Department of Mathematics **The City College of New York**

Math 203 **Final Exam** **Spring 2008**

No calculators permitted. Answers may be left in terms of radicals, π, e, etc and do not need to be simplified unless stated otherwise. Each question is worth 10 points.

Part I. Answer all 7 questions

1. (a) A curve is given parametrically by the equations $x = \sin(4t); y = \cos(4t); z = 2t^{3/2}$. Compute the parametric equations of the tangent line at the point when $t = 1$.

(b) Write an iterated integral, using polar coordinates, whcich can be used to find the surface area of the portion of the graph of $z = xy^3$ which is above the region in the first quadrant of the xy-plane inside the circle $x^2 + y^2 = 1$. YOU DO NOT HAVE TO EVALUATE THE INTEGRAL.

2. For parts (a),(b) and (c) let $f(x, y) = x^2 - e^{xy}$.

(a) At the point $(3, 0)$ compute the unit vector in the direction of maximum increase of the function f and compute the rate of increase in that direction.

(b) Compute the directional derivative of the function f at the point $(3, 0)$ in the direction of the vector $< -5, 12 >$.

(c) Compute an equation for the plane tangent to the surface given by the equation $z = f(x, y)$ at the point in space with $x = 3$ and $y = 0$.

3. Sketch the region of integration for

$$\int_0^2 \int_{\sqrt{y/2}}^1 \sqrt{x^3 + 1}\, dxdy,$$

and evaluate the integral. Notice that it may help to change the order of integration.

4. Find and classify the critical points of $f(x, y) = x^2y^2 - 3x^4 - y^2 - 10x^2 + 5$.

5. Find the volume of the solid inside the cylinders $x^2+y^2 = 4$, below the surface $z = x^2+1$ and above the plane $z = 1$.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent. Credit will not be given unless the reasons for your conclusions are explicitly stated.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5n}{3n^2 - 2n + 1} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n^2} \qquad \sum_{n=1}^{\infty} \cos(\frac{1}{n!})$$

7. Find the interval of convergence (including possible endpoints) for the power series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{3n\ 2^n}.$$

Part II. Answer 3 of the following 5 questions

8. Find the absolute minimum and absolute maximum of $f(x, y) = y - 6x - 10$ on the region bounded by $y = x^2$ and $y = 16$.

9. (a) Compute the equation of the plane which contains the three points $(1, 0, 1), (0, 2, 1)$ and $(1, 3, 2)$.

(b) Evaluate the following limit or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2 + y^2}$$

10. (a) Let R be the solid in the first octant which is bounded by the sphere $x^2+y^2+z^2 = 4$ and the planes $y = 0, z = 0$ and $y = x$. The density at any point of R is equal to the distance of that point from the xy-plane. Express the mass of R as an iterated integral three ways: using rectangular, cylindrical and spherical coordinates.
(a) Express the integral $\int\int\int_R z\ dV$ three ways: using rectangular, cylindrical and spherical coordinates.

(b) Use one of the three integrals of part (a) to compute the common value.

11. Use a series expansion to obtain a numerical estimate of the integral

$$\int_0^{1/10} \frac{1}{1 + 10x^3}\ dx$$

which is accurate to within $1/1,000,000$. Show your work and explain how you estimated the size of the error.

12. (a) The diameter of a right circular cylinder is measured to be 6 centimeters and the height is measured to be 10 centimeters. Each measurement is accurate to within $1/10$ of a centimeter. Compute the volume of the cylinder and use differentials to estimate the error in the volume resulting from the original measurement errors.

(b) Use the integral test to prove that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.