THE CITY COLLEGE DEPARTMENT OF MATHEMATICS

Mathematics 392 Final Examination Spring 2005

Instructions: Show all work. Calculators may not be used.

PART ONE: ANSWER SIX COMPLETE QUESTIONS (12 points each)

1. a) Write down a system of equations whose augmented matrix is given by the following matrix. Then use Gaussian elimination to get the general solution of the system.

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & -2 \\ 1 & 3 & -2 & 0 & 0 & -1 & 0 \\ 2 & 6 & -4 & 1 & 0 & 0 & 0 \\ 1 & 3 & -2 & 1 & 0 & 1 & 0 \end{pmatrix}$$

b) Compute the vector which describes the direction of greatest increase for the function $f(x,y) = x^2y^3$ at the point with coordinates (2,1).

2. a) Find the inverse of the matrix $\begin{pmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{pmatrix}$

b) Use your answer to part (a) to solve $\begin{cases} 3x + 4y - z = 10 \\ x + 3z = 5 \\ 2x + 5y - 3z = -5 \end{cases}$

3. a) Find the eigenvalues and eigenvectors of the matrix A = $\begin{pmatrix} 4 & -1 \\ 5 & -2 \end{pmatrix}$

b) Use your answer from a) to solve the following system of differential equations for $y_1(t)$ and $y_2(t)$ subject to the initial conditions $y_1(0) = 4$ and $y_2(0) = 2$:

$$y'_1 = 4y_1 - y_2$$

$$y'_2 = 5y_1 - 2y_2$$

4. a) Use Cramer's Rule to solve for *w* ONLY:

$$\begin{aligned} 2x + 4y + 2w &= 0 \\ x + 3y + w &= 0 \\ x + 4y + z + w &= 1 \\ 3x + 4z + w &= 2 \end{aligned}$$

b) Find the equation of the tangent plane for the surface given by the equation $z = x^2y^3$ at the point with (x,y) = (2,1).

Part I continues on the other side of this page.

5. a) Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ contained in the first octant $x \geq 0; y \geq 0; z \geq 0$.

b) Compute the directional derivative of the function f(x,y) = x^2y^3 in the direction from (1,2) to (4,-2).

6. a) For the vectorfield **F** = $(ye^{xy} - z\sin(xz))$ **i** + $(xe^{xy} + y^2)$ **j** + $(-x\sin(xz))$ **k** compute a potential function U(x,y,z) so the ∇U = **F**.

b) Use your answer to part (a) to compute the line integral $\int_C$ **F** • **dr** from (0,1,0) to $(1,2,\pi)$ along the path parametrized by < t , $t^2 + 1$, 2 arcsin(t) > with $0 \leq t \leq 1$.

7. a) For the path parametrized by **r**(t) = < t, sin(t), e^{2t} >, compute parametric equations for the tangent line at the point (π, 0, $e^{2\pi}$).

b) Compute the line integral $\int_C$ **F** • **dr** along the path given in part (a) from t = 0 to t = π where **F** = $y\cos(x)$ **i** + y^2 **j** + z^2 **k.**

PART TWO: ANSWER TWO COMPLETE QUESTIONS (14 POINTS EACH)

8. Let R be the region $x + 2y \leq 4$; $x \geq 0; y \geq 0$ in the *x,y*- plane. Let *C* be the boundary of *R*, oriented counterclockwise. Evaluate $\int_C \left(\sin(x) + y^2\right)dx + 2y\,dy$

a) directly as a line integral, and
b) as a double integral, by using Green's Theorem.

9. Let *S* be the portion of the plane $z = 2 - 2x - y$ which lies in the first octant. Let C be the boundary curve of *S*, oriented counterclockwise as seen from above and let $\vec{F}$ be the vector field x i + y **j** + xyz **k**. Evaluate $\int_C \vec{F} \cdot d\vec{r}$

a) directly as a line integral, and
b) as a surface integral, by using Stokes' Theorem.

10. Let *T* be the solid $x^2 + y^2 \leq 1$; $0 \leq z \leq 1$. Let S be the surface (including the top, bottom, and side) of *T* and let $\vec{n}$ be the outward pointing unit normal vector. Let $\vec{w}$ be the vector field y^2 **i** + x^2 **j** + $\sin^2(\pi z)$ **k** . Evaluate $\iint_S \vec{w} \cdot d\boldsymbol{S}$

a) directly as a surface integral, and
b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Make sure you answered 6 complete questions from Part I and 2 complete questions from Part II.