Department of Mathematics **The City College of New York**

Math 203 **Final Exam** **Fall 2006**

No calculators permitted. Answers may be left in terms of radicals, π, e, etc and do not need to be simplified unless stated otherwise. Each question is worth 10 points.

Part I. Answer all 7 questions

1. a) Compute an equation for the plane which contains the point (1,0,1) and the line given parametrically by the equations $x = 2t; y = 2 + t; z = 2 - t$.

b) Compute the directional derivative of the function $F(x, y, z) = xz^2 - (2y - 1)^2$ at the point $(1, 2, 3)$ in the direction of the vector $< 1, 0, -1 >$.

2. For parts (a) and (b) let $f(x, y) = e^x y - 3\tan(x)$.
a) Compute the unit vector pointing in the direction of greatest increase of the function f at the point $(0, -1)$ and compute the rate of increase in that direction.

b) Compute an equation for the plane tangent to the surface given by the equation $z = f(x, y)$ at the point in space with $x = 0$ and $y = -1$.

3. Evaluate $\iint_T x^2 y \, dA$, where T is the first quadrant region bounded by the curve with equations $y = x^3$ and lines $y = 8$ and $y = 8x$. Include a labeled sketch of the region T.

4. Find and classify all of the critical points of $f(x, y) = xy^2 - 2xy + x^2 + 3$.

5. Find the volume of the solid in the first octant, which is bounded by the coordinate planes, the cylinder $x^2 + y^2 = 9$, and the plane $y + z = 4$.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent. Credit will not be given unless the reasons for your conclusions are explicitly stated.

(a) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))}$ (c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}\, e^n}{n!}$

7. Find the interval of convergence (including possible endpoints) for the power series

$$\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n+1}.$$

(over)

Part II. Answer 3 of the following 5 questions

8. You plan to build an aquarium with a rectangular base and a volume of 12 cubic feet. The base is to be made of slate and the four sides will be glass. It is open at the top. The slate costs three times as much per square foot as the glass. Find the dimensions of the aquarium that minimize the total cost of the materials.

9. a) For the curve given parametrically by the equations $x = \cos(\pi t); y = 2e^{-t}; z = e^{2t}$, compute parametric equations of the tangent line at the point $(1, 2, 1)$.

b) For the surface given by the equation $z = x^4$, set up an iterated integral whose value is the surface area of the portion of the surface which lies over the region in the xy plane bounded by the curves $y = x^2$ and $y = x + 2$. DO NOT TRY TO EVALUATE YOUR INTEGRAL.

10. Let G be the solid above the xy plane which is contained above the cone given by $z^2 = x^2 + y^2$ and inside the sphere given by $x^2 + y^2 + z^2 = 4$. Evaluate $\iiint_G z \, dV$. Include a labeled sketch of the solid G.

11. a) Determine whether the following limit exists and if it exists compute its value. Justify your answer:

$$\lim_{(x,y) \to (0,0)} \frac{xy\cos(y)}{x^2 + y^2}$$

b) Compute the vector projection of the vector $< 2, 0, -1 >$ onto the vector $< 1, 1, -1 >$.

12. a) Use a known power series to find a power series expansion for the function $\frac{x}{1 + x^4}$.

b) Use your answer in part a) to find the first three nonzero terms of a numerical series that converges to $\int_0^{0.1} \frac{x}{1 + x^4} \, dx$.

Give an upper bound for the error of the approximation given by the sum of the first three terms.