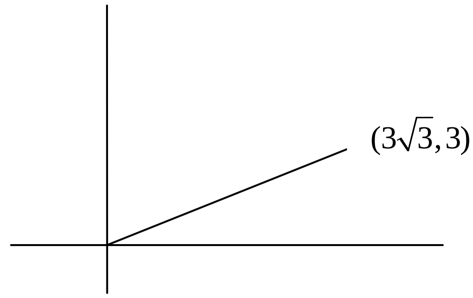
6) $r=\sqrt{x^2+y^2}\quad \theta=\tan^{-1}\dfrac{y}{x}$



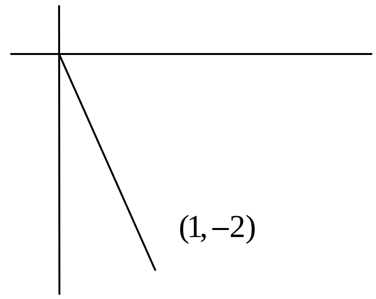
Part a) $(3\sqrt{3},3)$

$$r=\sqrt{(3\sqrt{3})^2+(3)^2}=\sqrt{3^2(3+1)}=3\sqrt{4}=6$$

(i) $$\theta=\tan^{-1}\left(\frac{3\sqrt{3}}{3}\right)=\tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{6}$$

$$(6,\tfrac{\pi}{6})$$

(ii) for $r<0$, $r=-6$ and $\theta=\dfrac{\pi}{6}+\pi=\dfrac{7\pi}{6}$ thus $(-6,\tfrac{7\pi}{6})$



Part b) $(1,-2)$

$$r=\sqrt{(1)^2+(-2)^2}=\sqrt{1+4}=\sqrt{5}$$

(i) $$\theta=\tan^{-1}\left(\frac{-2}{1}\right)=\tan^{-1}(-2)$$

but θ is in QIV; therefore $\theta=2\pi-\tan^{-1}2$

$(\sqrt{5},2\pi-\tan^{-1}2)$

(ii) for $r<0$, $r=-\sqrt{5}$ and $\theta=(2\pi-\tan^{-1}2)-\pi=\pi-\tan^{-1}2$ thus $(-\sqrt{5},\pi-\tan^{-1}2)$

14) $\pi=\dfrac{\pi}{3}$

$$\frac{y}{x}=\tan\theta=\tan\frac{\pi}{3}=\sqrt{3}\quad\Rightarrow\quad\frac{y}{x}=\sqrt{3}\quad\Rightarrow\quad y=\sqrt{3}x$$

Since we don't have any restriction for *r*, this expression $y=\sqrt{3}x$ with domain $-\infty<x<\infty$.

16) $r=\tan\theta\sec\theta$

$x=r\cos\theta$

$y=r\sin\theta$

$$r=\tan\theta\sec\theta$$
$$r=\frac{\sin\theta}{\cos^2\theta}\quad\Rightarrow$$
$$r\cos^2\theta=\sin\theta$$

$$(r\cos\theta)^2=r\sin\theta$$
$$(x)^2=y$$
$$y=x^2$$

18) $4y^2=x$

$x=r\cos\theta$

$y=r\sin\theta$

$$4y^2=x$$
$$4(r\sin\theta)^2=r\cos\theta\quad\Rightarrow$$
$$4r^2\sin^2\theta=r\cos\theta$$

$$\frac{r^2}{r}=\frac{\cos\theta}{4\sin^2\theta}$$
$$r=\frac{1}{4}\left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)=\frac{1}{4}\csc\theta\cot\theta$$

20) $xy=4$

$x=r\cos\theta$

$y=r\sin\theta$

$$xy=4$$
$$(r\cos\theta)(r\sin\theta)=4\quad\Rightarrow\quad r^2\left(\frac{1}{2}\sin(2\theta)\right)=4\qquad r^2=\frac{8}{\sin(2\theta)}=8\csc(2\theta)$$
$$r^2\sin\theta\cos\theta=4$$

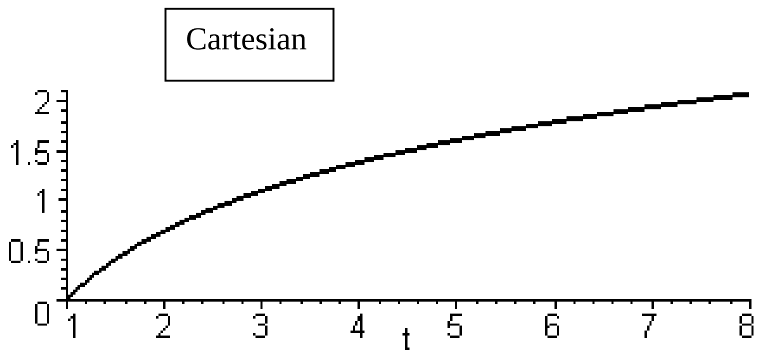
22) a) $r = 5$ center $= (2,3)$

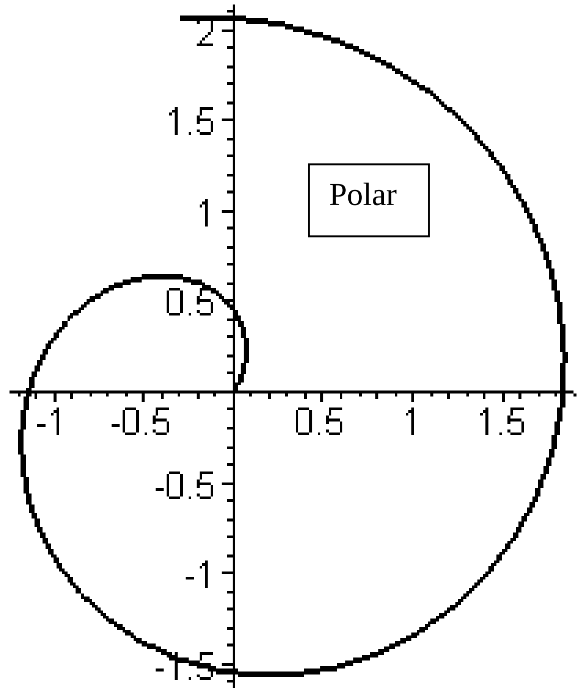
Cartesian is easier: $(x-2)^2 + (y-3)^2 = (5)^2$

b) $r = 4$ center $= (0,0)$

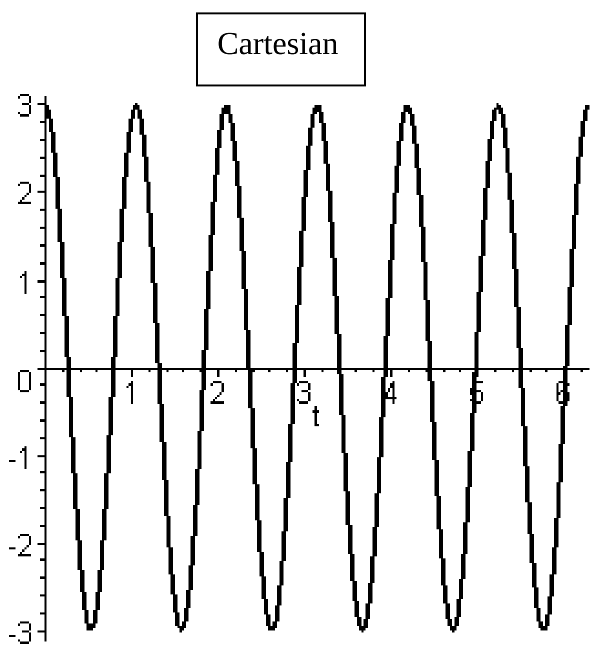
Polar is easier $r = 4$ instead of Cartesian $x^2 + y^2 = (4)^2$

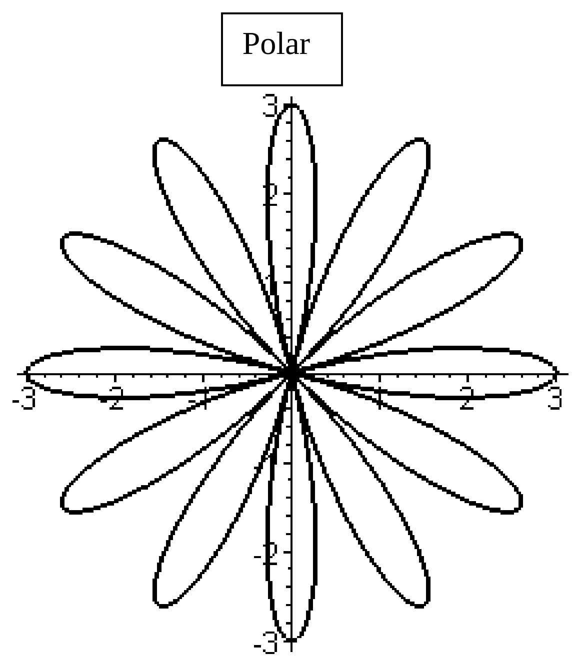
28) $r = \ln\theta \quad \theta \geq 1$





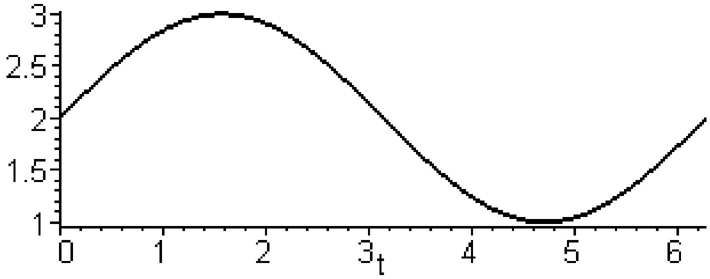
32) $r = 3\cos 6\theta$



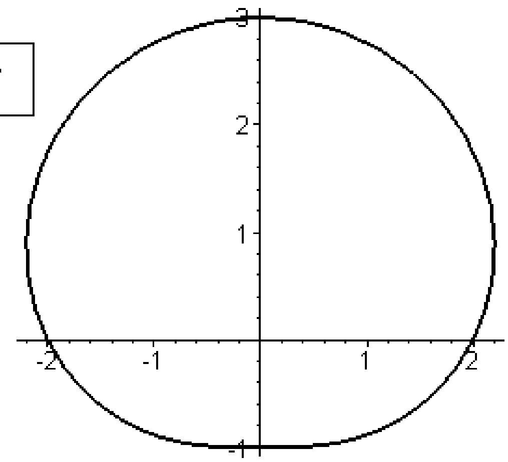


34) $r = 2 + \sin\theta$

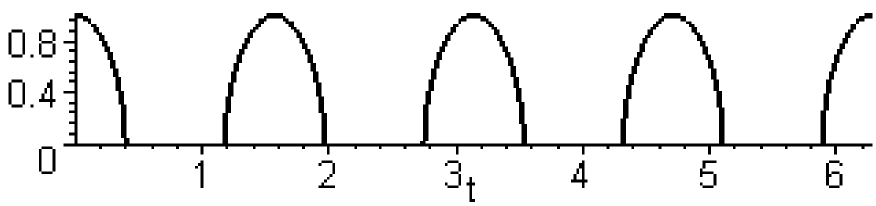
Cartesian



Polar

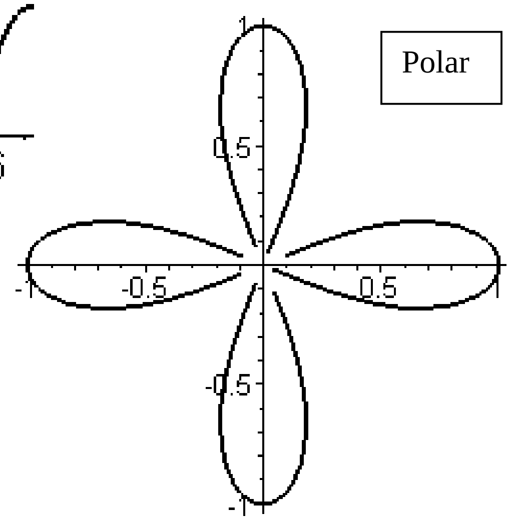


36) $r^2 = \cos 4\theta$



Cartesian

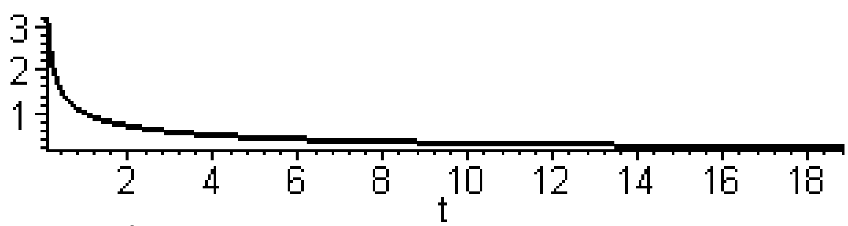
Polar



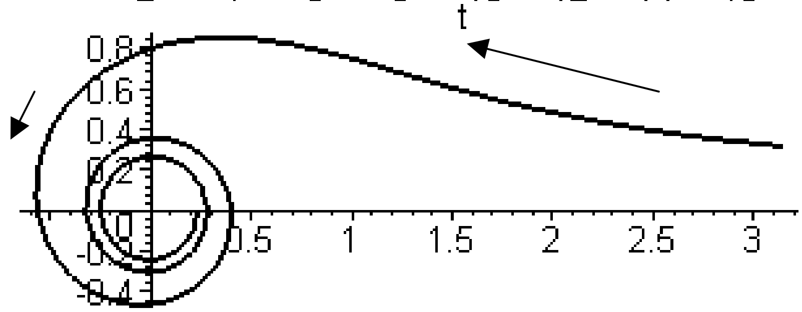
Each loop of the Polar plot to the right should have been drawn to the origin.

38) $r^2\theta = 1$

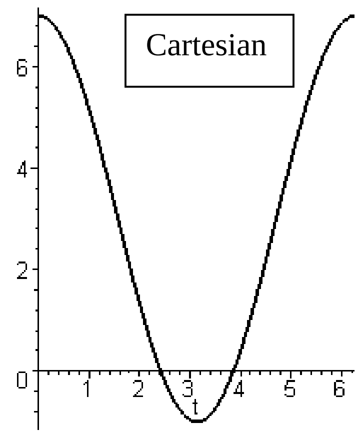
Cartesian

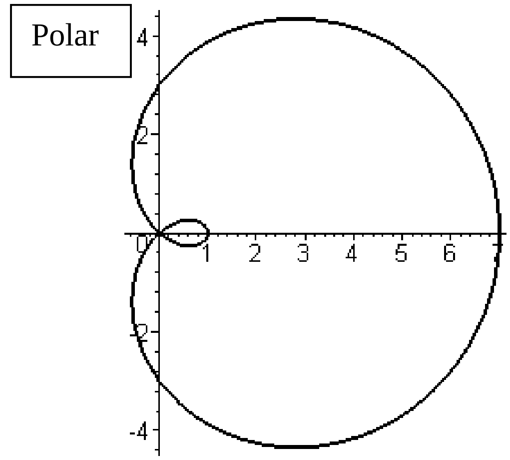


Polar



40) $r = 3 + 4\cos\theta$





48) $r = 2 - \sin\theta \quad \theta = \frac{\pi}{3}$

$$x = r\cos\theta = (2-\sin\theta)\cos\theta = 2\cos\theta - \sin\theta\cos\theta = 2\cos\theta - \frac{1}{2}\sin(2\theta)$$

$$y = r\sin\theta = (2-\sin\theta)\sin\theta = 2\sin\theta - \sin^2\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta - \frac{1}{2}\cos(2\theta)(2) = -2\sin\theta - \cos(2\theta) \quad \frac{dy}{d\theta} = 2\cos\theta - 2\sin\theta\cos\theta = 2\cos\theta - \sin(2\theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos\theta - \sin(2\theta)}{-2\sin\theta - \cos(2\theta)} \quad m = \frac{2\cos\left(\frac{\pi}{3}\right) - \sin\left(2\left(\frac{\pi}{3}\right)\right)}{-2\sin\left(\frac{\pi}{3}\right) - \cos\left(2\left(\frac{\pi}{3}\right)\right)} = \frac{2\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)}{-2\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{-1}{2}\right)} = \frac{\frac{2-\sqrt{3}}{2}}{\frac{1-2\sqrt{3}}{2}} = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$$

50) $r = \cos\left(\frac{\theta}{3}\right) \quad \theta = \pi$

$$x = r\cos\theta = \left(\cos\left(\frac{\theta}{3}\right)\right)\cos\theta = \cos\left(\frac{\theta}{3}\right)\cos\theta \quad y = r\sin\theta = \left(\cos\left(\frac{\theta}{3}\right)\right)\sin\theta = \cos\left(\frac{\theta}{3}\right)\sin\theta$$

$$\frac{dx}{d\theta} = \left[\frac{-1}{3}\sin\left(\frac{\theta}{3}\right)\right](\cos\theta) + \left(\cos\left(\frac{\theta}{3}\right)\right)[-\sin\theta] \quad \frac{dy}{d\theta} = \left[\frac{-1}{3}\sin\left(\frac{\theta}{3}\right)\right](\sin\theta) + \left(\cos\left(\frac{\theta}{3}\right)\right)[\cos\theta]$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\left[\frac{-1}{3}\sin\left(\frac{\theta}{3}\right)\right](\sin\theta) + \left(\cos\left(\frac{\theta}{3}\right)\right)[\cos\theta]}{\left[\frac{-1}{3}\sin\left(\frac{\theta}{3}\right)\right](\cos\theta) + \left(\cos\left(\frac{\theta}{3}\right)\right)[-\sin\theta]}$$

$$m = \frac{\left[\frac{-1}{3}\sin\left(\frac{(\pi)}{3}\right)\right](\sin(\pi)) + \left(\cos\left(\frac{(\pi)}{3}\right)\right)[\cos(\pi)]}{\left[\frac{-1}{3}\sin\left(\frac{(\pi)}{3}\right)\right](\cos(\pi)) + \left(\cos\left(\frac{(\pi)}{3}\right)\right)[-\sin(\pi)]} = \frac{\left[\frac{-1}{3}\left(\frac{\sqrt{3}}{2}\right)\right](0) + \left(\frac{1}{2}\right)[1]}{\left[\frac{-1}{3}\left(\frac{\sqrt{3}}{2}\right)\right](1) + \left(\frac{1}{2}\right)[0]} = \frac{\frac{-1}{2}}{\frac{\sqrt{3}}{3(2)}} = \frac{-3}{\sqrt{3}} = -\sqrt{3}$$