MATH 392 Final Exam, May 16, 2008

Answer all questions 1–7 and 3 **of the** 4 **questions 8–11**.

Question 1 (a) Find the inverse of the following 3 x 3 matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ -2 & 3 & -1 \end{pmatrix}$$

(b) Use the inverse matrix found in part (a) to solve the system of equations:

$$\begin{aligned} x + y + 4z &= 4 \\ y + z &= 8 \\ -2x + 3y - z &= -6 \end{aligned}$$

Question 2 (a) Calculate the eigenvalues and corresponding eigenvectors of

$$\mathbf{A} = \begin{pmatrix} 6 & 3 \\ -1 & 2 \end{pmatrix}$$

(b) Give the general solution to the system of the ordinary equations:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Question 3 Evaluate the line integral

$$\int_C 6x^2\, ds,$$

where C is the part in the first quadrant of the circle of radius 4 in the xy plane centered at the origin.

Question 4 Let

$\vec{F} = (y^2e^{xy}+6xy^2z)\vec{i}+(e^{xy}+xye^{xy}+12z+6x^2yz)\vec{j}+(12y+3x^2y^2)\vec{k}$

(a) Show that $\vec{F}$ is conservative; that is, find a scalar function f such that $\vec{F} = \nabla f$.

(b) Find the work done by $\vec{F}$ along the path from $(0, 1, 0)$ to $(3, 4, -1)$ to $(1, -1, 3)$ along two straight segments.

Question 5 (a) Calculate the determinant of 4×4 matrix $\mathbf{A}$:

$$\begin{pmatrix} 0 & -2 & -2 & 4 \\ 1 & 4 & 6 & -3 \\ 1 & 6 & -4 & 5 \\ 2 & 12 & 1 & 6 \end{pmatrix}.$$

(b) Calculate the determinant of the matrix $\mathbf{B} = -3\mathbf{A}^3$.

Question 6 Find the general solution to the following linear system:

$$\begin{array}{rrrrrrl} 3u & -6v & +2w & +4x & -y & & = 2 \\ u & -2v & +w & +x & & & = 1 \\ u & -2v & & +2x & +y & +z & = 6 \\ u & -2v & & +2x & & & = 3 \end{array}$$

Express the general solution as a linear combination of column vectors.

Question 7 Calculate the surface integral to find the flux

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS,$$

where S is the part of the sphere $x^2+y^2+z^2 = 9$ with $z \leq 0, y \geq 0$ and

$\vec{F} = x\vec{i}+y\vec{j}+(z+2)\vec{k}$, and $\vec{n}$ is the unit normal vector field along S directed away from the origin.

Question 8 Evaluate the surface integral

$$\int\int_S 4yz\, dS$$

where S is part of an elliptic paraboloid parameterized as $\vec{r}(u,v) =< u^2, u\sin(v), u\cos(v) >$, with $0 \le u \le 2$ and $0 \le v \le \pi$.

Question 9 Find the work done by the vector field

$$\vec{F} = (e^x + x^2 y)\vec{i} + (e^y - xy^2)\vec{j}$$

around the circle of radius 3 centered at the origin travelled clockwise.

Question 10 Let C be the intersection curve of the surfaces $z = 3x - 7$ and $x^2 + y^2 = 1$, oriented clockwise as seen from above. Let $\vec{F} = (4z-1)\vec{i} + 2x\vec{j} + (5y+1)\vec{k}$. Compute the work integral $\int_c \vec{F} \cdot d\vec{r} = \int_c \vec{F} \cdot \vec{T}\, ds$ two ways:
(a) directly as a line integral
(b) as a double integral, using Stokes' Theorem.

Question 11 Let T be the part of the surface $z = 9 - x^2 - y^2$ which lies above the xy plane, with upward unit normal. Let B be the disk of radius 3 in the xy plane centered at the origin with downward unit normal. Find the total combined flux of the vector field $\vec{F} = (2x + 9y - 2yz)\vec{i} + (3x + y - 5z)\vec{j} + (x^2 + y^2 + 2z^2)\vec{k}$ across T and B in the directions of their given normals.