NAME (printed) ______________________________

NAME (signed) ______________________________

MATH 203 Final Exam

May 21, 2018

Circle your section (for example, XX, Instructor, Days, Hours):

BB, Camacho, M, W 9–10:40
DD, Shell, W 12–1:40, F 1–2:40
EE, Diotte, M, W 2–3:40
LL, Paolillo, T, Th, 9–10:40
LM, Islam, T, Th, 10–11:40
MM, Gallobit, T, F, 11–12:40
PP, Hernandez, T, Th, 2–3:40

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Instructions: Answer ALL 10 questions (10 points each). Show all work.

No calculators or other electric devices may be used. Answers are to be left in terms of $\sqrt{7}$, π, $\ln 3$, etc. when these can not be simplified. You have **2 hours and 15 minutes to complete the exam.**

Answer ALL questions (10 points each). Show all work.

1. Let Pl_1 and Pl_2 be the planes

$$Pl_1: \quad x + y + z = 4 \qquad\qquad Pl_2: \quad 3x + z = 0$$

(a) Are Pl_1 and Pl_2 parallel, perpendicular or neither?

(b) Find parametric equations for the line of intersection of Pl_1 and Pl_2.

2. Let $f(x,y) = \dfrac{1}{2x^2} + \sin(4y)$.

(a) Find ∇f.

(b) Find the rate at which the f is changing per unit change at the point $(1/2, \pi)$ in the direction towards $(3/2, 2\pi)$.

(c) If (r, θ) are the polar coordinates, $x = r\cos\theta$ and $y = r\sin\theta$, use the chain rule to find $\dfrac{\partial f}{\partial r}$.

3. Find all local maxima, local minima and saddle points of the graph of $f(x, y) = x^3 - 3x + 3xy^2$.

4. (a) Find the volume of the region bounded by $z = (x^2 + y^2)^2$ and $z = 16$.

(b) Use differentials (linear approximation) to approximate $\dfrac{1.9^2}{e^{.1}}$.

5. (a) Find the x-coordinate of the center of mass of the triangular lamina bounded by $y = x$, $y = 0$ and $x = 1$ and having density $\delta(x, y) = 2xy$.

(b) Find an equation of the tangent plane to the graph of $1 + z = x \ln(2y + z)$ at the point $(3, 1, -1)$.

6. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{2^n}$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{2n^2+1}$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{2n^3+1}$

7. (a) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)(x+2)^n}{(n+3)^2}$.

Remember to check the endpoints, if applicable.

(b) Find the limit or show it does not exist: $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^4}$

8. (a) Find a unit vector in the direction of the tangent vector at the point $(0, 1, 2)$ to the curve with vector representation $\mathbf{r}(t) = \langle \ln(t^2 + 1), t + \cos t, t^2 - t + 2 \rangle$.

(b) Graph the equation $9y^2 + z^2 - 9x^2 - 6z = 0$, labelling the coordinates of the z-intercepts.

9. Do part (a) **OR** (b). If you do both parts, only part (a) will be graded. Mark clearly, by crossing out any work you do want graded.

(a) Find the mass of the region in space bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 3$ and having density $\delta(x, y, z) = 2z$.

(b) Find the mass of the region above the xy-plane and inside both the cone $\phi = \pi/4$ and the sphere $\rho = 2$ which has density $\delta(x, y, z) = z$.

10. (a) Let $f(x) = x \sin x$.

(i) Find the first four nonzero terms of the Maclaurin series (i.e., the power series centered at 0) representation of $f(x)$.

(ii) Use the result in (i) to find $f(1/2)$ with an error less than or equal .01. Justify that your answer has the required accuracy.

(b) Find the surface area of the portion of the surface $z = 2x + y$ which is above the rectangle in the xy-plane $0 \leq x \leq 1$ and $0 \leq y \leq 2$.