Part I. Answer ALL questions. Total 64 points.

1. [13 Points] Solve the initial value problem:

\[ y'' - 4y' + 4y = x^2 + 12e^{2x}, \quad y(0) = 1, \ y'(0) = 0. \]

2. [8 Points] Solve

\[ \left( y \cos(xy) + \frac{y}{2x} \right) dx + \left( x \cos(xy) + \frac{1}{2} \ln(x) + \frac{1}{e^y} \right) dy = 0. \]

3. [9 Points] Find the general solution to

\[ y'' - 2y' + y = \frac{e^x}{x}. \]

4. [7 Points] Solve \( xy' - 2y = xy + xe^x \).

5. [13 Points] For the equation \( 2xy'' - y' + y = 0 \),

(a) Show \( x = 0 \) is a regular singular point.

(b) Find the indicial equation and the recurrence relation corresponding to the larger root.

(c) Find the first four terms of the series solution valid near \( x > 0 \) corresponding to the larger root.

6. [4 Points] Use separation of variables to replace the partial differential equation:

\[ xtu_{xx} + u_{xt} + tu_x = 0, \]

where \( u \) is a function of \( x \) and \( t \), by two ordinary differential equations.

7. [10 Points] Use the Laplace Transform method to solve:

\[ y'' + 4y = 2, \quad y(0) = 1, \ y'(0) = 3. \]

Note that:

\[ \mathcal{L}\{e^{at}\} = \frac{1}{s - a}; \quad \mathcal{L}\{\sin at\} = \frac{a}{s^2 + a^2}; \quad \mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2}. \]

Part II begins on the back.
Part II. Answer any THREE (3) COMPLETE questions. Total: 36 points.

8. [12 Points] Find the Fourier series for

\[ f(x) = \begin{cases} 
  x + 2 & \text{if } -2 < x \leq 0; \\
  2 - x & \text{if } 0 < x \leq 2,
\end{cases} \]

where \( f(x + 4) = f(x) \) for all \( x \).

9. [12 Points] Find the terms of the power series solution through \( x^5 \) of

\[ y'' - y' + xy = 0, \quad y(0) = 1, \quad y'(0) = 2. \]

10. (a) [4 Points] Solve \( 2x^2y'' + xy' - y = 0 \).

(b) [8 Points] A mass weighing two pounds stretches a spring 6 inches. The mass is pulled down 3 inches and given an upward velocity of 1 ft/sec. Find \( u(t) \), the displacement of the mass in feet from its equilibrium position at time \( t \) seconds after release. Assume that the acceleration due to gravity is 32 ft/sec\(^2\) and that air resistance is negligible.

11. A 200 gallon tank is half full of pure water. A salt solution with a concentration 5 lb/gal is flowing into the tank at the rate of 4 gal/min while the well-mixed solution is flowing out at the rate of 2 gal/min.

(a) [9 Points] Find \( Q(t) \), the amount of salt in lbs in the tank at time \( t \) minutes.

(b) [3 Points] Find the concentration of salt in the tank when the tank overflows.