

Math 34600 Final Exam, Fall 2006

1. Invert the following matrices, if possible.

(a) (5 points)

$$A = \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$$

(b) (5 points)

$$A = \begin{pmatrix} 4 & -8 \\ 1 & -2 \end{pmatrix}$$

2. Let

$$A = \begin{pmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{pmatrix}.$$

(a) Compute A^{-1} , if possible. (5 points)

(b) Find the rank and nullity of A . (5 points)

(c) Find all solutions of $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. (5 points)

3. Let

$$A = \begin{pmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{pmatrix}.$$

(a) Find all solutions of $Ax = \begin{pmatrix} -13 \\ -28 \\ 43 \end{pmatrix}$. (5 points)

(b) Find a basis of $N(A)$. (5 points)

(c) Find the rank and nullity of A . (5 points)

4. Let

$$A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}.$$

- (a) Compute $\det A$. (5 points)
- (b) Is A invertible? Is A surjective? Why or why not? (5 points)

5. Let

$$A = \begin{pmatrix} 8 & -10 \\ 3 & -3 \end{pmatrix}.$$

- (a) Find the eigenvalues of A . (5 points)
- (b) Find the corresponding eigenvectors of A . (5 points)
- (c) Find a diagonal matrix D as well as an invertible matrix P such that $P^{-1}AP = D$. (5 points)

6. Are the following sets subspaces of P_2 ? Why or why not?

- (a) $U = \{p \in P_2 \mid p(3) = 0\}$ (5 points)
- (b) $W = \{a_0 + a_1x + a_2x^2 \mid a_0 + 2a_2 = 1\}$ (5 points)

7. Let $p_1 = 1 - 3x + 2x^2$, $p_2 = 1 + x + 4x^2$, and $p_3 = 3 - x + 10x^2$. Find a basis of the subspace of P_2 spanned by p_1, p_2 , and p_3 . (5 points)