Department of Mathematics **The City College of New York**

Math 203 **Final Exam** **Fall 2005**

No calculators permitted. Answers may be left in terms of radicals, π, e, etc. and do not need to be simplified unless stated otherwise. Each question is worth 10 points.

Part I. Answer all 7 questions

1. a) For the curve given parametrically by the equations $x = \sin(\pi t); y = e^t; z = e^{-t}$, compute the parametric equations of the tangent line at the point $(0, 1, 1)$.
b) Compute <u>all</u> unit vectors normal to the plane which contains the points $(0, 1, 1), (1, -1, 0)$, and $(1, 0, 2)$ and compute an equation of the plane.

2. For parts a), b) and c) let $f(x, y) = x^2 - 3xy$.
a) At the point with $x = 1$ and $y = -1$ compute the unit vector pointing in the direction of greatest increase of the function $f(x, y)$ and compute the rate of increase in that direction.
b) Compute an equation for the plane tangent to the surface given by the equation $z = f(x, y)$ at the point in space with $x = 1$ and $y = -1$.
c) Find the rate at which $f(x, y)$ is changing at $(1, -1)$ in the direction toward the point $(5, 2)$.

3. A triangular plate (a lamina) with density $\rho = 1$ has vertices at the points $(-3, 0), (0, 3)$ and $(3, 0)$. Compute $\bar{y}$, which is the y coordinate of the center of gravity. You may be able to compute the mass without integrating although you may use integration if you wish.

4. Find and classify the critical points of $f(x, y) = yx^2 + 2x^2 - y^2 - 6y$.

5. Find the volume of the solid, contained in the first octant, which is bounded by the cylinder $x^2 + y^2 = 4$, the paraboloid $z = 9 - x^2 - y^2$ and the planes $x = 0$, $z = 0$ and $y = x$.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent. Credit will not be given unless the reasons for your conclusions are explicitly stated.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n^2 + n + 1}$ b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^2}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n2^n}{n!}$

7. Find the interval of convergence (including possible endpoints) for the power series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}.$$

Part II. Answer 3 of the following 5 questions

8. Find the absolute minimum and absolute maximum of $f(x, y) = x^2 - xy - 2y - 1$ on the rectangle with vertices at $(0, 0), (3, 0), (3, 2)$ and $(0, 2)$.

9. a) Find the points, if any, on the surface given by $x^2 + y^2 = xz - z$ at which the tangent plane is parallel to the plane with equation $2y + z = 1$.
b) Write an iterated integral for the surface area of the portion of the surface given by $z = 3 + \sin(x + 2y)$ which lies over the region in the xy plane bounded by $y = x^2$ and $y = x + 2$. Do not attempt to compute the integral.

10. a) Change the following triple integral to cylindrical coordinates and then to spherical coordinates:

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$$

b) Use one of the three integrals of part (a) to compute the common value.

11. a) Find the Maclaurin series (that is, the Taylor series centered at 0) for $f(x) = \dfrac{1}{1 + x^2}$. Express your answer with summation notation.
b) Use your answer to part (a) to obtain the Maclaurin series for $g(x) = \arctan(x)$.
c) Estimate $\arctan(.1)$ to the nearest thousandth. Show the work used to guarantee your answer has the required accuracy.

12. a) Show that the following limit does not exist:

$$\lim_{(x,y) \to (0,0)} \frac{x^2 \cos(x + y)}{x^2 + y^2}$$

b) Explain carefully and precisely why the harmonic series diverges.