MATH 203 Final Exam
May 23, 2007

PART I: Answer all parts of Questions 1-8 (points as indicated).

Question 1 (8 points) Given the two lines $\frac{x-2}{3} = \frac{y}{4} = \frac{z+1}{5}$ and $\frac{x+2}{6} = \frac{y-1}{8} = \frac{z}{10}$
(a) Are they parallel? Explain.
(b) Find an equation of the plane containing the two lines.

Question 2 (8 points) (a) Find an equation of the tangent plane at $(2, 1, 3)$ to the surface $z = xy + \sqrt{y}$.
(b) Find the parametric equations of the normal line; that is, the line perpendicular to the tangent plane, passing through $(2, 1, 3)$.

Question 3 (8 points) Consider the function $T(x, y, z) = \dfrac{9}{1 + x^2 + y^2} - 3z$.
(a) Find the rate of change of T at the point $(1, 1, 1)$ in the direction towards the point $(11, 21, 6)$.
(b) In what direction does T increase most rapidly at the point $(1, 1, 1)$?
(c) At what rate is T increasing in the direction given by the answer to part (b)?

Question 4 (8 points) Let $f(x, y, z) = \dfrac{\sqrt{x}}{yz^2}$. Use linear approximation (differentials) to approximate $f(.98, 1.03, 1.01)$.

Question 5 (8 points) Find the area of the portion of the surface $z = x^3 + y$ that lies above the region in the xy-plane bounded by $y = x^3$, $x = 1$ and the x-axis.

Question 6 (10 points) Consider the region R bounded on top by $z = 1 - x$, on the side by $y = 1 - x^2$ and which lies in the first octant.
a) Sketch the region R.
b) Find the volume of the region R.

Question 7 (10 points) Find the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)2^n}$.
Remember to check convergence at the endpoints, if applicable.

Question 8 (10 points) State, for each series, whether it converges absolutely, converges conditionally or diverges. Justify each answer. Find the sum of one of the series which is convergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n}{1+n^2}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{5(-3)^n}{2^{2n}}$ (d) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n(2^n+1)}{2^{n+1}}$

PART I : Answer all parts of any three of Questions 9-12 (10 points each).

Question 9 Find the center of mass of the lamina which occupies the portion of the circle $x^2 + y^2 \leq 1$ which is in the first quadrant and has density $\delta(x, y) = xy$.

Question 10 Find all local maxima and minima and all saddle points of the function $f(x, y) = \frac{9}{2}x^2 + y^2 + xy^2$.

Question 11 Find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 18$, outside the cone $z^2 = x^2 + y^2$, and above the xy-plane.

Question 12 (i) Using a known Maclaurin series find the terms through x^8 for a representation of the function $f(x) = x^2 e^{-x^2}$.

(ii) Using your answer to (i), approximate the integral $\int_0^{1/10} x^2 e^{-x^2}\, dx$ as a sum of fractions. You do not have to evaluate the sum.

(iii) From the result in (ii) it follows that the integral is approximately 1/3000. Obtain an upper bound for the error in this estimate and provide a justification for your assertion.