

$$2) \quad r = \cos \theta \quad 0 \leq \theta \leq \frac{\pi}{6}$$

$$L = \int_a^b \frac{1}{2} r^2 d\theta \quad r^2 = (\cos \theta)^2 = \cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$A = \int_0^{\frac{\pi}{6}} \frac{1}{2} \left( \frac{1}{2}(1 + \cos(2\theta)) \right) d\theta = \frac{1}{4} \left[ \theta + \frac{1}{2} \sin(2\theta) + C \right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{4} \left\{ \left[ \left( \frac{\pi}{6} \right) + \frac{1}{2} \sin \left( 2 \left( \frac{\pi}{6} \right) \right) + C \right] - \left[ (0) + \frac{1}{2} \sin(2(0)) + C \right] \right\} = \frac{1}{4} \left\{ \left[ \frac{\pi}{6} + \frac{1}{2} \left( \frac{\sqrt{3}}{2} \right) \right] - [0] \right\} = \frac{\pi}{24} + \frac{\sqrt{3}}{16}$$

$$6) \quad r = 1 + \cos \theta \quad 0 \leq \theta \leq \pi$$

$$L = \int_a^b \frac{1}{2} r^2 d\theta \quad r^2 = (1 + \cos \theta)^2 = 1 + 2 \cos \theta + \cos^2 \theta = 1 + 2 \cos \theta + \frac{1}{2}(1 + \cos(2\theta))$$

$$= \frac{3}{2} + 2 \cos \theta + \frac{1}{2} \cos(2\theta)$$

$$A = \int_0^{\pi} \frac{1}{2} \left( \frac{3}{2} + 2 \cos \theta + \frac{1}{2} \cos(2\theta) \right) d\theta = \frac{1}{2} \left[ \frac{3}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin(2\theta) + C \right]_0^{\pi}$$

$$= \frac{1}{2} \left\{ \left[ \frac{3}{2} (\pi) + 2 \sin(\pi) + \frac{1}{4} \sin(2(\pi)) + C \right] - \left[ \frac{3}{2} (0) + 2 \sin(0) + \frac{1}{4} \sin(2(0)) + C \right] \right\}$$

$$= \frac{1}{2} \left\{ \left[ \frac{3\pi}{2} + 0 + 0 \right] - [0] \right\} = \frac{1}{2} \left\{ \frac{3\pi}{2} \right\} = \frac{3\pi}{4}$$

$$8) \quad r = \sin 2\theta \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$L = \int_a^b \frac{1}{2} r^2 d\theta \quad r^2 = (\sin(2\theta))^2 = \sin^2(2\theta) = \frac{1}{2}(1 - \cos(4\theta))$$

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2} \left( \frac{1}{2}(1 - \cos(4\theta)) \right) d\theta = \frac{1}{4} \left[ \theta - \frac{1}{4} \sin(4\theta) + C \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} \left\{ \left[ \left( \frac{\pi}{2} \right) - \frac{1}{4} \sin \left( 4 \left( \frac{\pi}{2} \right) \right) + C \right] - \left[ (0) - \frac{1}{4} \sin(4(0)) + C \right] \right\} = \frac{1}{4} \left\{ \left[ \frac{\pi}{2} - (0) \right] - [0] \right\} = \frac{\pi}{8}$$

$$16) \quad r^2 = \sin(2\theta)$$

a loop starts at  $r = 0$  and ends back at  $r = 0$ , so  $0 = \sin(2\theta) \Rightarrow$

$2\theta = 0$	$2\theta = \pi$	interval:
$\theta = 0$	$\theta = \frac{\pi}{2}$	$0 \leq \theta \leq \frac{\pi}{2}$

$$A = \int_0^{\frac{\pi}{2}} \frac{1}{2} (\sin(2\theta)) d\theta = \frac{1}{2} \left[ -\frac{1}{2} \cos(2\theta) + C \right]_0^{\frac{\pi}{2}} = \frac{1}{2} \left\{ \left[ -\frac{1}{2} \cos 2 \left( \frac{\pi}{2} \right) + C \right] - \left[ -\frac{1}{2} \cos(2(0)) + C \right] \right\}$$

$$= \frac{1}{2} \left\{ \left[ -\frac{1}{2} (-1) \right] - \left[ -\frac{1}{2} (1) \right] \right\} = \frac{1}{2} \left\{ \left[ \frac{1}{2} \right] - \left[ -\frac{1}{2} \right] \right\} = \frac{1}{2}$$

18)  $r = 2 \cos \theta - \sec \theta$

$$\begin{aligned} 0 &= 2 \cos \theta - \sec \theta & 1 &= 2 \cos^2 \theta & \sqrt{2} \cos \theta + 1 &= 0 & \sqrt{2} \cos \theta - 1 &= 0 \\ \sec \theta &= 2 \cos \theta & \Rightarrow 0 &= 2 \cos^2 \theta - 1 & \Rightarrow \cos \theta &= \frac{-1}{2} & \cos \theta &= \frac{1}{2} \\ \frac{1}{\cos \theta} &= 2 \cos \theta & 0 &= (\sqrt{2} \cos \theta + 1)(\sqrt{2} \cos \theta - 1) & \theta &= \frac{3\pi}{4} & \theta &= \frac{\pi}{4} \end{aligned}$$

But  $\sec\left(\frac{\pi}{2}\right)$  is undefined. Therefore, our interval needs to be modified to  $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

$$r^2 = (2 \cos \theta - \sec \theta)^2 = 4 \cos^2 \theta - 4 + \sec^2 \theta = 4 \left( \frac{1}{2} (1 + \cos(2\theta)) \right) - 4 + \sec^2 \theta = 2 \cos(2\theta) - 2 + \sec^2 \theta$$

$$\begin{aligned} A &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} (2 \cos(2\theta) - 2 + \sec^2 \theta) d\theta = \frac{1}{2} [\sin(2\theta) - 2\theta + \tan \theta + C]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\ &= \frac{1}{2} \left\{ \left[ \sin\left(2\left(\frac{\pi}{4}\right)\right) - 2\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right) + C \right] - \left[ \sin\left(2\left(-\frac{\pi}{4}\right)\right) - 2\left(-\frac{\pi}{4}\right) + \tan\left(-\frac{\pi}{4}\right) + C \right] \right\} \\ &= \frac{1}{2} \left\{ \left[ (1) - \frac{\pi}{2} + (1) \right] - \left[ (-1) + \frac{\pi}{2} + (-1) \right] \right\} = \frac{1}{2} \left\{ \left[ 2 - \frac{\pi}{2} \right] - \left[ -2 + \frac{\pi}{2} \right] \right\} = \frac{1}{2} \{ 4 - \pi \} = 2 - \frac{\pi}{2} \end{aligned}$$

20)  $r = 1 - \sin \theta$   $r = 1$

$$1 = 1 - \sin \theta \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0 \quad \theta = \pi$$

We need the interval to be inside 1<sup>st</sup> and outside 2<sup>nd</sup> curve.

Therefore, the interval must be  $\pi \leq \theta \leq 2\pi$

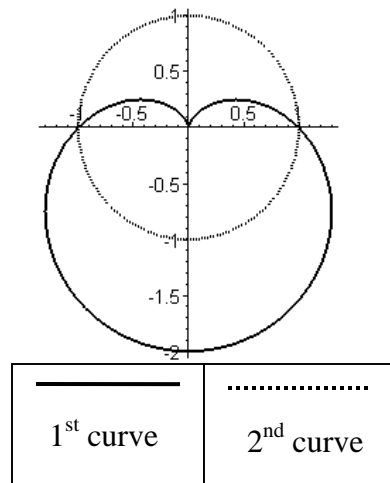
$$r_i^2 = (1)^2 = 1$$

$$\begin{aligned} A &= \int_{\pi}^{2\pi} \frac{1}{2} (1) d\theta = \left[ \frac{1}{2} \theta + C \right]_{\pi}^{2\pi} = \left[ \frac{1}{2} (2\pi) + C \right] - \left[ \frac{1}{2} (\pi) + C \right] \\ &= \left[ \frac{2\pi}{2} \right] - \left[ \frac{\pi}{2} \right] = \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} r_o^2 &= (1 - \sin \theta)^2 = 1 - 2 \sin \theta + \sin^2 \theta \\ &= 1 - 2 \sin \theta + \frac{1}{2} (1 - \cos(2\theta)) = \frac{3}{2} - 2 \sin \theta - \frac{1}{2} \cos(2\theta) \end{aligned}$$

$$\begin{aligned} A_o &= \int_{\pi}^{2\pi} \frac{1}{2} \left( \frac{3}{2} - 2 \sin \theta - \frac{1}{2} \cos(2\theta) \right) d\theta \\ &= \frac{1}{2} \left[ \frac{3}{2} \theta + 2 \cos \theta - \frac{1}{4} \sin(2\theta) + C \right]_{\pi}^{2\pi} \\ &= \frac{1}{2} \left\{ \left[ \frac{3}{2} (2\pi) + 2 \cos(2\pi) - \frac{1}{4} \sin(2(2\pi)) + C \right] - \left[ \frac{3}{2} (\pi) + 2 \cos(\pi) - \frac{1}{4} \sin(2(\pi)) + C \right] \right\} \\ &= \frac{1}{2} \left\{ \left[ \frac{6\pi}{2} + 2(1) - \frac{1}{4} (0) \right] - \left[ \frac{3\pi}{2} + 2(-1) - \frac{1}{4} (0) \right] \right\} = \frac{1}{2} \left\{ \frac{3\pi}{2} + 4 \right\} = \frac{3\pi}{4} + 2 \end{aligned}$$

$$A = A_o - A_i = \left( \frac{3\pi}{4} + 2 \right) - \left( \frac{\pi}{2} \right) = \frac{\pi}{4} + 2$$



24)  $r = 1 + \cos \theta$   $r = 1 - \cos \theta$

The region is possible to be cut in 4 identical pieces. For simplicity, we use symmetry and the interval

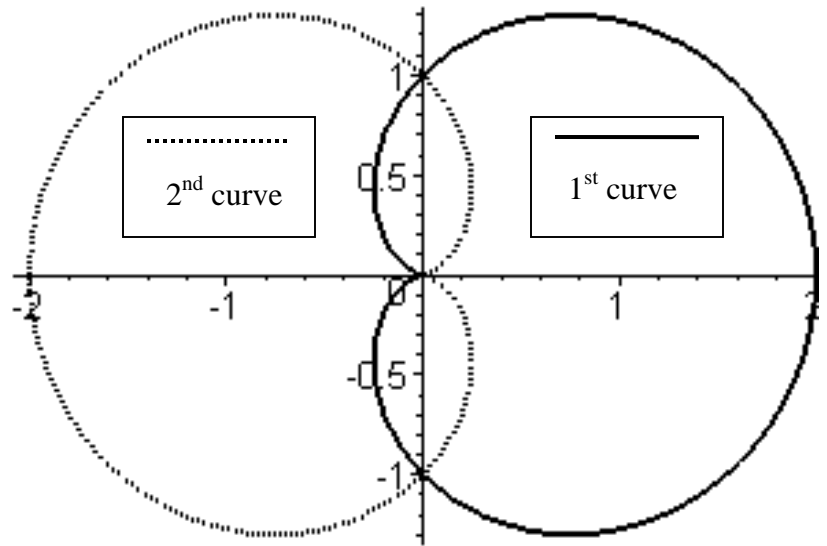
$0 \leq \theta \leq \frac{\pi}{2}$  with the curve  $r = 1 - \cos \theta$ .

Once we get the answer for this we can multiply by 4 to obtain the area inside both curves.

$$\begin{aligned} r^2 &= (1 - \cos \theta)^2 \\ &= 1 - 2 \cos \theta + \cos^2 \theta \\ &= 1 - 2 \cos \theta + \frac{1}{2}(1 + \cos(2\theta)) \\ &= \frac{3}{2} - 2 \cos \theta + \frac{1}{2} \cos(2\theta) \end{aligned}$$

$$\begin{aligned} A_1 &= \int_0^{\frac{\pi}{2}} \frac{1}{2} \left( \frac{3}{2} - 2 \cos \theta + \frac{1}{2} \cos(2\theta) \right) d\theta = \frac{1}{2} \left[ \frac{3}{2} \theta - 2 \sin \theta - \frac{1}{4} \sin(2\theta) + C \right]_0^{\frac{\pi}{2}} \\ &= \frac{1}{2} \left\{ \left[ \frac{3}{2} \left( \frac{\pi}{2} \right) - 2 \sin \left( \frac{\pi}{2} \right) - \frac{1}{4} \sin \left( 2 \left( \frac{\pi}{2} \right) \right) + C \right] - \left[ \frac{3}{2} (0) - 2 \sin(0) - \frac{1}{4} \sin(2(0)) + C \right] \right\} \\ &= \frac{1}{2} \left\{ \left[ \frac{3\pi}{4} - 2(1) - (0) \right] - [0] \right\} = \frac{1}{2} \left\{ \frac{3\pi}{4} - 2 \right\} = \frac{3\pi}{8} - 1 \end{aligned}$$

$$A = 4A_1 = 4 \left( \frac{3\pi}{8} - 1 \right) = \frac{3\pi}{2} - 4$$



30)  $r = \cos 3\theta$   $r = \sin 3\theta$

$\cos 3\theta = \sin 3\theta$

$1 = \frac{\sin 3\theta}{\cos 3\theta} = \tan 3\theta \Rightarrow$

$3\theta = \frac{\pi}{4}$

$3\theta = \frac{\pi}{4} + k\pi$

$\theta = \frac{\pi}{12} + \frac{k}{3}\pi$

We have 3 intersection points:

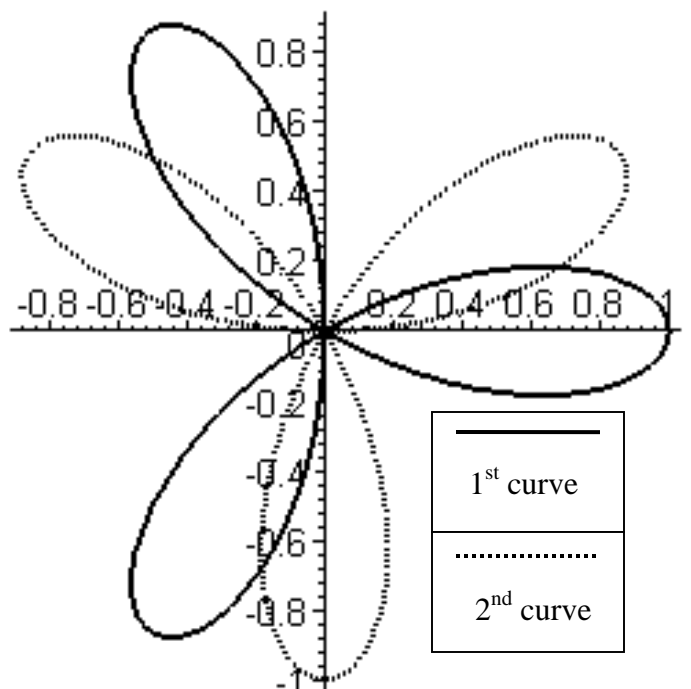
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

So our answer here is:

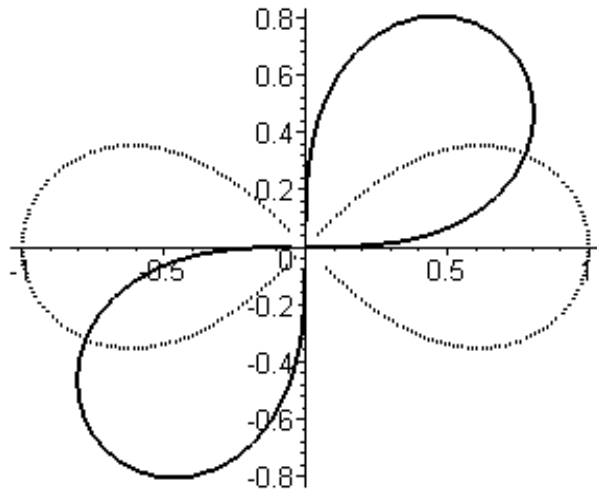
$(\frac{1}{\sqrt{2}}, \frac{\pi}{12})$

$(\frac{-1}{\sqrt{2}}, \frac{5\pi}{12})$

$(\frac{1}{\sqrt{2}}, \frac{3\pi}{4})$



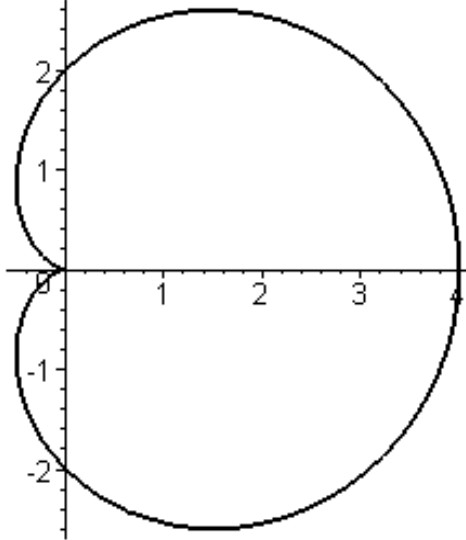
32)  $r^2 = \sin 2\theta$   $r^2 = \cos 2\theta$   
 $\sin 2\theta = \cos 2\theta$   
 $\frac{\sin 2\theta}{\cos 2\theta} = 1$   $\Rightarrow$   $2\theta = \frac{\pi}{4} + k\pi$   
 $\tan 2\theta = 1$   $\Rightarrow$   $\theta = \frac{\pi}{8} + \frac{k}{2}\pi$   
 $2\theta = \frac{\pi}{4}$   
 $r^2 = \sin\left(2\left(\frac{\pi}{8}\right)\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$   
 $r = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt[4]{2}}$



—	⋯
1 <sup>st</sup> curve	2 <sup>nd</sup> curve

Since both equations must be positive in equations we have the following angles  $\theta = \frac{\pi}{8}, \frac{9\pi}{8}$   
 $(\frac{1}{\sqrt[4]{2}}, \frac{\pi}{8})$   $(\frac{1}{\sqrt[4]{2}}, \frac{9\pi}{8})$

36)  $r = 2(1 + \cos \theta)$   
 This is a cardioid, the actual interval is  $0 \leq \theta \leq 2\pi$  but this graph is symmetric about x-axis and we can analyze  $0 \leq \theta \leq \pi$  and multiply the result by 2.



$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \quad \frac{dr}{d\theta} = 2(-\sin \theta) = -2 \sin \theta$$

$$\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} = \sqrt{(2(1 + \cos \theta))^2 + (-2 \sin \theta)^2}$$

$$= \sqrt{4(1 + 2 \cos \theta + \cos^2 \theta) + 4 \sin^2 \theta}$$

$$= \sqrt{4 + 8 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta}$$

$$= \sqrt{4 + 8 \cos \theta + 4(\cos^2 \theta + \sin^2 \theta)}$$

$$= \sqrt{4 + 8 \cos \theta + 4} = \sqrt{8 + 8 \cos \theta} = \sqrt{8(1 + \cos \theta)}$$

$$= \sqrt{8 \left(2 \cos^2 \left(\frac{\theta}{2}\right)\right)} = \sqrt{16 \cos^2 \left(\frac{\theta}{2}\right)} = 4 \cos \left(\frac{\theta}{2}\right)$$

$$L = \int_0^{2\pi} 4 \cos \left(\frac{\theta}{2}\right) d\theta$$

$$L = 2 \int_0^{\pi} 4 \cos \left(\frac{\theta}{2}\right) d\theta = 8 \left[ 2 \sin \left(\frac{\theta}{2}\right) + C \right]_0^{\pi} = 8 \left\{ \left[ 2 \sin \left(\frac{\pi}{2}\right) + C \right] - \left[ 2 \sin \left(\frac{0}{2}\right) + C \right] \right\}$$

$$= 8 \{ [2(1)] - [0] \} = 16$$