8) $y^2 = 4(x+4)^3 \quad 0 \le x \le 2 \quad y > 0$

$$L = \int_a^b \sqrt{1^2 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y^2 = 4(x+4)^3 \quad \Rightarrow \quad y = \sqrt{4(x+4)^3} = 2(x+4)^{\frac{3}{2}} \qquad \frac{dy}{dx} = 3(x+4)^{\frac{1}{2}}(1) = 3\sqrt{x+4}$$

$$\sqrt{1^2 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1^2 + \left(3\sqrt{x+4}\right)^2} = \sqrt{1+9(x+4)} = \sqrt{9x+37}$$

$$p = 9x+37 \quad dp = 9\,dx \quad \frac{1}{9}\,dp = dx$$

$$\int \sqrt{9x+37}\,dx = \int \sqrt{p}\left(\frac{1}{9}\,dp\right) = \frac{1}{9}\left(\frac{2}{3}p^{\frac{3}{2}}\right) + C = \frac{2}{27}\left(\sqrt{9x+37}\right)^3 + C$$

$$L = \int_0^2 \sqrt{9x+37}\,dx = \left[\frac{2}{27}\left(\sqrt{9x+37}\right)^3 + C\right]_0^2 = \left[\frac{2}{27}\left(\sqrt{9(2)+37}\right)^3 + C\right] - \left[\frac{2}{27}\left(\sqrt{9(0)+37}\right)^3 + C\right]$$

$$= \frac{2}{27}\left\{\left[\left(\sqrt{55}\right)^3\right] - \left[\left(\sqrt{37}\right)^3\right]\right\} = \frac{2}{27}\left\{55\sqrt{55} - 37\sqrt{37}\right\}$$

14) $y = 3 + \frac{1}{2}\cosh 2x \quad 0 \le x \le 1$

$$\frac{dy}{dx} = \frac{1}{2}\left(\sinh(2x)(2)\right) = \sinh(2x) \qquad \cosh^2\theta - \sinh^2\theta = 1$$

$$\cosh^2\theta = 1 + \sinh^2\theta$$

$$\sqrt{1^2 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1^2 + \left(\sinh(2x)\right)^2} = \sqrt{1+\sinh^2(2x)} = \sqrt{\cosh^2(2x)} = \cosh(2x)$$

$$L = \int_0^1 \cosh(2x)\,dx = \left[\frac{1}{2}\sinh(2x) + C\right]_0^1 = \left[\frac{1}{2}\sinh(2(1)) + C\right] - \left[\frac{1}{2}\sinh(2(0)) + C\right] = \frac{1}{2}\sinh 2$$

16) $y = \sqrt{x - x^2} + \sin^{-1}\left(\sqrt{x}\right)$

The domain for the radical part is $[0,1]$ because for any other values we have complex number. The domain for the $\sin^{-1}\left(\sqrt{x}\right)$ is also $[0,1]$ because the maximum value of sine is 1. Therefore, our interval is $0 \le x \le 1$.

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x-x^2}}(1-2x) + \frac{1}{\sqrt{1^2 - \left(\sqrt{x}\right)^2}}\left(\frac{1}{2\sqrt{x}}\right) = \frac{1-2x}{2\sqrt{(x)(1-x)}} + \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1-2x}{2\sqrt{x}\sqrt{1-x}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{2-2x}{2\sqrt{x}\sqrt{1-x}} = \frac{2(1-x)}{2\sqrt{x}\sqrt{1-x}} = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}}$$

$$\sqrt{1^2 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1^2 + \left(\sqrt{\frac{1-x}{x}}\right)^2} = \sqrt{1 + \frac{1-x}{x}} = \sqrt{\frac{x}{x} + \frac{1-x}{x}} = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$$

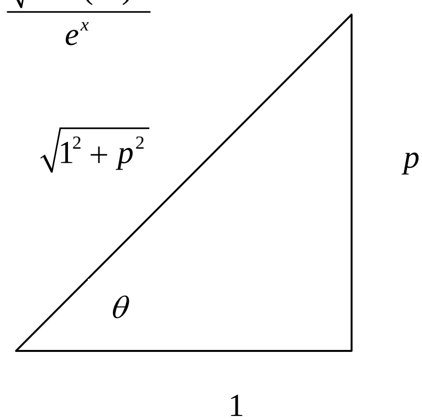
$$L = \int_0^1 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x} + C\right]_0^1 = \left[2\sqrt{(1)} + C\right] - \left[2\sqrt{(0)} + C\right] = 2$$

18) $y = 1 + e^{-x} \quad 0 \le x \le 2$

$$\frac{dy}{dx} = e^{-x}(-1) = -e^{-x} = \frac{-1}{e^x}$$

$$\sqrt{1^2 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1^2 + \left(\frac{-1}{e^x}\right)^2} = \sqrt{1 + \frac{1}{e^{2x}}} = \sqrt{\frac{e^{2x}}{e^{2x}} + \frac{1}{e^{2x}}} = \sqrt{\frac{e^{2x}+1}{e^{2x}}} = \frac{\sqrt{1^2 + (e^x)^2}}{e^x}$$

$$p = e^x \quad \Rightarrow \quad x = \ln p \quad dx = \frac{1}{p}\,dp$$



$$\frac{p}{1} = \tan\theta \qquad \frac{\sqrt{1^2 + p^2}}{1} = \sec\theta$$

$$p = \tan\theta$$

$$dp = \sec^2\theta\, d\theta \qquad \sqrt{1^2 + p^2} = \sec\theta$$

$$\int \frac{\sqrt{1^2 + (e^x)^2}}{e^x}\,dx = \int \frac{\sqrt{1^2 + p^2}}{p}\left(\frac{1}{p}\,dp\right)$$

$$= \int \frac{\sqrt{1^2 + p^2}}{p^2}\,dp$$

$$= \int \frac{(\sec\theta)}{(\tan\theta)^2}(\sec^2\theta\, d\theta)$$

$$= \int \frac{\sec\theta}{\tan^2\theta}(1 + \tan^2\theta)\, d\theta$$

$$= \int \left(\frac{\sec\theta}{\tan^2\theta} + \sec\theta\right) d\theta$$

$$= \int \left(\frac{\cos\theta}{\sin^2\theta} + \sec\theta\right) d\theta$$

$$= \frac{-1}{\sin\theta} + \ln\left|\sec\theta + \tan\theta\right| + C = -\csc\theta + \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{-\sqrt{1^2 + p^2}}{p} + \ln\left|\sqrt{1^2 + p^2} + p\right| + C = \frac{-\sqrt{1^2 + (e^x)^2}}{e^x} + \ln\left|\sqrt{1^2 + (e^x)^2} + e^x\right| + C$$

$$L = \int_0^2 \frac{\sqrt{1^2 + (e^x)^2}}{e^x}\,dx = \left[\frac{-\sqrt{1^2 + (e^x)^2}}{e^x} + \ln\left|\sqrt{1^2 + (e^x)^2} + e^x\right| + C\right]_0^2$$

$$= \left[\frac{-\sqrt{1^2 + (e^{(2)})^2}}{e^{(2)}} + \ln\left|\sqrt{1^2 + (e^{(2)})^2} + e^{(2)}\right| + C\right] - \left[\frac{-\sqrt{1^2 + (e^{(0)})^2}}{e^{(0)}} + \ln\left|\sqrt{1^2 + (e^{(0)})^2} + e^{(0)}\right| + C\right]$$

$$= \left[\frac{-\sqrt{1 + e^4}}{e^2} + \ln\left|\sqrt{1 + e^4} + e^2\right|\right] - \left[\frac{-\sqrt{1 + (1)^2}}{1} + \ln\left|\sqrt{1^2 + (1)^2} + 1\right|\right]$$

$$= \ln(\sqrt{1 + e^4} + e^2) - \ln(\sqrt{2} + 1) - \frac{\sqrt{1 + e^4}}{e^2} + \sqrt{2}$$