THE CITY COLLEGE DEPARTMENT OF MATHEMATICS
MATHEMATICS 34600 FINAL EXAMINATION SPRING 2005

PART I: ANSWER ALL THREE QUESTIONS (40%)

1. (15) Let $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$.

a. Find the eigenvalues and bases for the eigenspaces of A.
b. Find a diagonal matrix D and an invertible matrix P such that $P^{-1}AP = D$.
c. Use the result of part b. to compute A^5.

2. (10) Let T: $\mathbf{V} \rightarrow \mathbf{W}$ be a linear transformation.
a. Define the kernel of T and prove that it is subspace of **V**.
b. Define the range of T and prove that it is subspace of **W**.

3. (15) Suppose L: $\mathbf{P}_2 \rightarrow \mathbf{P}_2$ is given by L (p(t)) = 2p(t) – 3p'(t). Given the basis $\mathbf{B} = \{1, 3t + 1, 2t^2\}$ for $\mathbf{P}_2$,
a. Find the **B**-coordinates of $p(t) = 5 - 6t + 4t^2$.
b. Find $[L]_B$, the **B**-matrix for L.
c. Using the results of parts a. and b., compute $[L(5 - 6t + 4t^2)]_B$.

PART II: ANSWER FIVE OUT OF SEVEN QUESTIONS (OMIT TWO) (60%)

4. (12) Suppose that $\mathbf{S} = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a linear independent set and that $\mathbf{v} \notin \text{Span}(\mathbf{S})$. Prove that $\{\mathbf{v}, \mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set.

5. (12) Let $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$.

a. Find a basis for the nullspace of A.
b. Find a basis for the column space of A.
c. What is the rank of A?

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6. (12) a. Find A^{-1} if $A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$.

b. Solve $A\mathbf{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ using your answer from part a.

7. (12) Let B be the reduced row echelon form of an $m \times n$ matrix A. For each of the following statements, if it is true, explain why. If it is false, give a counterexample.
a. Row space of A = Row space of B
b. Column space of A = Column space of B

8. (12) Use Cramer's Rule (no credit for any other method) to solve the system

$$\begin{aligned} 4x_1 + 5x_2 &= 2 \\ 11x_1 + x_2 + 2x_3 &= 3 \\ x_1 + 5x_2 + 2x_3 &= 1 \end{aligned}$$

9.(12) Suppose T: $\mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation. Explain your answers to the following questions:
a. Under what conditions on m and n can one be sure that T is not one-to-one?
b. Under what conditions on m and n can one be sure that T is not onto?
c. If T is one-to-one and onto, what can one conclude about m and n?

10.(12) Let $A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$.
a. Find AB and BA.
b. Show that {A, B, AB, BA} is a basis for $\mathbf{M}_{2x2}$, the vector space of all 2×2 real matrices.