MATH 203 Final Exam December 19, 2007

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

Question 1 a) Find an equation of the plane that contains both the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the point $(1, 2, 0)$.
b) Find parametric equations describing the line through the point $(1, 2, 3)$ which is perpendicular to the plane $z = x + 2y - 4$.

Question 2 a) Given the curve $\vec{r}(t) =< \sqrt{t}, \frac{4}{t}, \frac{t^2}{2} >$, find a parameterization of the tangent line at $(2, 1, 8)$.

b) Find an equation of the tangent plane to the surface $\frac{x^3}{z^2} + 4(y-1)^2 = 5$ at the point $(1, 2, -1)$.

Question 3 Consider the function $f(x, y) = \frac{x}{2x + 3y}$.
a) Find the directional derivative of $f(x, y)$ at the point P $(2, -1)$ in the direction toward the point Q $(-1, 1)$.
b) Find the directional derivative in the direction of maximum increase of $f(x, y)$ at the point P.

Question 4
a) Let R be the region bounded by $y = x$, $y = 2x$ and $y = 6$. Write $\int\int_R f(x, y)\, dA$ as an iterated integral.

(b) Reverse the order of integration in the integral $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$

Note for both parts: You CAN NOT evaluate the integrals, since no function $f(x, y)$ has been specified.

Question 5 Let the region R in space be above the plane $z = 0$, inside $x^2 + y^2 + z^2 = 4$ and outside $x^2 + y^2 = 1$.
a) Sketch the region
b) Use a triple integral to find its volume.

Question 6 State, for each series, whether it converges absolutely, converges conditionally or diverges. Justify each answer.

(a) $\sum_{n=1}^{\infty} (-1)^n (\frac{n}{n+1})^n$ (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n^2)}{n^2 + 1}$

Question 7 (10 points) Find the set of all points x for which the following series converges: $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n^2)2^n}$.
Remember to check convergence at the endpoints, if applicable.

PART II : Answer all parts of any three of Questions 8-12 (10 points each).

Question 8 Find all critical points of the following function and classify each critical point (as a local maximum, local minimum, saddle point, etc.)
$f(x, y) = xy^2 - 2xy + x^2$.

Question 9 Let E be the region bounded the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and the planes $z = x$ and $z = 4$. Evaluate $\int\int\int_E z\, dV$.

Question 10 Recall that $\int_0^x \frac{1}{1+t^2}\, dt = \arctan(x)$.
a) Find a formula for the nth term of the power series centered at $x = 0$ for the function $\arctan(x)$
b) What is the minimum number of terms that you need in the series for $\arctan(x)$ to compute $\arctan(\frac{1}{10})$ with an error of less than $\frac{1}{10,000}$? Justify your answer.

Question 11 a) Show that the following limit does not exist:

$$\lim_{(x,y) \to (0,0)} \frac{x^2 + xy^2}{x^2 + y^2}$$

b) Graph the quadric surface $x^2 - 2x + y^2 - z^2 - 2z + 1 = 0$, labelling the coordinates of all vertices, if there are any. Show the trace of the graph in the coordinate planes.

Question 12 Let S be that part of the surface $z = 4 - x^2 - y^2$ that lies above $z = 1$. Find the **area** of S.