= f_{xx}f_{yy} - f_{xy}^2 = 32(y+1) - 4x^2; \\ D(0,0) &= 32, \quad f_{yy} = 16 > 0, \text{ so } (0,0) \text{ is a minimum.} \\ D(\pm 4, -1) &= -4^3 < 0, \text{ so } (\pm 4, -1) \text{ are saddle points.} \end{aligned}$$

10. (a) The surface of the hemisphere may be described in spherical coordinates by the equation $\rho = 5$, and the density is ρ^2. The mass is $\int\int\int_H \rho^2\, dV$, where H is the hemisphere. Iterating the integral, we obtain

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^5 \rho^2(\rho^2 \sin\phi)\, d\rho\, d\phi\, d\theta = \int_0^{2\pi} d\theta \int_0^{\pi/2} \sin\phi\, d\phi \int_0^5 \rho^4\, d\rho = 2\pi[0 - (-1)]\frac{\rho^5}{5}\Big|_0^5 = 1250\pi.$$

(b) The centers of the four squares are $(1,1)$, $(1,3)$, $(3,1)$, and $(3,3)$, and the area $\Delta x \Delta y$ of each square is 4. The Riemann sum is $4(1^2 + 1^2 + 3^3 + 3^2) = 80$. The exact value of the integral is $\int_0^4 x^2\, dx \int_0^4 dy = \frac{256}{3}$. Thus, the Riemann sum and the integral differ by $\frac{256}{3} - 80 = \frac{16}{3}$.

11. (a) Let S be the surface area, and let A be the area of the rectangle R above which the surface lies.

$S = \int\int_R \sqrt{1 + z_x^2 + z_y^2}\, dA = \int\int_R \sqrt{1+4+1}\, dA = \sqrt{6}A = 2\sqrt{6}$.

(b) The region of integration is the roughly triangular shaped region in the first quadrant bounded on the right by the graph of $y = x^3$, on the left by the y-axis, and above by the line $y = 8$. Reversing the order of integration yields $\int_0^2 \int_{x^3}^8 f(x,y)\, dy\, dx$.

12. (a) One should know from memory the Maclaurin series $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 \pm \ldots$; alternately, one can calculate the Taylor coefficients: $c_n = f^{(n)}(0)/n!$.

(b) $\sin x^2 = x^2 - \frac{1}{3!}(x^2)^3 + \frac{1}{5!}(x^2)^5 \pm \cdots = x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} \pm \ldots$;

(c) $\int_0^{1/2} \sin(x^2)\, dx = \int_0^{1/2} x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} \pm \ldots\, dx = [\frac{x^3}{3} - \frac{x^7}{42} + \frac{x^{11}}{11\cdot 120} \pm \ldots]_0^{1/2} = \frac{1}{3\cdot 2^3} - \frac{1}{42\cdot 2^7} + \frac{1}{11\cdot 120\cdot 2^{11}} \pm \ldots$

Since estimating the sum of an alternating series by summing through nth term has error with absolute value at most the absolute value of the $(n+1)$st term, the integral is approximately $\frac{1}{24}$ (the first term of the series), with error at most $\frac{1}{42\cdot 128} < \frac{1}{100}$.