Department of Mathematics **The City College of NY**
Math 20300 **Final Exam** **Spring 2006**

No calculators permitted. Answers may be left in terms of radicals, π, e, etc. and do not need to be simplified unless stated otherwise. Show all work.

Part I. Answer all 7 questions. Each is 10 points.

1. Given the points $P(1,1,0)$, $Q(2,-1,2)$, and $R(-2,2,1)$.
   a) Find the equation of the plane determined by the given points P, Q, R.
   b) Find the equation of the line, in symmetric form, which is perpendicular to the plane in (a) at the point P.
   c) Find the area of the triangle PQR.

2. Given that $u = f(x,y,z) = x^3 y + y^2 e^z$.
   a) Evaluate $grad\ u$.
   b) Find the rate of change of u at $(1,1,0)$ in the direction of $(3,-2,6)$.
   c) Find the equation of the tangent plane to the surface $x^3 y + y^2 e^z = 2$ at the point $(1,1,0)$.

3. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent. Credit will not be given unless the reasons for your conclusion are explicitly stated.

   a) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n-1}}{n(n+1)}$ b) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$ c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{4n^2+20}}$

4. Using a triple integral in cylindrical coordinates, find the volume of the ellipsoid
$$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1.$$

5. Find the interval of convergence (including possible endpoints) for the power series $\sum_{n=1}^{\infty} \frac{4^n (x-3)^n}{n(n+1)}$.

6. Find and classify the critical points of $f(x,y) = xy^2 - 6x^2 - 3y^2$.

7. A planar lamina with density $\rho = \frac{\pi}{2}$ is bounded by $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, and $x = 1$. Find the x–coordinate of the center of mass.

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Part II. Answer 3 of the following 5 questions. Each is 10 points.

8. a) Use a known series to approximate $\int_0^{0.1} e^{-t^2}\,dt$ to 5 decimal places.

   Explain how you know your result has the desired accuracy.

   b) Write the equation of the tangent line to the curve

   $x = \cos t,\ y = \sin t,\ z = \ln \sin t$ at $t = \dfrac{\pi}{6}$ in parametric form.

9. a) Let $f(x) = x^2 + 3x + 4$. Expand $f(x)$ in a Taylor series about $a = 1$, and verify your result algebraically.

   b) Find the surface area of the portion of the cone $x^2 + y^2 = 4z^2$ lying above the xy-plane and inside the cylinder $x^2 + y^2 = 16$.

10. a) Use the chain rule to find $\dfrac{\partial u}{\partial t}$ if $u = \dfrac{x^2}{x^2 + y^2}$, and $x = \cos t$ and $y = te^s$.

    You may leave your answer in terms of x, y, s, and t.

    b) Find the volume of the region bounded by $z = 1 - x^2$, $y = x$, $z = 0$ and $y = 0$; sketch the region.

11. a) i) Sketch the region of integration and then change the order of integration for $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\,dx\,dy$.

    ii) Evaluate the double integral you obtain in i).

    b) Does $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 + 2x^2y^2 + y^4}{x^4 + y^4}$ exist? Justify your answer.

12. a) Use spherical coordinates to determine the volume of the smaller region bounded by the sphere of radius 2 centered at the origin and the cone $\varphi = \dfrac{\pi}{3}$.

    b) Does the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin(n^2)}{n^2 + 1}$ converge or diverge? Explain carefully.