Math 34600 Final Exam Fall 2005

Instructions: Answer any seven questions. Omit one.

1a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$.

b) Using A^{-1} solve the system of equations: $\begin{aligned} x - y &= 1 \\ -x + 2y - z &= 2 \\ -y + 2z &= -2 \end{aligned}$.

2a) Let $B = \begin{pmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 3 & 0 & 3 \end{pmatrix}$. Find $\det(B)$.

b) Find $\det(B^2)$. c) Find $\det(3B)$.

3a) Find all solutions of the system $\begin{aligned} x + y + z &= 0 \\ -4x + 2y - z &= 3 \\ -5x + y - 2z &= 3 \end{aligned}$.

b) Explain why you cannot use Cramer's Rule to answer problem 3a).

4) Find all eigenvalues and all associated eigenvectors for the matrix $A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$.

5) Let $\mathbb{P}_2 = \{\text{quadratic polynomials}\}$. Which of the following subsets is a subspace of $\mathbb{P}_2$?

a) {quadratic polynomials $p(x)$ with $p(0) = 1$}.

b) {quadratic polynomials $p(x)$ with $p(1) = 0$}.

c) For each of the subsets that are subspaces, find a basis for the subspace.

6) Prove or disprove the following statements about $n \times n$ matrices A and B.

a) If A and B are invertible, so is AB.

b) If A and B are invertible, so is $A + B$.

c) If λ is an eigenvalue for A, then λ^2 is an eigenvalue for A^2.

7) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be described by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.

a) Find a basis for $\ker T$.

b) Is $T(\mathbb{R}^3) = \mathbb{R}^2$?

c) Find a basis for the column space of A.

8) Suppose L is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ for which $L\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}5\\0\\0\end{pmatrix}$,

$L\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}6\\6\\0\end{pmatrix}$, and $L\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}7\\7\\7\end{pmatrix}$.

a) Find $L\begin{pmatrix}2\\3\\4\end{pmatrix}$

b) Find the matrix of L with respect to the basis vectors $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$