THE CITY COLLEGE **DEPARTMENT OF MATHEMATICS**
Mathematics 39200 **Final Examination** **December 2006**

INSTRUCTIONS: Answer five questions from Part I and two questions from Part II. Show all work. Calculators are not permitted.

PART I. Answer five complete questions from this part. (14 points each)

1. Let A be the matrix $\begin{pmatrix} 2 & 0 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{pmatrix}$.

a) Find A^{-1}. Use the method of your choice.

b) Use your answer in part a) to solve $\begin{cases} 2x & & +2z & =2 \\ 2x & +y & +z & =5 \\ 3x & +2y & +2z & =8 \end{cases}$

c) Solve the system in b) for *x* (not *y* or *z*) by using Cramer's Rule.

2. Solve the simultaneous differential equations $\begin{cases} y'_1 = 3y_1(t) + y_2(t) \\ y'_2 = y_1(t) + 3y_2(t) \end{cases}$ for $y_1(t)$ and $y_2(t)$ subject to initial conditions $y_1(0) = 1$ and $y_2(0) = 2$. First find eigenvalues and eigenvectors of an appropriate matrix. No credit for any other method!

3. Use Gaussian Elimination to solve the following linear systems:

a) $\begin{cases} w + x + y + z = 1 \\ 2w \quad + 2y \quad = 4 \\ \qquad y + z = 6 \end{cases}$ **b)** $\begin{cases} w + x + y + z = 1 \\ 2w \quad + 2y \quad = 4 \\ 3w + x + 3y + z = 6 \end{cases}$ c) $\begin{cases} 2x & & +2z & =2 \\ 2x & +y & +z & =5 \\ 3x & +2y & +2z & =8 \end{cases}$

4. Let $\vec{F}(x,y,z) = < ye^z + y,\ xe^z + x + 1,\ xye^z + 1 >$

a) Find a potential function $f(x,y,z)$ with $\nabla f = \vec{F}$

b) Let C be the straight line segment joining (1,1,1) to (2,2,0).
Use the result of a) to evaluate $\int_C \vec{F} \cdot d\vec{r}$

5. Let T be the solid ball $x^2 + y^2 + z^2 \le 4$. Let T_1 be the part of T that lies above the cone $z = \sqrt{x^2 + y^2}$. Let T_2 be the part of T with $x \le 0$ and $z \le 0$. Use spherical co-ordinates (ρ, ϕ, θ) to set up bounds of integration for **a)** $\iiint_{T_1} (x^2 + y^2 + z^2)\, dV$ and **b)** $\iiint_{T_2} (x^2 + y^2 + z^2)\, dV$

Then evaluate *one* of the definite integrals **a)** or **b).**

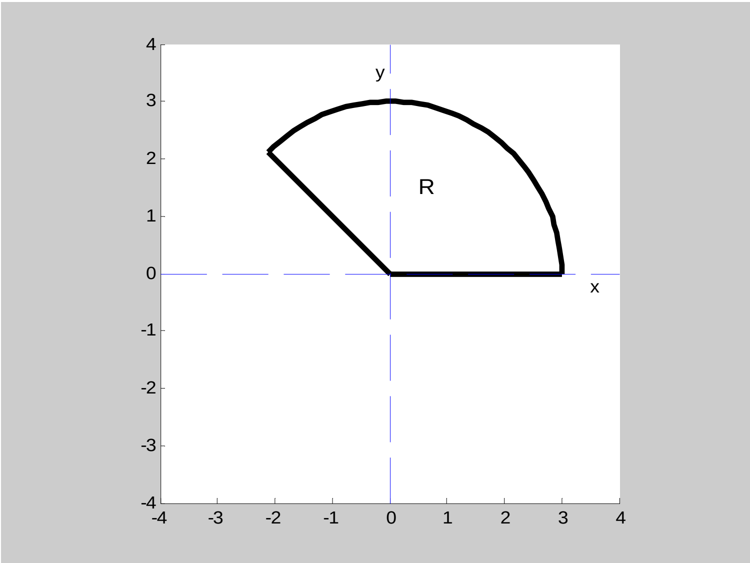
Please turn the page for the rest of Part I and for Part II.

6. **a)** Find the surface area of the part of the surface S: $z = x^2 + y^2$ with $1 \le z \le 4$.
b) Find an equation of the tangent plane to S at the point (1,1,2).

End of Part I. Make sure you answered five complete questions from this part.

PART II: Answer 2 complete questions from this part (15 points each).

7. Let R be the region shown below bounded by the line $y = -x$, the circle $x^2 + y^2 = 9$, and the line $y = 0$.



Suppose the boundary C of R is oriented counterclockwise.

Evaluate $\int_C -ydx + xdy$

a) directly, as a line integral, *and*
b) as a double integral, by using Green's Theorem.

8. Let S be that part of the surface $z = 1 - x^2$ in the first octant with $0 \le y \le 2$. Let C be the boundary of S, oriented counterclockwise when viewed from above.

If $\vec{F} = <1, 0, y^2>$, Calculate $\int_C \vec{F} \cdot d\vec{r}$

a) directly as a line integral, *and*
b) as a surface integral, by using Stokes' Theorem.

9. Let T be the solid bounded below by $z = x^2 + y^2$ and above by $z = 4$, and let S be the boundary surface of T, with outward pointing unit normal vector.
Let $\vec{F}$ be the vector field $< x, y, 1 >$.
Calculate $\iint_S \vec{F} \cdot d\vec{S}$

a) directly as a surface integral, *and*
b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Make sure that you answered five complete questions from Part I and two complete questions from Part II.