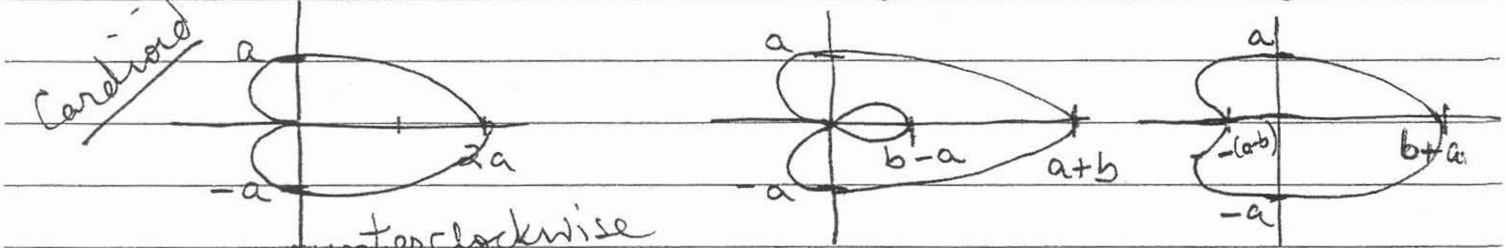
Polar Graphs

1. **Limaçons:** $r = a + b\cos\theta$; $a, b > 0$

$a = b$ $\qquad$ $a < b$ $\qquad$ $a > b$



$r = a - b\cos\theta$, rotate counterclockwise by π; $r = a \pm b\sin\theta$, by $\frac{\pi}{2}$, $\frac{3\pi}{2}$ resp.

2. **Lemniscates:** $r^2 = a\cos 2\theta$; $a > 0$

$r^2 \geq 0 \Rightarrow$
2 quadrants left-out

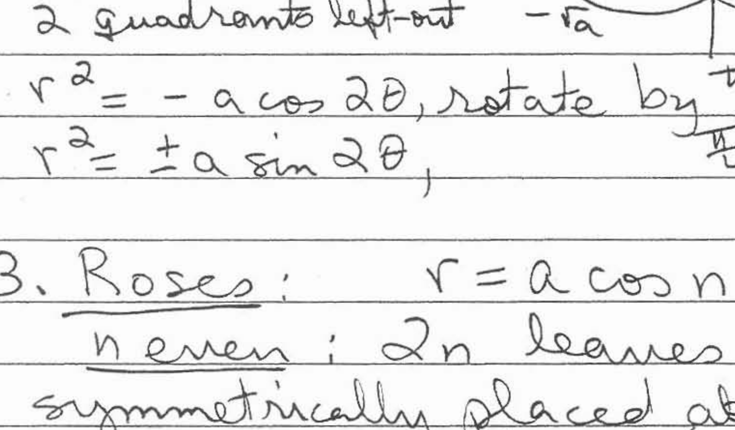
$r^2 = -a\cos 2\theta$, rotate by $\pi/2$ counterclockwise
$r^2 = \pm a\sin 2\theta$, $\qquad$ $\frac{\pi}{4}$, $3\pi/4$, respectively

3. **Roses:** $r = a\cos n\theta$, $r = a\sin n\theta$

n even: $2n$ leaves; **n odd**: n leaves
symmetrically placed about origin, of length a,
centered on lines $(2\pi/\#\text{leaves})$ apart

$a\cos n\theta$: one leaf is centered on x-axis
$a\sin n\theta$: $\qquad \theta = \pi/2n$

ex: $n = 2$ $\qquad$ $n = 3$

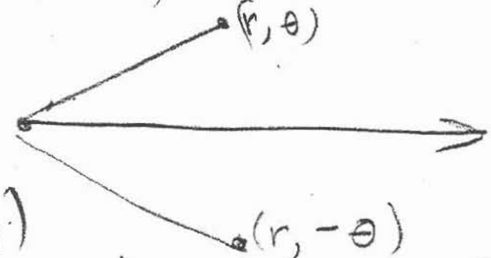


$n = 4$: 8 leaves,
stuck ½ way between
those in $n = 2$

etc.

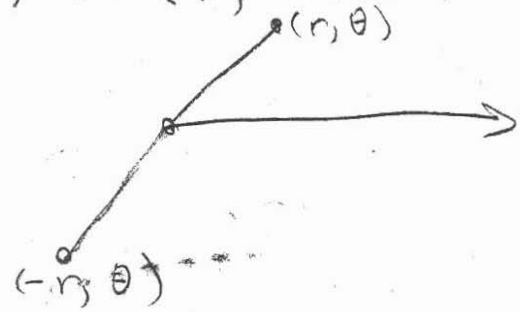
Symmetry: NOTE: Sufficient, not necessary cond are:

1. sym. about polar (x) axis if replacing (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$ doesn't change anything



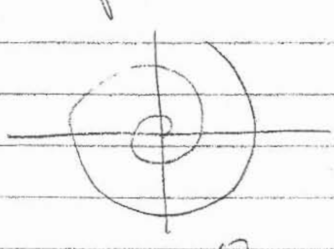
2. sym. about $\theta = \pi/2$ (y-axis) if replacing (r, θ) by $(-r, -\theta)$ or $(r, \pi - \theta)$ doesn't change anything

3. sym. about origin (pole) if replacing (r, θ) by $(-r, \theta)$ or $(r, \pi + \theta)$ doesn't change anything



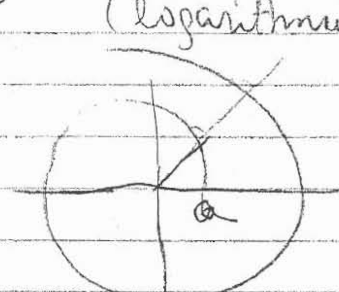
Types of Graphs:

4. Spirals $r = a\theta$ (Archimedes)
$r = ae^{b\theta}$ (logarithmic $\Leftrightarrow \log a + b\theta$)



$r = a\theta$

(calc. where crosses x-axis, i.e. $a\theta = 0, \pi, 2\pi, 3\pi$, etc.)



$r = ae^{b\theta}$ $(a = 1)$

Ex: $y = 2$ (horiz. st. line)

$2 = r\sin\theta$, $r = \dfrac{2}{\sin\theta}$

In gen: $\boxed{\text{line}}$ $r = \dfrac{d}{\cos(\theta-\theta_0)}$ where d = dist. of line to pole, $\theta_0 = \angle$(polar axis, ray $\perp$ to line)

—or— $\theta = \theta_0$

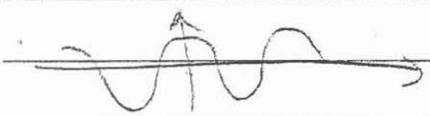
—OR— $\theta = \text{const.} = \theta_0$ if goes thru pole

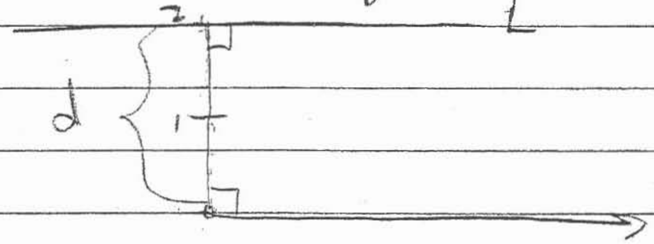
In ex. above, $d = 2$ clearly

& $\theta_0 = \pi/2$;

recall

$\sin\theta = \cos(\theta - \pi/2)$

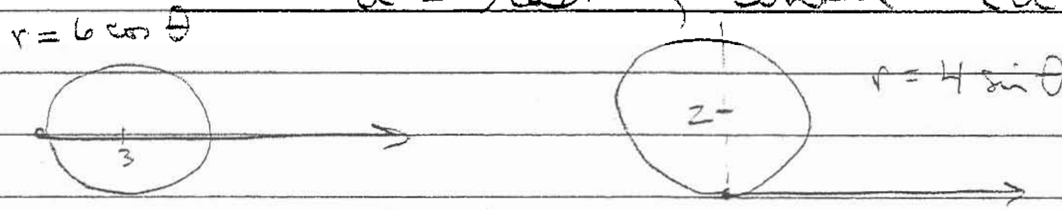




$\boxed{\text{circle}}$ $r = 2a\cos(\theta-\theta_0)$ —or— $r = \text{const} = a$ if centered at pole

where

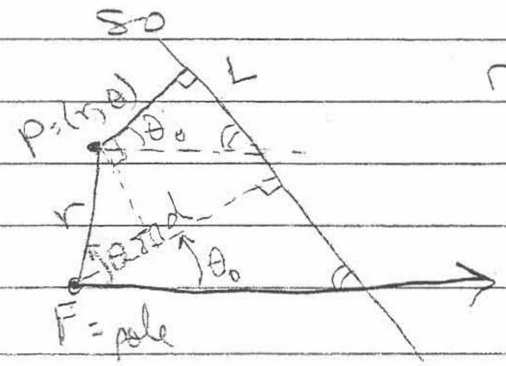
a = rad. , center $= (a, \theta_0)$



$\boxed{\text{conics}}$ put pole at focus F, d = dist (F, directrix L)

$P = (r, \theta)$ pt. on curve, $|PF| = e\,|PL|$

so



$$r = e\left(d - r\cos(\theta-\theta_0)\right)$$

$$r = \frac{ed}{1 + e\cos(\theta-\theta_0)}$$