PART I. Answer 5 complete questions from this part. (14 points each)

1. (a) Find the inverse of the matrix 
\[ A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \]

(b) Use the matrix \( A^{-1} \) that you found in (a) to solve the system:
\[
\begin{align*}
    x - y &= 1 \\
    -x + 2y - z &= 2 \\
    -y + 2z &= -2
\end{align*}
\]

No credit for any other method!

2. Solve the following simultaneous differential equations for \( y_1(t) \) and \( y_2(t) \) subject to initial conditions \( y_1(0) = 1 \) and \( y_2(0) = 3 \). First find the eigenvalues and eigenvectors of an appropriate matrix. No credit for any other method!

\[
\begin{align*}
    y_1'(t) &= y_1(t) - 2y_2(t) \\
    y_2'(t) &= -2y_1(t) + y_2(t)
\end{align*}
\]

3. Let \( B \) be the matrix 
\[
\begin{pmatrix}
    0 & 2 & 1 & 1 \\
    1 & 0 & -1 & 0 \\
    0 & 1 & 0 & 1 \\
    2 & 3 & 0 & 1
\end{pmatrix}
\]
Use the method of your choice to:

(a) find the determinant of \( B \)

(b) find the determinant of the matrix \( 2B^3 \).

4. (a) Solve the following system of equations:
\[
\begin{align*}
    x + y + z &= 0 \\
    -4x + 2y - z &= 3 \\
    -5x + y - 2z &= 3
\end{align*}
\]

(b) Can Cramer’s Rule be used to solve the system in (a)? Explain your answer.

(c) Suppose the coefficient matrix of a system of linear equations is a \( 4 \times 5 \) matrix with rank 4. Is it possible that the system is inconsistent? Explain your answer.

5. (a) Find the area of the part of the surface \( z = xy - 1 \) that lies inside the cylinder \( x^2 + y^2 = 4 \).

(b) Find a function \( f(x, y, z) \) with gradient \( \nabla f = (2xy + 2xz) \mathbf{i} + (x^2 + z) \mathbf{j} + (x^2 + y) \mathbf{k} \).

Please turn the page for the rest of Part I and for the rest of the exam.
Part I, continued

6. (a) Let S be the surface \( z = \sqrt{x^2 + y^2} \), and let P be the point (3,4,5) on S.
   i) Find a normal vector to the surface S at point P. Then
   ii) Find an equation of the tangent plane to the surface S at point P.

(b) Let \( C \) be the curve parametrized by \( x(t) = 3t + 1, \quad y(t) = e^t, \quad z(t) = 1 \).
    Find parametric equations of the tangent line to the curve \( C \) at the point (1,1,1).

7. (a) Find the length of the parametrized curve given by position vector
    \( r(t) = \sqrt{2} \mathbf{i} + \cos^2 t \mathbf{j} + \sin^2 t \mathbf{k} \) with \( 0 \leq t \leq \pi / 2 \).
    (b) If \( T \) is the solid contained above \( z = x^2 + y^2 \) and below \( z = 2 \), find \( \iiint_T (x^2 + y^2) dV \)

End of Part I. Make sure you answered 5 complete questions from this part.

PART II: Answer 2 complete questions from this part (15 points each).

8. Let \( C \) be the boundary curve of the triangle with vertices \( P(-1,0), Q(0,1), \) and \( R(1,1) \).
    Let \( C \) be oriented counter-clockwise. Draw triangle PQR and find \( \int_C y^2 dx - x^2 dy \)
    (a) directly as a line integral AND
    (b) as a double integral, by using Green's Theorem.

9. Let \( C \) be the curve of intersection of the cone \( x^2 + y^2 = z^2 \) and the plane \( z = 3 \).
    Let \( F(x,y,z) = y \mathbf{i} + z \mathbf{j} - x \mathbf{k} \). Let \( C \) be oriented clockwise as seen from above.
    Calculate \( \int_C F \cdot dr \)
    (a) directly as a line integral AND
    (b) as a double integral, by using Stokes' Theorem.

10. Let \( T \) be the solid described by \( \begin{cases} x^2 + y^2 + z^2 \leq 4 \quad \text{and} \quad z \geq 1 \end{cases} \).
    Let \( S \) be the (two-part) boundary surface of \( T \). Let \( F(x,y,z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \).
     Use the outward pointing unit normal vector to calculate \( \iint_S F \cdot dS \)
    (a) directly as a surface integral AND
    (b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Make sure that you answered 5 complete questions from Part I and 2 complete questions from Part II. On the cover of your answer booklet write the numbers of the 7 problems you want graded.