1a: Take cross products of the normal to get the direction of the line:

\[ \text{Cross}[(2, -1, 1), (1, 0, -3)] \]
\[ (3, 7, 1) \]

Then parameterize the line using the point and the direction to get:

\[ x = 4 + 3t, \ y = -7t, \ z = -3 + t \]

1b: The desired plane contains the normals of the two specified plane, so its normal is the same cross product as in 1a. That gives an equation of the plane as \( 3(x - 4) + 7(y + 2) + (z - 3) = 0 \)

2a: the gradient is:

\[ f = x^2 y + y^2 z \]
\[ x^2 y + y^2 z \]
\[ \{D[f, x], D[f, y], D[f, z]\} \]
\[ \{2xy, x^2 + 2yz, y^2\} \]

2b: the gradient will give a normal vector, so evaluating at the point gives:

\[ \{2xy, x^2 + 2yz, y^2\} / \{x \rightarrow 2, \ y \rightarrow -1, \ z \rightarrow 3\} \]
\[ \{-4, -2, 1\} \]

So an equation of the tangent plane is \(-4(x - 2) + -2(y + 1) + (z - 3) = 0\)

2c: we just want \(D, f = \nabla f \cdot v\) to be negative, so there are many correct answers. For example, moving in the direction given by \(i\) gives:

\(\{-4, -2, 1\}, \{1, 0, 0\}\)

\(-4\)

3: First, sketch the triangular region to reverse the order, giving an integral which goes easily by substitution:

\[ \int_0^1 \int_0^{\sqrt{1+x^2}} \, dy \, dx \]
\[ \frac{1}{3} \left( -1 + 2 \sqrt{2} \right) \]

4a: Absolutely convergent by the ratio test
4b: Convergent by the alternating series test, and then absolutely convergent by the integral test:

\[ \int_2^\infty \frac{1}{x \log^2 x} \, dx \]
\[ \frac{1}{\log[2]} \]
4c: Convergent by the alternating series test, and then divergent by limit comparison with $\sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$ so it is conditionally convergent

5: use the ratio test to get convergence on the interval $-2<x<2$. For $x=2$, divergent by comparison with the harmonic series. For $x=-2$, convergent by the alternating series test.

6a: Taking the first partials and setting them to zero and solving gives:

\[
T = x^3 + y^3 - 3 \times xy
x^3 - 3 \times xy + y^3
\]

\[
\text{Solve}\left[\{D[T, x] = 0, D[T, y] = 0\}\right]
\]

\[
\left\{\{x \to 0, y \to 0\}, \{x \to 1, y \to 1\}, \{x \to (-1)^{1/3}, y \to (-1)^{2/3}\}, \{x \to (1)^{2/3}, y \to -(1)^{1/3}\}\right\}
\]

At the two points with real coordinates (0,0) and (1,1), we evaluate the discriminant to get:

\[
D[T, x, x] D[T, y, y] - D[T, x, y]^2
-9 + 36xy
-9 + 36xy /. \{x \to 0, y \to 0\}
-9
-9 + 36xy /. \{x \to 1, y \to 1\}
27
D[T, x, x] /. \{x \to 1, y \to 1\}
6
\]

So there is a relative min at (1,1) and a saddle at (0,0).

6b: we consider the two points from part a) together with the edges:

\[
T /. \{x \to 0, y \to t\}
\]

\[
\text{Solve}\left[D[T /. \{x \to 0, y \to t\}, t] = 0\right]
\]

\[
\left\{\{t \to 0\}\right\}
\]

\[
\text{Solve}\left[D[T /. \{x \to 2, y \to t\}, t] = 0\right]
\]

\[
\left\{\{t \to -\sqrt{2}\}, \{t \to \sqrt{2}\}\right\}
\]

\[
\text{Solve}\left[D[T /. \{x \to t, y \to 0\}, t] = 0\right]
\]

\[
\left\{\{t \to 0\}\right\}
\]

\[
\text{Solve}\left[D[T /. \{x \to t, y \to 2\}, t] = 0\right]
\]

\[
\left\{\{t \to -\sqrt{2}\}, \{t \to \sqrt{2}\}\right\}
\]
That gives potential critical points at the four corners, the interior point \((1,1)\) and the points \((\sqrt{2}, 2)\) and \((\sqrt{2}, 0)\). Checking all of them, we get:

\[
\begin{align*}
T/ & \to \{x \to 1, y \to 1\} \\
T/ & \to \{x \to 0, y \to 3\} \\
T/ & \to \{x \to 0, y \to 0\} \\
T/ & \to \{x \to 2, y \to 0\} \\
T/ & \to \{x \to \sqrt{2}, y \to 3\} \\
T/ & \to \{x \to \sqrt{2}, y \to 0\} \\
-1 & \\
27 & \\
17 & \\
8 & \\
0 & \\
27 - 7\sqrt{2} & \\
2\sqrt{2} & \\
\end{align*}
\]

So the hottest point is \((0,3)\) with temperature 27.

7: set up the integral in polar, to get

\[
\int_0^{2\pi} \int_0^a r \left( a - \sqrt{r^2} \right) \, dr \, d\theta
\]

\[
\frac{1}{3} a^2 \pi \left( 3 a - 2 \text{Abs}[a] \right)
\]

8. So \(f(x,y)\) is of the form \(f(r) = f(x^2 + y^2)\). Using the chain rule, we get \(f_x = 2x f_r\) and \(f_y = 2y f_r\) and the desired result.

9a. Use the geometric series with \(r = -t^2\) to get the function as \(1 - t^2 + t^4 - t^6 + \ldots\) which converges for \(-1 < t < 1\). 

9b. Integrating gives \(t - \frac{t^3}{3} + \frac{t^5}{5} - \ldots\) and then evaluating gives \(\frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}^3} + \frac{1}{5\sqrt{3}^5} - \ldots\) 

For the desired error, using the alternating series analysis, we want \(1/k (\text{Sqrt}[3])^{2k+1}\) to be less than .01, which happens when \(k=3\), so the first 3 terms are sufficient

\[
\text{Table}\left[\frac{1}{(2k+1) (\sqrt{3})^{2k+1}}, \{k, 1, 4\}\right]
\]

\[
\{0.06415, 0.01283, 0.00305476, 0.000791976\}
\]

10. To find the mass, we integrate the density which is \(\rho = \sqrt{x^2 + y^2} = r\) to get, using polar coordinates:
For the x-coordinate of the center of mass:

\[ \bar{x} = \frac{1}{8 \pi} \int_{\frac{\pi}{3}}^{\pi} \int_{0}^{2} r \cos(\theta) r \, dr \, d\theta \]

\[ \bar{x} = \frac{3}{\pi} \]

\[ \bar{y} = \frac{1}{8 \pi} \int_{\frac{\pi}{3}}^{\pi} \int_{0}^{2} r \sin(\theta) r \, dr \, d\theta \]

\[ \bar{y} = 0 \]

11. Using the shadow technique and integrating \( \sqrt{1 + f_x^2 + f_y^2} \), we get:

\[ f = \sqrt{9 - x^2 - y^2} \]

\[ \sqrt{9 - x^2 - y^2} \]

Simplify \( 1 + D[f, x]^2 + D[f, y]^2 \)

\[ 9 \]

\[ -9 + x^2 + y^2 \]

Converting to polar and doubling (there is a top piece and a bottom piece):

\[ 2 \int_{0}^{2} \int_{0}^{\frac{2}{3}} \sqrt{\frac{9}{9 - x^2}} \, r \, dr \, d\theta \]

\[ -12 \left( -3 + \sqrt{5} \right) \pi \]

12. a) changing to polar gives the quantity as \( \frac{r^2 \cos(\theta) \sin(\theta)}{r^2} \) which goes to zero as \( r \) goes to zero, independant of \( \theta \). So the limit exists and is zero.

12. b) Taking derivatives and evaluating:

\[ \text{Table} \left[ D \left[ \frac{1}{x}, \{x, i\} \right], \{i, 1, 6\} \right] \]

\[ \left\{ \frac{1}{x^2}, \frac{2}{x^3}, \frac{6}{x^4}, \frac{24}{x^5}, \frac{120}{x^6}, \frac{720}{x^7} \right\} \]

which gives the series:
Knowing that $e$ is between 2 and 3 gives an error estimate of:

\[
R_6(e) = f^{(6)}(z) \frac{(e-2)^6}{(5+1)!}
\]

which is less than if we let $z=3$ and note that $(e-2)^6 < 1$:

\[
\left| R_6(e) \right| < \frac{720}{2^7} \frac{1}{128} = \frac{1}{128}
\]