2) $F(x) = \cos\left(\frac{\pi}{3}x\right)$ $W = \int_a^b F(x)\,dx = \int_a^b \cos\left(\frac{\pi}{3}x\right)dx = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}x\right) + C\right]_a^b$

$$W_{1-2} = \int_1^2 \cos\left(\frac{\pi}{3}x\right)dx = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}x\right) + C\right]_1^2 = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(2)\right) + C\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(1)\right) + C\right]$$

$$= \left[\frac{3}{\pi}\sin\left(\frac{2\pi}{3}\right)\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}\right)\right] = \left[\frac{3}{\pi}\left(\frac{\sqrt{3}}{2}\right)\right] - \left[\frac{3}{\pi}\left(\frac{\sqrt{3}}{2}\right)\right] = 0 \text{ J}$$

$$W_{1-1.5} = \int_1^{1.5} \cos\left(\frac{\pi}{3}x\right)dx = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}x\right) + C\right]_1^{1.5} = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(1.5)\right) + C\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(1)\right) + C\right]$$

$$= \left[\frac{3}{\pi}\sin\left(\frac{\pi}{2}\right)\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}\right)\right] = \left[\frac{3}{\pi}(1)\right] - \left[\frac{3}{\pi}\left(\frac{\sqrt{3}}{2}\right)\right] = \frac{3}{\pi}\left(1 - \frac{\sqrt{3}}{2}\right) \text{ J}$$

$$W_{1.5-2} = \int_{1.5}^{2} \cos\left(\frac{\pi}{3}x\right)dx = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}x\right) + C\right]_{1.5}^{2} = \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(2)\right) + C\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}(1.5)\right) + C\right]$$

$$= \left[\frac{3}{\pi}\sin\left(\frac{2\pi}{3}\right)\right] - \left[\frac{3}{\pi}\sin\left(\frac{\pi}{2}\right)\right] = \left[\frac{3}{\pi}\left(\frac{\sqrt{3}}{2}\right)\right] - \left[\frac{3}{\pi}(1)\right] = \frac{3}{\pi}\left(\frac{\sqrt{3}}{2} - 1\right) \text{ J}$$

The work from 1.5 to 2 is equal in magnitude than 1 to 1.5 but in opposite direction.

8) let L be the natural length of the spring in meters. $F(x) = kx$ $\int kx\,dx = \frac{k}{2}x^2 + C$

	Start length	Start displacement	End length	End displacement	Work
1	10 cm	$(0.10 - L)$ m	12 cm	$(0.12 - L)$ m	6 J
2	12 cm	$(0.12 - L)$ m	14 cm	$(0.14 - L)$ m	10 J

$$W_1 = \int_{(0.10-L)}^{(0.12-L)} kx\,dx = \left[\frac{k}{2}x^2 + C\right]_{(0.10-L)}^{(0.12-L)} = \left[\frac{k}{2}(0.12 - L)^2 + C\right] - \left[\frac{k}{2}(0.10 - L)^2 + C\right]$$

$$= \frac{k}{2}\left\{\left[0.0144 - 0.24L - L^2\right] - \left[0.01 - 0.2L - L^2\right]\right\} = \frac{k}{2}\left\{0.0044 - 0.04L\right\}$$

$$W_1 = 6 = \frac{k}{2}\left\{0.0044 - 0.04L\right\} \qquad 12 = 0.0001k\left\{44 - 400L\right\}$$

$$12 = k\left\{0.0044 - 0.04L\right\} \quad \Rightarrow \quad 120000 = k\left\{44 - 400L\right\}$$

$$W_2 = \int_{(0.12-L)}^{(0.14-L)} kx\,dx = \left[\frac{k}{2}x^2 + C\right]_{(0.12-L)}^{(0.14-L)} = \left[\frac{k}{2}(0.14 - L)^2 + C\right] - \left[\frac{k}{2}(0.12 - L)^2 + C\right]$$

$$= \frac{k}{2}\left\{\left[0.0196 - 0.28L - L^2\right] - \left[0.0144 - 0.24L - L^2\right]\right\} = \frac{k}{2}\left\{0.0052 - 0.04L\right\}$$

$$W_2 = 10 = \frac{k}{2}\left\{0.0052 - 0.04L\right\} \qquad 20 = 0.0001k\left\{52 - 400L\right\}$$

$$20 = k\left\{0.0052 - 0.04L\right\} \quad \Rightarrow \quad 200000 = k\left\{52 - 400L\right\}$$

Now we have 2 equations with 2 unknowns. Since we want L we solve W_1 and get $k = \dfrac{120000}{44-400L}$.

$$200000 = \left(\frac{120000}{44-400L}\right)\{52-400L\}$$
$$\frac{200000}{120000} = \frac{52-400L}{44-400L}$$
$$\frac{5}{3} = \frac{52-400L}{44-400L}$$
$$\Rightarrow \begin{aligned} 5(44-400L) &= 3(52-400L) \\ 220-2000L &= 156-1200L \\ 64 &= 800L \end{aligned} \Rightarrow \begin{aligned} L &= \frac{64}{800} = \frac{8}{100}\text{ m} \\ &= 0.08\text{ m} \\ &= 80\text{ cm} \end{aligned}$$

12) Since the weight of the bucket does not change, $W_B = (4\text{ lbs/ft})(80\text{ ft}) = 320\text{ ft-lbs}$.
We are loosing water at a constant rate and the bucket is moving at a constant speed but we need the rate of water lost per distance traveled in order for us to compute our work.
The bucket is being pulled up at a rate of $2\text{ ft/sec} = \dfrac{2\text{ ft}}{1\text{ sec}}$ and the water leaking is $0.2\text{ lbs/sec} = \dfrac{0.2\text{ lbs}}{1\text{ sec}}$.

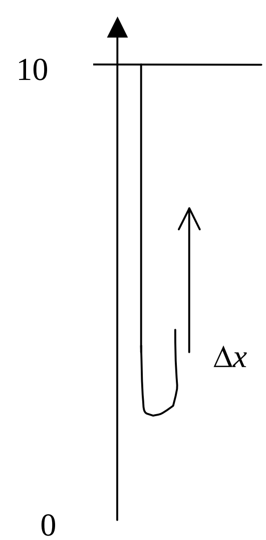
Therefore, the rate of water lost over the distance is $\dfrac{\frac{0.2\text{ lbs}}{1\text{ sec}}}{\frac{2\text{ ft}}{1\text{ sec}}} = 0.1\text{ lbs/ft} = \dfrac{1}{10}\text{ lbs/ft}$ and for any arbitrary distance (x ft) traveled be the bucket, we lost $\left(\dfrac{1}{10}\text{ lbs/ft}\right)(x\text{ ft}) = \dfrac{1}{10}x\text{ lbs}$.

We started with 40 lbs of water and at a height x ft, we have $\left(40-\dfrac{1}{10}x\right)\text{lbs}$ of water left in the bucket.

$$W_{water} = \int_0^{80}\left(40-\frac{1}{10}x\right)dx = \left[40x-\frac{1}{10}x^2+C\right]_0^{80} = \left[40(80)-\frac{1}{10}(80)^2+C\right]-\left[40(0)-\frac{1}{10}(0)^2+C\right]$$
$$= \left[40(80)-\frac{1}{10}(80)(80)\right]-[0] = (3200-320)\text{ ft-lbs}$$

$$W = W_B + W_{water} = (320)+(3200-320) = 3200\text{ ft-lbs}$$

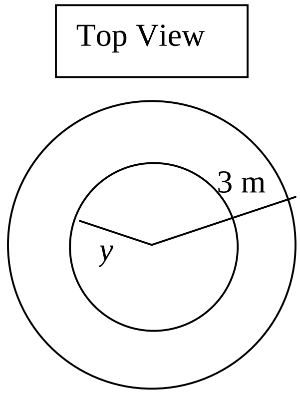
14) The rate of chain weight per foot is $\dfrac{25\text{ lbs}}{10\text{ ft}} = 2.5\text{ lbs/ft}$. Let's analyze by link and Δx be the length of the link. The lowest link of chain must travel $10\text{ ft} = 10-0(\Delta x)\text{ ft}$. The next link does not reach the top because only the lowest link occupies this space. So, 2nd link travels $(10-2(\Delta x))$ ft and 3rd link travels $(10-2(2\Delta x)) = (10-4(\Delta x))$ ft. This continues until we fold the chain which is 5 ft. Therefore, our interval is $0 \le x \le 5$ and we can formulate the amount of chain that will affect the force:
$F(x) = (2.5\text{ lbs/ft})((10-2x)\text{ ft}) = (25-5x)\text{ lbs}$ for any $0 \le x \le 5$

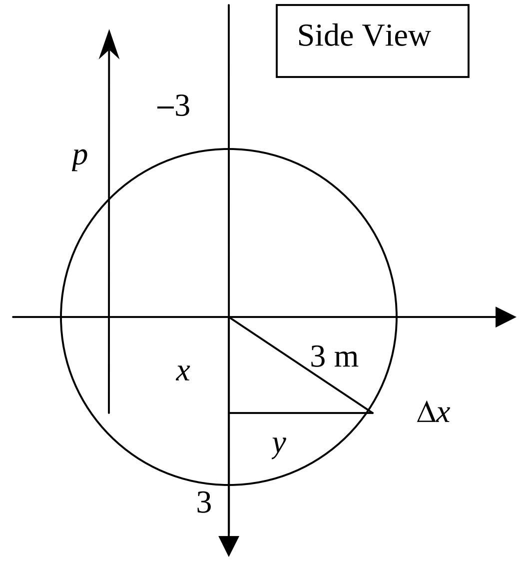


$$W = \int_0^5 (25-5x)\,dx = \left[25x-\frac{5}{2}x^2+C\right]_0^5 = \left[25(5)-\frac{5}{2}(5)^2+C\right]-\left[25(0)-\frac{5}{2}(0)^2+C\right] = 125\left[1-\frac{1}{2}\right] = \frac{125}{2}\text{ ft-lbs}$$

18) pumping distance: $p = (x+4)$

$$x^2 + y^2 = (3)^2$$
$$y^2 = 9 - x^2$$
$$y = \pm\sqrt{9-x^2}$$
$$y = \sqrt{9-x^2}$$
$$A = \pi y^2 = \pi\left(\sqrt{9-x^2}\right)^2$$
$$= \pi(9-x^2) \text{ m}^2$$
$$\Delta V = A\Delta x$$
$$= \pi(9-x^2)\Delta x \text{ m}^3$$





a) There is an error in this part, the density here should be $\delta_{water} = 1000 \text{ kg/m}^3$ because the measurement of the tank is metric. The interval is $-3 \le x \le 3$.

$$\Delta m = (\delta)(\Delta V) = (1000 \text{ kg/m}^3)(\pi(9-x^2)\Delta x \text{ m}^3) = 1000\pi(9-x^2)\Delta x \text{ kg}$$
$$\Delta F = (\Delta m)(a) = (1000\pi(9-x^2)\Delta x \text{ kg})(9.8 \text{ m/sec}^2) = 9800\pi(9-x^2)\Delta x \text{ N}$$
$$\Delta W = (\Delta F)(p) = (9800\pi(9-x^2)\Delta x \text{ N})((x+4)\text{ m}) = 9800\pi(36+9x-4x^2-x^3)\Delta x \text{ J}$$
$$W = \int_{-3}^{3} 9800\pi(36+9x-4x^2-x^3)\,dx = 9800\pi\left[36x + \frac{9}{2}x^2 - \frac{4}{3}x^3 - \frac{1}{4}x^4 + C\right]_{-3}^{3}$$
$$= 9800\pi\left\{\left[36(3) + \frac{9}{2}(3)^2 - \frac{4}{3}(3)^3 - \frac{1}{4}(3)^4 + C\right] - \left[36(-3) + \frac{9}{2}(-3)^2 - \frac{4}{3}(-3)^3 - \frac{1}{4}(-3)^4 + C\right]\right\}$$
$$= 9800\pi\left\{3^2\left[12(1) + \frac{9}{2} - 4 - \frac{9}{4}\right] - \left[12(-1) + \frac{9}{2} + 4 - \frac{9}{4}\right]\right\} = 9800\pi\left\{3^2\left[8 + \frac{9}{2} - \frac{9}{4}\right] - \left[-8 + \frac{9}{2} - \frac{9}{4}\right]\right\}$$
$$= 9800\pi(9)(16) = 1411200\pi \text{ J}$$

b) The oil has density of $\delta_{oil} = 900 \text{ kg/m}^3$ and the tank is only half full. The interval is $0 \le x \le 3$.

$$\Delta m = (\delta)(\Delta V) = (900 \text{ kg/m}^3)(\pi(9-x^2)\Delta x \text{ m}^3) = 900\pi(9-x^2)\Delta x \text{ kg}$$
$$\Delta F = (\Delta m)(a) = (900\pi(9-x^2)\Delta x \text{ kg})(9.8 \text{ m/sec}^2) = 8820\pi(9-x^2)\Delta x \text{ N}$$
$$\Delta W = (\Delta F)(p) = (8820\pi(9-x^2)\Delta x \text{ N})((x+4)\text{ m}) = 8820\pi(36+9x-4x^2-x^3)\Delta x \text{ J}$$
$$W = \int_{0}^{3} 8820\pi(36+9x-4x^2-x^3)\,dx = 8820\pi\left[36x + \frac{9}{2}x^2 - \frac{4}{3}x^3 - \frac{1}{4}x^4 + C\right]_{0}^{3}$$
$$= 8820\pi\left\{\left[36(3) + \frac{9}{2}(3)^2 - \frac{4}{3}(3)^3 - \frac{1}{4}(3)^4 + C\right] - \left[36(0) + \frac{9}{2}(0)^2 - \frac{4}{3}(0)^3 - \frac{1}{4}(0)^4 + C\right]\right\}$$
$$= 8820\pi\left\{3^2\left[12(1) + \frac{9}{2} - 4 - \frac{9}{4}\right] - [0]\right\} = 8820\pi\left\{3^2\left[8 + \frac{18}{4} - \frac{9}{4}\right]\right\} = 8820\pi\left\{3^2\left[8 + \frac{9}{4}\right]\right\}$$
$$= 8820\pi(9)\left(\frac{41}{4}\right) = (9)(41)(2205)\pi = 813645\pi \text{ J}$$