1. Invert the following matrices, if possible.

(a) \( A = \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix} \) (5 points)

(b) \( A = \begin{pmatrix} 4 & -8 \\ 1 & -2 \end{pmatrix} \) (5 points)

2. Let \( A = \begin{pmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{pmatrix} \).

(a) Compute \( A^{-1} \), if possible. (5 points)

(b) Find the rank and nullity of \( A \). (5 points)

(c) Find all solutions of \( Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \). (5 points)

3. Let \( A = \begin{pmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{pmatrix} \).

(a) Find all solutions of \( Ax = \begin{pmatrix} -13 \\ -28 \\ 43 \end{pmatrix} \). (5 points)

(b) Find a basis of \( N(A) \). (5 points)

(c) Find the rank and nullity of \( A \). (5 points)

4. Let \( A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix} \).
(a) Compute $\det A$. 

(b) Is $A$ invertible? Is $A$ surjective? Why or why not?

5. Let 

$$A = \begin{pmatrix} 8 & -10 \\ 3 & -3 \end{pmatrix}.$$ 

(a) Find the eigenvalues of $A$. 

(b) Find the corresponding eigenvectors of $A$. 

(c) Find a diagonal matrix $D$ as well as an invertible matrix $P$ such that $P^{-1}AP = D$. 

6. Are the following sets subspaces of $P_2$? Why or why not?

(a) $U = \{p \in P_2 \mid p(3) = 0\}$ 

(b) $W = \{a_0 + a_1x + a_2x^2 \mid a_0 + 2a_2 = 1\}$ 

7. Let $p_1 = 1 - 3x + 2x^2$, $p_2 = 1 + x + 4x^2$, and $p_3 = 3 - x + 10x^2$. Find a basis of the subspace of $P_2$ spanned by $p_1, p_2, \text{ and } p_3$. 
