**MATH 203** **FINAL EXAMINATION** **SPRING 2005**

**PART I: Answer ALL questions in this part (10 points each)**
*Show all work and simplify all answers.* **NO CALCULATORS!!!**

**1**. (a) Sketch the graph of $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{64}=1$. Also graph the traces of this graph in the $xy$- and $xz$-planes.

(b) Find an equation of the tangent plane at the point $(1,-1,4\sqrt{2})$ to the graph in Part (a).

**2**. For the line with parametric equations

$$\begin{aligned} x &= 3+t \\ y &= 2+2t \\ z &= -1-3t \end{aligned}$$

and the plane with equation $z=x+2y$:

(a) Show the line is not perpendicular to the plane.

(b) Find the point of intersection.

(c) Find the symmetric form of the equations of the line which is perpendicular to the above plane and which passes through the $z$-axis at $z=4$.

**3**. Evaluate $\iint_R x^2y\,dx\,dy$, where $R$ is the planar region bounded by $y=x^2$ and $y=4$.

**4**. For the function $f(x,y,z)=e^{-xyz}+x$, find:

(a) The directional derivative at $(2,0,1)$ in the direction $\mathbf{v}=3\mathbf{i}+\mathbf{j}+\mathbf{k}$.

(b) (i) The direction and (ii) the magnitude of greatest increase at $(2,0,1)$.

**5**. Find the volume of the region bounded by $z=4-x^2$, $x=y$, $y=0$, and $z=0$.

**6**. Find the interval of convergence of the series $\sum_{n=0}^{\infty}\frac{2^n(x-1)^n}{n+1}$. (Remember to check the endpoints.)

**7**. State, for each series, whether it converges absolutely, converges conditionally or diverges. Justify each answer. Find the numerical value of one of the series which is convergent.

(a) $\sum_{n=2}^{\infty}\frac{\cos n}{n(\ln n)^2}$; (b) $\sum_{n=1}^{\infty}\frac{(-2)^n}{3^{2n}}$; (c) $\sum_{n=1}^{\infty}\frac{(-2)^n}{2^n+n}$

**CONTINUED ON BACK**

**Part II: Answer any THREE COMPLETE questions (10 points each).**
WRITE THE NUMBERS of the three questions you answered
on the cover of your answer booklet.

**8**. (a) Find the following limits, or show they do not exist:

(i) $\lim_{(x,y)\to(0,0)} \dfrac{(x+y)^2}{x^2+y^2}$; (ii) $\lim_{(x,y)\to(0,0)} \dfrac{x+y-1}{e^{x^2+y^2}}$

(b) If $f(u,v) = u^2 + 2v^2$, $u = 3x + y$ and $v = x + 4y$, find the value of $\dfrac{\partial f}{\partial x}$ when $x = 1$ and $y = -1$.

**9**. Find all local maxima, local minima and saddle points (if any) of the function

$$f(x,y) = x^2y + x^2 + 8y^2.$$

**10**. (a) Find the mass of a hemisphere of radius 5, given that the density at each point is equal to the square of the distance from the center (where the center means the point from which all radii emanate).

(b) Use the midpoint rule to approximate $\int_0^4 \int_0^4 x^2\,dx\,dy$ with $\Delta x = \Delta y = 2$.
(This means find the Riemann sum when the region of integration (a square) is divided into four equal size smaller squares and the centers of each of the smaller squares are used to calculate the Riemann sum.) How much does the approximation differ from the exact value of the integral?

**11**. (a) Find the area of the portion of the surface $z = 2x + y$ which is above the rectangle with sides $0 \le x \le 1$ and $0 \le y \le 2$ in the $xy$-plane.

(b) Sketch the region of integration and reverse the order of integration:

$$\int_0^8 \int_0^{y^{1/3}} f(x,y)\,dx\,dy.$$

[Note: The function $f$ is not given explicitly, so it is not possible to evaluate the integral.]

**12**. (a) Write the first three nonzero terms of the Maclaurin series of $\sin x$.

(b) Write the first three nonzero terms of the Maclaurin series for $\sin(x^2)$.

(c) Find the value of $\int_0^{1/2} \sin(x^2)\,dx$, accurate to the nearest hundredth. Explain how you know your answer has the required accuracy.