Department of Mathematics **The City College of New York**
Math 39200 **Final Examination** **Fall 2005**

Instructions: Show all work. Calculators are not permitted.
Answer 5 questions from part I and 2 questions from Part II.

PART I. Answer 5 complete questions from this part. (14 points each)

1. (a) Find the inverse of the matrix A = $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$.

(b) Use the matrix A^{-1} that you found in (a) to solve the system $\begin{cases} x - y = 1 \\ -x + 2y - z = 2 \\ -y + 2z = -2 \end{cases}$

No credit for any other method!

2. Solve the following simultaneous differential equations for $y_1(t)$ and $y_2(t)$ subject to initial conditions $y_1(0) = 1$ and $y_2(0) = 3$. First find the eigenvalues and eigenvectors of an appropriate matrix. No credit for any other method!

$$\begin{cases} y_1'(t) = y_1(t) - 2y_2(t) \\ y_2'(t) = -2y_1(t) + y_2(t) \end{cases}$$

3. Let B be the matrix $\begin{pmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix}$. Use the method of your choice to:

(a) find the determinant of B *and*
(b) find the determinant of the matrix $2B^3$.

4. (a) Solve the following system of equations: $\begin{cases} x + y + z = 0 \\ -4x + 2y - z = 3 \\ -5x + y - 2z = 3 \end{cases}$

(b) Can Cramer's Rule be used to solve the system in (a)? Explain your answer.
(c) Suppose the coefficient matrix of a system of linear equations is a 4×5 matrix with rank 4. Is it possible that the system is inconsistent? Explain your answer.

5. (a) Find the area of the part of the surface $z = xy - 1$ that lies inside the cylinder $x^2 + y^2 = 4$.

(b) Find a function $f(x, y, z)$ with gradient $\nabla f = (2xy + 2xz)\,\mathbf{i} + (x^2 + z)\,\mathbf{j} + (x^2 + y)\,\mathbf{k}$.

Please turn the page for the rest of Part I and for the rest of the exam.

Part I, continued

6. **(a)** Let S be the surface $z = \sqrt{x^2 + y^2}$, and let P be the point (3,4,5) on S.

i) Find a normal vector to the surface S at point P. Then
ii) Find an equation of the tangent plane to the surface S at point P.

(b) Let C be the curve parametrized by $x(t) = 3t + 1, \quad y(t) = e^t, \quad z(t) = 1$.
Find parametric equations of the tangent line to the curve C at the point (1,1,1).

7. **(a)** Find the length of the parametrized curve given by position vector
$\mathbf{r}(t) = \sqrt{2}\ \mathbf{i} + \cos^2 t\,\mathbf{j} + \sin^2 t\,\mathbf{k}$ with $0 \le t \le \pi/2$.

(b) If T is the solid contained above $z = x^2 + y^2$ and below $z = 2$, find $\iiint_T (x^2 + y^2)\,dV$

End of Part I. Make sure you answered 5 complete questions from this part.

PART II: Answer 2 complete questions from this part (15 points each).

8. Let C be the boundary curve of the triangle with vertices P$(-1,0)$, $Q(0,1)$, and R(1,1).
Let C be oriented counter-clockwise. Draw triangle PQR and find $\int_C y^2 dx - x^2 dy$

(a) directly, as a line integral **AND**
(b) as a double integral, by using Green's Theorem.

9. Let C be the curve of intersection of the cone $x^2 + y^2 = z^2$ and the plane $z = 3$. Let $\mathbf{F}(x,y,z)$ be the vector field $y\mathbf{i} + z\mathbf{j} - x\,\mathbf{k}$. Let C be oriented clockwise as seen from above.
Calculate $\int_C \mathbf{F} \cdot d\boldsymbol{r}$

(a) directly as a line integral **AND**
(b) as a double integral, by using Stokes' Theorem.

10. Let T be the solid described by $\begin{cases} x^2 + y^2 + z^2 \le 4 \\ z \ge 1 \end{cases}$. Let S be the (two-part) boundary surface of T. Let $\mathbf{F}(x,y,z)$ be the vector field $x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.
Use the outward pointing unit normal vector to calculate $\iint_S \mathbf{F} \cdot d\boldsymbol{S}$

(a) directly as a surface integral **AND**
(b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Make sure that you answered 5 complete questions from Part I and 2 complete questions from Part II. On the cover of your answer booklet write the numbers of the 7 problems you want graded.