Series Convergence Tests

Convergence Test:	When it can be used:	Conclusions:
Geometric Series [redacted]	$\sum_{k=0}^{\infty} ar^k$ or $\sum_{n=1}^{\infty} ar^{n-1}$	For $\lvert r\rvert < 1$, converges to $\frac{a}{1-r}$ For $\lvert r\rvert \geq 1$, diverges
A Test for Divergence	All series	If $\lim_{k\to\infty} a_k \neq 0$, the series diverges.
The Integral Test	$\sum_{n=1}^{\infty} a_n$ where $a_n = f(n)$ for f continuous and decreasing, and $f(x) \geq 0$	$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both converge or both diverge
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges for $p > 1$, diverges for $p \leq 1$.
The Comparison Test	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ where $0 \leq a_n \leq b_n$	If $\sum_{n=1}^{\infty} b_n$ converges, $\sum_{n=1}^{\infty} a_n$ converges. If $\sum_{n=1}^{\infty} a_n$ diverges, $\sum_{n=1}^{\infty} b_n$ diverges
The Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ where $a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge
The Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ where $a_n > 0$ for all n	If $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for all n, then the series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.
Absolute Convergence	Any series with some positive and some negative terms	If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely. (This implies convergence.)
The Ratio Test	Any series (especially those with exponentials or factorials)	For $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$, if $L < 1$, $\sum_{n=1}^{\infty} a_n$ converges absolutely, if $L > 1$ or the limit is ∞, $\sum_{n=1}^{\infty} a_n$ diverges, if $L = 1$, no conclusion.
The Root Test	Any series (especially those with exponentials)	For $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$, if $L < 1$, $\sum_{n=1}^{\infty} a_n$ converges absolutely, if $L > 1$ or the limit is ∞, $\sum_{n=1}^{\infty} a_n$ diverges, if $L = 1$, no conclusion.