Instructions: Show all work. Calculators may not be used.

PART ONE: ANSWER SIX COMPLETE QUESTIONS (12 points each)

1. a) Write down a system of equations whose augmented matrix is given by the following matrix. Then use Gaussian elimination to get the general solution of the system.

\[
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 1 & -2 \\
1 & 3 & -2 & 0 & 0 & -1 & 0 \\
2 & 6 & -4 & 1 & 0 & 0 & 0 \\
1 & 3 & -2 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

b) Compute the vector which describes the direction of greatest increase for the function \( f(x,y) = x^2y^3 \) at the point with coordinates (2,1).

2. a) Find the inverse of the matrix

\[
\begin{pmatrix}
-4 & 5 & 2 \\
3 & 0 & 1 \\
1 & 4 & 3 \\
\end{pmatrix}
\]

b) Use your answer to part (a) to solve

\[
\begin{cases}
3x + 4y - z = 10 \\
x + 3z = 5 \\
2x + 5y - 3z = -5
\end{cases}
\]

3. a) Find the eigenvalues and eigenvectors of the matrix \( A = \begin{pmatrix} 4 & -1 \\ 5 & -2 \end{pmatrix} \)

b) Use your answer from a) to solve the following system of differential equations for \( y_1(t) \) and \( y_2(t) \) subject to the initial conditions \( y_1(0) = 4 \) and \( y_2(0) = 2 \):

\[
\begin{align*}
y_1' &= 4y_1 - y_2 \\
y_2' &= 5y_1 - 2y_2
\end{align*}
\]

4. a) Use Cramer’s Rule to solve for \( w \) ONLY:

\[
\begin{align*}
2x + 4y + 2w &= 0 \\
x + 3y + w &= 0 \\
x + 4y + z + w &= 1 \\
3x + 4z + w &= 2
\end{align*}
\]

b) Find the equation of the tangent plane for the surface given by the equation \( z = x^2y^3 \) at the point with \((x,y) = (2,1)\).

Part I continues on the other side of this page.
5. a) Find the surface area of the part of the paraboloid \( z = 4 - x^2 - y^2 \) contained in the first octant \( x \geq 0; y \geq 0; z \geq 0 \).

b) Compute the directional derivative of the function \( f(x,y) = x^2y^3 \) in the direction from (1,2) to (4,-2).

6. a) For the vectorfield \( \mathbf{F} = (ye^{xy} - z \sin(xz)) \mathbf{i} + (xe^{xy} + y^2) \mathbf{j} + (-x \sin(xz)) \mathbf{k} \) compute a potential function \( U(x,y,z) \) so the \( \nabla U = \mathbf{F} \).

b) Use your answer to part (a) to compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) from (0,1,0) to (1,2,\( \pi \)) along the path parametrized by \( < t, t^2 + 1, 2 \arcsin(t) > \) with \( 0 \leq t \leq 1 \).

7. a) For the path parametrized by \( \mathbf{r}(t) = < t, \sin(t), e^{2t} > \), compute parametric equations for the tangent line at the point \( (\pi, 0, e^{2\pi}) \).

b) Compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) along the path given in part (a) from \( t = 0 \) to \( t = \pi \) where \( \mathbf{F} = y \cos(x) \mathbf{i} + y^2 \mathbf{j} + z^2 \mathbf{k} \).

**PART TWO: ANSWER TWO COMPLETE QUESTIONS (14 POINTS EACH)**

8. Let \( R \) be the region \( x + 2y \leq 4; x \geq 0; y \geq 0 \) in the \( x,y \)-plane. Let \( C \) be the boundary of \( R \), oriented counterclockwise. Evaluate \( \int_C (\sin(x) + y^2) \, dx + 2y \, dy \)

a) directly as a line integral, and

b) as a double integral, by using Green’s Theorem.

9. Let \( S \) be the portion of the plane \( z = 2 - 2x - y \) which lies in the first octant. Let \( C \) be the boundary curve of \( S \), oriented counterclockwise as seen from above and let \( \vec{F} \) be the vector field \( x \mathbf{i} + y \mathbf{j} + xyz \mathbf{k} \). Evaluate \( \int_C \vec{F} \cdot d\vec{r} \)

a) directly as a line integral, and

b) as a surface integral, by using Stokes’ Theorem.

10. Let \( T \) be the solid \( x^2 + y^2 \leq 1; 0 \leq z \leq 1 \). Let \( S \) be the surface (including the top, bottom, and side) of \( T \) and let \( \vec{n} \) be the outward pointing unit normal vector. Let \( \vec{w} \) be the vector field \( y^2 \mathbf{i} + x^2 \mathbf{j} + \sin^2(\pi z) \mathbf{k} \). Evaluate \( \iint_S \vec{w} \cdot d\mathbf{S} \)

a) directly as a surface integral, and

b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Make sure you answered 6 complete questions from Part I and 2 complete questions from Part II.