

6) $y = \frac{1}{4}x^2$ $y = 5 - x^2$; about x -axis

$$\frac{1}{4}x^2 = 5 - x^2$$

$$\frac{5}{4}(x+2)(x-2) = 0$$

$$\frac{5}{4}x^2 - 5 = 0 \quad x+2=0 \quad x-2=0$$

$$\frac{5}{4}(x^2 - 4) = 0 \quad x = -2 \quad x = 2$$

interval: $-2 \leq x \leq 2$

$$r_o = (5 - x^2) + 0 = (5 - x^2) \quad r_i = \left(\frac{1}{4}x^2\right) + 0 = \left(\frac{1}{4}x^2\right)$$

$$A_o = \pi(5 - x^2)^2 = \pi(25 - 10x^2 + x^4) \quad A_i = \pi\left(\frac{1}{4}x^2\right)^2 = \pi\left(\frac{1}{16}x^4\right)$$

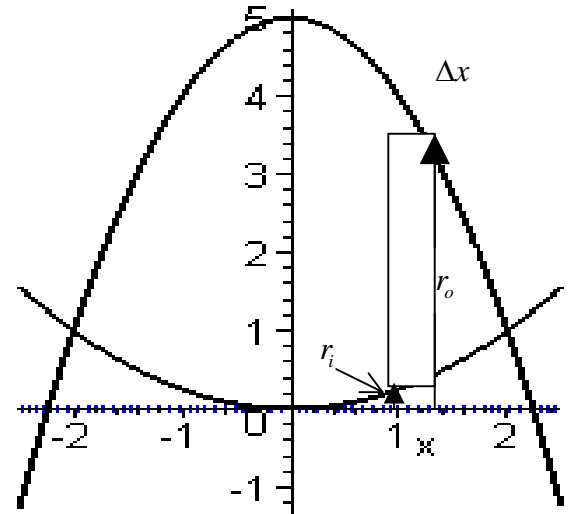
$$A = A_o - A_i = \pi(25 - 10x^2 + x^4) - \pi\left(\frac{1}{16}x^4\right) = \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right)$$

$$\Delta V = A\Delta x = \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right)\Delta x$$

$$V = \int_{-2}^2 \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right) dx = \pi\left[25x - \frac{10}{3}x^3 + \frac{3}{16}x^5 + C\right]_{-2}^2$$

$$= \pi\left\{\left[25(2) - \frac{10}{3}(2)^3 + \frac{3}{16}(2)^5 + C\right] - \left[25(-2) - \frac{10}{3}(-2)^3 + \frac{3}{16}(-2)^5 + C\right]\right\}$$

$$= \pi\left\{\left[50 - \frac{80}{3} + 6\right] - \left[-50 + \frac{80}{3} - 6\right]\right\} = \pi\left\{100 - \frac{160}{3} + 12\right\} = \pi\left\{\frac{300}{3} - \frac{160}{3} + \frac{36}{3}\right\} = \frac{176\pi}{3} \text{ units}^3$$



8) $y = \frac{1}{4}x^2$ $x = 2$ $y = 0$; about y -axis

$$y = \frac{1}{4}(2)^2 = 1$$

interval: $0 \leq y \leq 1$

$$y = \frac{1}{4}x^2 \Rightarrow 4y = x^2 \quad x = 2$$

$$x = \sqrt{4y}$$

$$r_o = (2) + 0 = (2) \quad r_i = (\sqrt{4y}) + 0 = (\sqrt{4y})$$

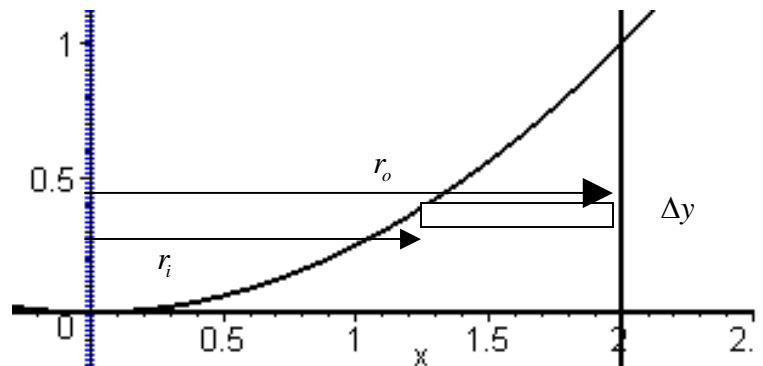
$$A_o = \pi(2)^2 = \pi(4) \quad A_i = \pi(\sqrt{4y})^2 = \pi(4y)$$

$$A = A_o - A_i = \pi(4) - \pi(4y) = \pi(4 - 4y)$$

$$\Delta V = A\Delta y = \pi(4 - 4y)\Delta y$$

$$V = \int_0^1 \pi(4 - 4y) dy = \pi[4y - 2y^2 + C]_0^1 = \pi\left\{\left[4(1) - 2(1)^2 + C\right] - \left[4(0) - 2(0)^2 + C\right]\right\}$$

$$= \pi\left\{[4 - 2] - [0]\right\} = 2\pi \text{ units}^3$$



10) $y = e^{-x}$ $y = 1$ $x = 2$; about $y = 2$

interval: $0 \leq x \leq 2$

$$r_o = 2 - (e^{-x})$$

$$r_i = 2 - (1) = (1)$$

$$= (2 - e^{-x})$$

$$A_o = \pi(2 - e^{-x})^2$$

$$A_i = \pi(1)^2$$

$$= \pi(4 - 4e^{-x} + e^{-2x})$$

$$= \pi(1)$$

$$A = A_o - A_i = \pi(4 - 4e^{-x} + e^{-2x}) - \pi(1)$$

$$= \pi(3 - 4e^{-x} + e^{-2x})$$

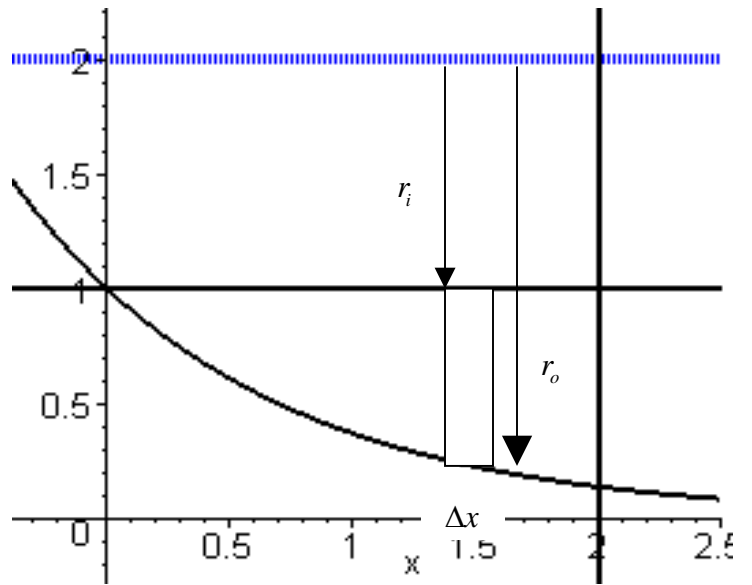
$$\Delta V = A \Delta x$$

$$= \pi(3 - 4e^{-x} + e^{-2x}) \Delta x$$

$$V = \int_0^2 \pi(3 - 4e^{-x} + e^{-2x}) dx = \pi \left[3x + 4e^{-x} - \frac{1}{2}e^{-2x} + C \right]_0^2 = \pi \left[3x + \frac{4}{e^x} - \frac{1}{2e^{2x}} + C \right]_0^2$$

$$= \pi \left\{ \left[3(2) + \frac{4}{e^{(2)}} - \frac{1}{2e^{2(2)}} + C \right] - \left[3(0) + \frac{4}{e^{(0)}} - \frac{1}{2e^{2(0)}} + C \right] \right\}$$

$$= \pi \left\{ \left[6 + \frac{4}{e^2} - \frac{1}{2e^4} \right] - \left[0 + \frac{4}{1} - \frac{1}{2(1)} \right] \right\} = \pi \left\{ 6 + \frac{4}{e^2} - \frac{1}{2e^4} - \frac{7}{2} \right\} = \pi \left(\frac{5}{2} + \frac{4}{e^2} - \frac{1}{2e^4} \right) \text{units}^3$$



12) $y = x$ $y = \sqrt{x}$; about $x = 2$

$$y = x \quad y = \sqrt{x}$$

$$x = y \quad x = y^2$$

$$y = y^2$$

$$0 = y^2 - y \Rightarrow y = 0 \quad y - 1 = 0$$

$$0 = y(y - 1) \quad y = 1$$

interval: $0 \leq y \leq 1$

$$r_o = 2 - (y^2) \quad r_i = 2 - (y)$$

$$= (2 - y^2) \quad = (2 - y)$$

$$A_o = \pi(2 - y^2)^2$$

$$A_i = \pi(2 - y)^2$$

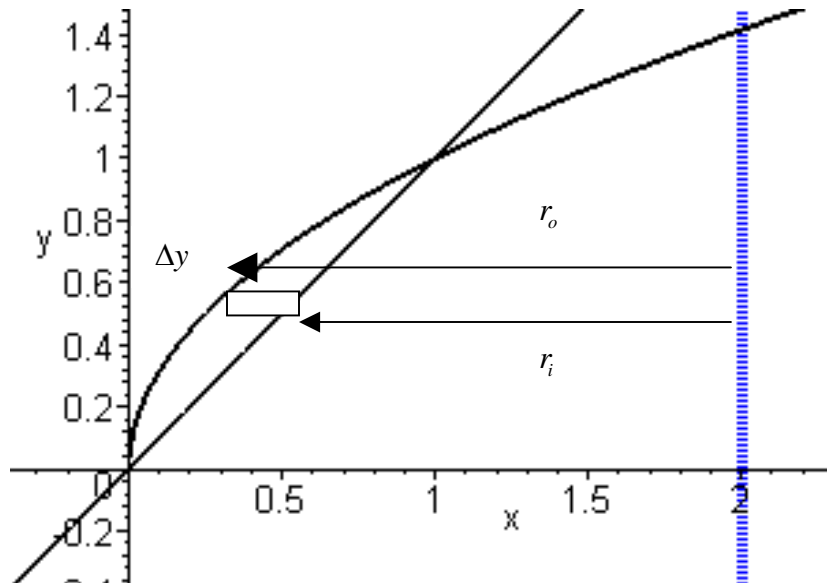
$$= \pi(4 - 4y^2 + y^4) \quad = \pi(4 - 4y + y^2)$$

$$A = A_o - A_i = \pi(4 - 4y^2 + y^4) - \pi(4 - 4y + y^2) = \pi(y^4 - 5y^2 + 4y)$$

$$\Delta V = A \Delta y = \pi(y^4 - 5y^2 + 4y) \Delta y$$

$$V = \int_0^1 \pi(y^4 - 5y^2 + 4y) dy = \pi \left[\frac{1}{5}y^5 - \frac{5}{3}y^3 + 2y^2 + C \right]_0^1$$

$$= \pi \left\{ \left[\frac{1}{5}(1)^5 - \frac{5}{3}(1)^3 + 2(1)^2 + C \right] - \left[\frac{1}{5}(0)^5 - \frac{5}{3}(0)^3 + 2(0)^2 + C \right] \right\} = \pi \left\{ \left[\frac{1}{5} - \frac{5}{3} + 2 \right] - [0] \right\} = \frac{8\pi}{15} \text{units}^3$$



14) $x = 2y - y^2$ $x = 0$; about y-axis

$$\begin{aligned} 0 &= 2y - y^2 & \Rightarrow & y = 0 & 2 - y &= 0 \\ 0 &= y(2 - y) & & & & y = 2 \end{aligned}$$

interval: $0 \leq y \leq 2$

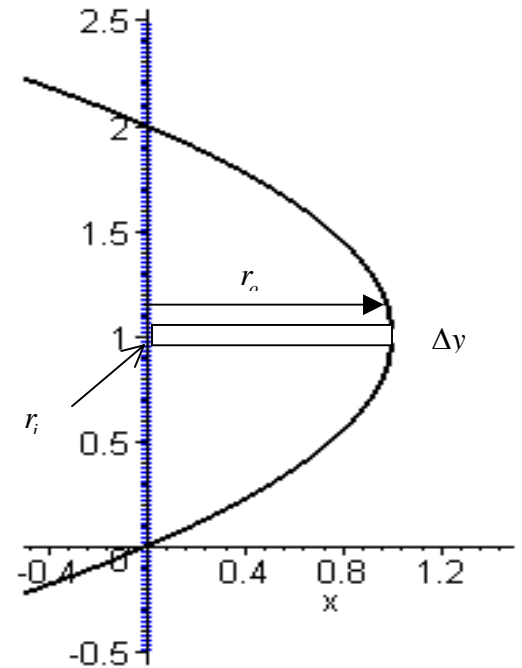
$$\begin{aligned} r_o &= (2y - y^2) + 0 & r_i &= (0) + 0 \\ &= (2y - y^2) & &= (0) \end{aligned}$$

$$\begin{aligned} A_o &= \pi(2y - y^2)^2 & A_i &= \pi(0)^2 \\ &= \pi(4y^2 - 4y^3 + y^4) & &= \pi(0) \end{aligned}$$

$$\begin{aligned} A &= A_o - A_i = \pi(4y^2 - 4y^3 + y^4) - \pi(0) \\ &= \pi(4y^2 - 4y^3 + y^4) \end{aligned}$$

$$\Delta V = A \Delta y = \pi(4y^2 - 4y^3 + y^4) \Delta y$$

$$\begin{aligned} V &= \int_0^2 \pi(4y^2 - 4y^3 + y^4) dy = \pi \left[\frac{4}{3}y^3 - y^4 + \frac{1}{5}y^5 + C \right]_0^2 \\ &= \pi \left\{ \left[\frac{4}{3}(2)^3 - (2)^4 + \frac{1}{5}(2)^5 + C \right] - \left[\frac{4}{3}(0)^3 - (0)^4 + \frac{1}{5}(0)^5 + C \right] \right\} \\ &= \pi \left\{ \left[\frac{2}{3}(2)^4 - (2)^4 + \frac{2}{5}(2)^4 \right] - [0] \right\} = \pi(2)^4 \left\{ \frac{2}{3} - 1 + \frac{2}{5} \right\} = \pi(2)^4 \left\{ \frac{10}{15} - \frac{15}{15} + \frac{6}{15} \right\} = \pi(2)^4 \left\{ \frac{1}{15} \right\} = \frac{16\pi}{15} \text{ units}^3 \end{aligned}$$



16) $x = y^2$ $x = 1$; about $x = 1$

$$\begin{aligned} 1 &= y^2 \\ 0 &= y^2 - 1 & \Rightarrow & y + 1 = 0 & y - 1 = 0 \\ 0 &= (y + 1)(y - 1) & & y = -1 & y = 1 \end{aligned}$$

interval: $-1 \leq y \leq 1$

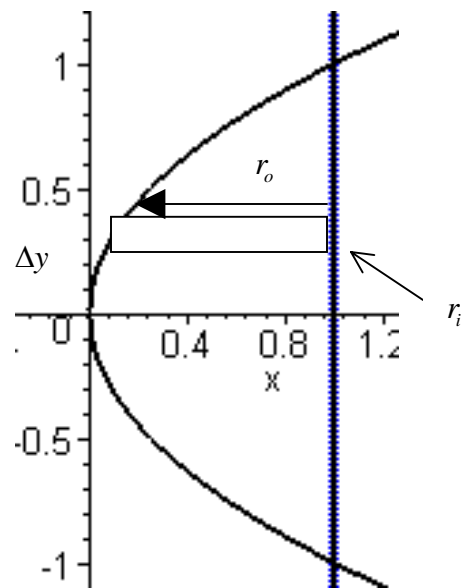
$$r_o = 1 - (y^2) = (1 - y^2) \quad r_i = 1 - (1) = (0)$$

$$A_o = \pi(1 - y^2)^2 = \pi(1 - 2y^2 + y^4) \quad A_i = \pi(0)^2 = \pi(0)$$

$$A = A_o - A_i = \pi(1 - 2y^2 + y^4) - \pi(0) = \pi(1 - 2y^2 + y^4)$$

$$\Delta V = A \Delta y = \pi(1 - 2y^2 + y^4) \Delta y$$

$$\begin{aligned} V &= \int_{-1}^1 \pi(1 - 2y^2 + y^4) dy = \pi \left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5 + C \right]_{-1}^1 \\ &= \pi \left\{ \left[(1) - \frac{2}{3}(1)^3 + \frac{1}{5}(1)^5 + C \right] - \left[(-1) - \frac{2}{3}(-1)^3 + \frac{1}{5}(-1)^5 + C \right] \right\} = \pi \left\{ \left[1 - \frac{2}{3} + \frac{1}{5} \right] - \left[-1 + \frac{2}{3} - \frac{1}{5} \right] \right\} \\ &= \pi \left\{ 2 - \frac{4}{3} + \frac{2}{5} \right\} = \pi \left\{ \frac{30}{15} - \frac{20}{15} + \frac{6}{15} \right\} = \frac{16\pi}{15} \text{ units}^3 \end{aligned}$$



18) $y = x^3$ $y = \sqrt{x}$; about $y = 1$

interval: $0 \leq x \leq 1$

$$r_o = 1 - (x^3) = (1 - x^3) \quad r_i = 1 - (\sqrt{x}) = (1 - \sqrt{x})$$

$$A_o = \pi(1 - x^3)^2 \quad A_i = \pi(1 - \sqrt{x})^2$$

$$= \pi(1 - 2x^3 + x^6) \quad = \pi(1 - 2\sqrt{x} + x)$$

$$A = A_o - A_i = \pi(1 - 2x^3 + x^6) - \pi(1 - 2\sqrt{x} + x)$$

$$= \pi(x^6 - 2x^3 - x + 2\sqrt{x})$$

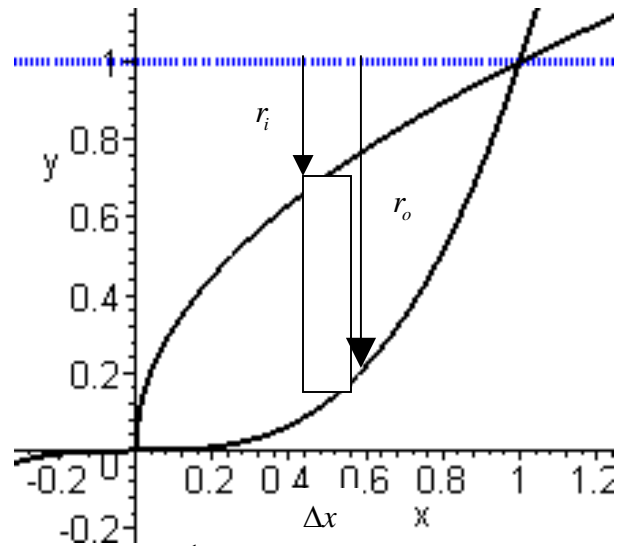
$$\Delta V = A\Delta x$$

$$= \pi(x^6 - 2x^3 - x + 2\sqrt{x})\Delta x$$

$$V = \int_0^1 \pi(x^6 - 2x^3 - x + 2\sqrt{x}) dx = \pi \left[\frac{1}{7}x^7 - \frac{1}{2}x^4 - \frac{1}{2}x^2 + \frac{4}{3}(\sqrt{x})^3 + C \right]_0^1$$

$$= \pi \left\{ \left[\frac{1}{7}(1)^7 - \frac{1}{2}(1)^4 - \frac{1}{2}(1)^2 + \frac{4}{3}(\sqrt{1})^3 + C \right] - \left[\frac{1}{7}(0)^7 - \frac{1}{2}(0)^4 - \frac{1}{2}(0)^2 + \frac{4}{3}(\sqrt{0})^3 + C \right] \right\}$$

$$= \pi \left\{ \left[\frac{1}{7} - \frac{1}{2} - \frac{1}{2} + \frac{4}{3} \right] - [0] \right\} = \pi \left\{ \frac{1}{7} - 1 + \frac{4}{3} \right\} = \pi \left\{ \frac{1}{7} + \frac{1}{3} \right\} = \pi \left\{ \frac{3}{21} + \frac{7}{21} \right\} = \frac{10\pi}{21} \text{ units}^3$$



20) $y = 0$ $y = \cos^2 x$ $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

a) about x -axis

interval: $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$r_o = (\cos^2 x) + 0 \quad r_i = (0) + 0$$

$$= (\cos^2 x) \quad = (0)$$

$$A_o = \pi(\cos^2 x)^2 \quad A_i = \pi(0)^2$$

$$= \pi(\cos^4 x) \quad = \pi(0)$$

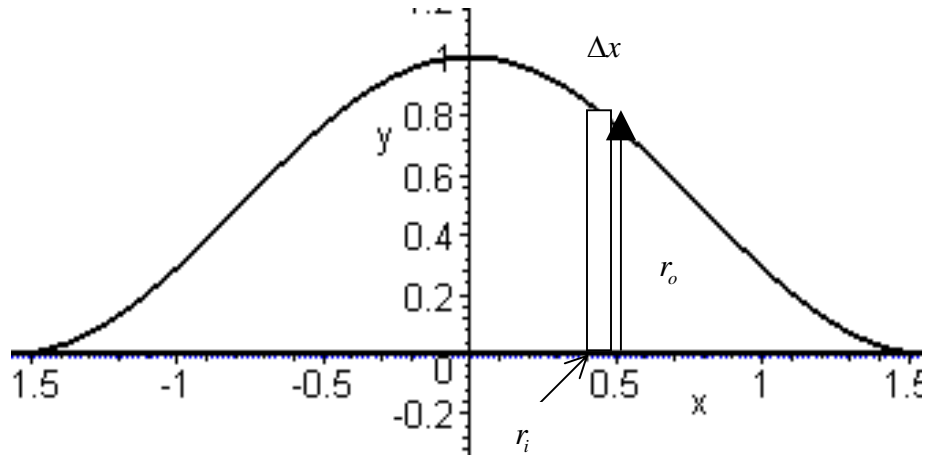
$$A = A_o - A_i = \pi(\cos^4 x) - \pi(0) = \pi(\cos^4 x)$$

$$\Delta V = A\Delta x = \pi(\cos^4 x)\Delta x = \pi(\cos^2 x)^2 \Delta x$$

$$= \pi \left(\frac{1}{2}(1 + \cos(2x)) \right)^2 \Delta x$$

$$= \frac{\pi}{4} (1 + 2\cos(2x) + \cos^2(2x))\Delta x = \frac{\pi}{4} \left(1 + 2\cos(2x) + \left(\frac{1}{2}(1 + \cos(4x)) \right) \right) \Delta x$$

$$= \frac{\pi}{4} \left(\frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x) \right) \Delta x$$



$$\begin{aligned}
 V &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\pi}{4} \left(\frac{3}{2} + 2 \cos(2x) + \frac{1}{2} \cos(4x) \right) dx = \frac{\pi}{4} \left[\frac{3}{2}x + \sin(2x) + \frac{1}{8} \sin(4x) + C \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{\pi}{4} \left\{ \left[\frac{3}{2} \left(\frac{\pi}{2} \right) + \sin \left(2 \left(\frac{\pi}{2} \right) \right) + \frac{1}{8} \sin \left(4 \left(\frac{\pi}{2} \right) \right) + C \right] - \left[\frac{3}{2} \left(\frac{-\pi}{2} \right) + \sin \left(2 \left(\frac{-\pi}{2} \right) \right) + \frac{1}{8} \sin \left(4 \left(\frac{-\pi}{2} \right) \right) + C \right] \right\} \\
 &= \frac{\pi}{4} \left\{ \left[\frac{3\pi}{4} \right] - \left[\frac{-3\pi}{4} \right] \right\} = \frac{\pi}{4} \left\{ \frac{6\pi}{4} \right\} = \frac{\pi}{4} \left\{ \frac{3\pi}{2} \right\} = \frac{3\pi^2}{8} \text{ units}^3
 \end{aligned}$$

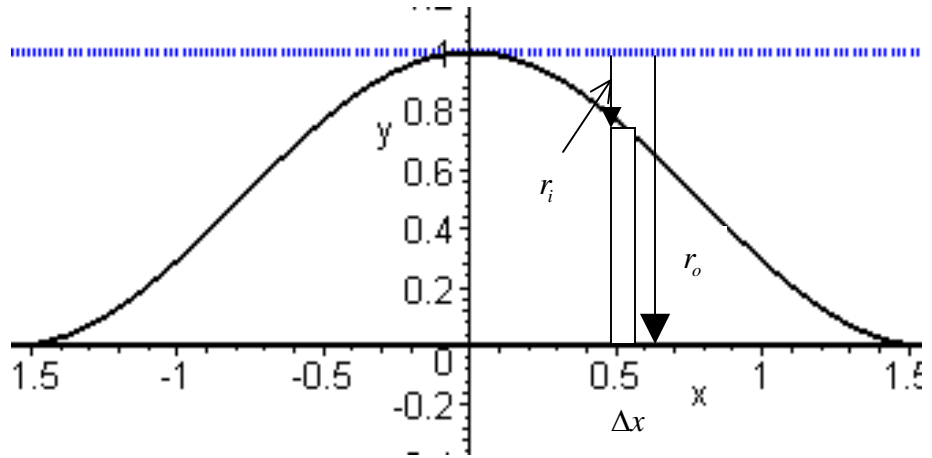
b) about $y = 1$

interval: $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$\begin{aligned}
 r_i &= 1 - (\cos^2 x) \\
 r_o &= 1 - (0) \\
 &= (1) \\
 &= \sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 A_o &= \pi(1)^2 & A_i &= \pi(\sin^2 x)^2 \\
 &= \pi(1) & &= \pi(\sin^4 x)
 \end{aligned}$$

$$\begin{aligned}
 A &= A_o - A_i = \pi(1) - \pi(\sin^4 x) \\
 &= \pi(1 - \sin^4 x)
 \end{aligned}$$



$$\begin{aligned}
 \Delta V &= A \Delta x = \pi(1 - \sin^4 x) \Delta x = \pi(1 - (\sin^2 x)^2) \Delta x = \pi \left(1 - \left(\frac{1}{2}(1 - \cos(2x)) \right)^2 \right) \Delta x \\
 &= \pi \left(1 - \frac{1}{4}(1 - 2 \cos(2x) + \cos^2(2x)) \right) \Delta x = \pi \left(\frac{3}{4} + \frac{1}{2} \cos(2x) - \frac{1}{4} \cos^2(2x) \right) \Delta x \\
 &= \pi \left(\frac{3}{4} + \frac{1}{2} \cos(2x) - \frac{1}{4} \left(\frac{1}{2}(1 + \cos(4x)) \right) \right) \Delta x = \pi \left(\frac{5}{8} + \frac{1}{2} \cos(2x) - \frac{1}{8} \cos(4x) \right) \Delta x \\
 &= \frac{\pi}{8} (5 + 4 \cos(2x) - \cos(4x)) \Delta x
 \end{aligned}$$

$$\begin{aligned}
 V &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\pi}{8} (5 + 4 \cos(2x) - \cos(4x)) dx = \frac{\pi}{8} \left[5x + 2 \sin(2x) - \frac{1}{4} \sin(4x) + C \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
 &= \frac{\pi}{8} \left\{ \left[5 \left(\frac{\pi}{2} \right) + 2 \sin \left(2 \left(\frac{\pi}{2} \right) \right) - \frac{1}{4} \sin \left(4 \left(\frac{\pi}{2} \right) \right) + C \right] - \left[5 \left(\frac{-\pi}{2} \right) + 2 \sin \left(2 \left(\frac{-\pi}{2} \right) \right) - \frac{1}{4} \sin \left(4 \left(\frac{-\pi}{2} \right) \right) + C \right] \right\} \\
 &= \frac{\pi}{8} \left\{ \left[\frac{5\pi}{2} \right] - \left[\frac{-5\pi}{2} \right] \right\} = \frac{\pi}{8} \left\{ \frac{10\pi}{2} \right\} = \frac{\pi}{8} \{ 5\pi \} = \frac{5\pi^2}{8} \text{ units}^3
 \end{aligned}$$

22) $y = x^2 \quad x^2 + y^2 = 1 \quad y \geq 0$

intersection points:

$$\begin{aligned}
 x^2 + (x^2)^2 &= 1 \\
 x^4 + x^2 - 1 &= 0 \\
 x^2 &= \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2} \\
 x^2 &= \frac{-1 - \sqrt{5}}{2} & x^2 &= \frac{-1 + \sqrt{5}}{2} \\
 \Rightarrow x &= \pm \sqrt{\frac{-1 - \sqrt{5}}{2}} & x &= \pm \sqrt{\frac{-1 + \sqrt{5}}{2}} \\
 \text{discard} & & x &= -\sqrt{\frac{\sqrt{5} - 1}{2}} \quad x = +\sqrt{\frac{\sqrt{5} - 1}{2}}
 \end{aligned}$$

a) about x -axis

interval: $-\sqrt{\frac{\sqrt{5}-1}{2}} \leq x \leq +\sqrt{\frac{\sqrt{5}-1}{2}}$

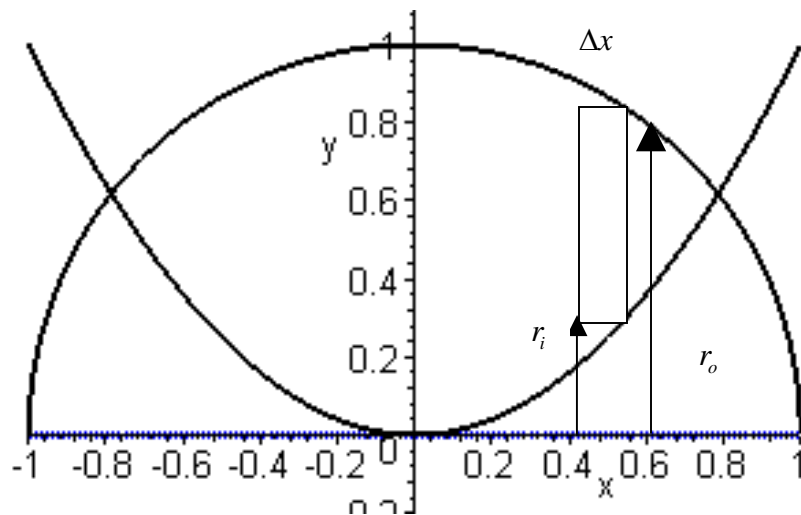
$y = x^2 \quad x^2 + y^2 = 1 \quad y \geq 0$
 $y = \sqrt{1-x^2}$

$r_o = (\sqrt{1-x^2}) + 0 \quad r_i = (x^2) + 0$
 $= (\sqrt{1-x^2}) \quad = (x^2)$

$A_o = \pi(\sqrt{1-x^2})^2 \quad A_i = \pi(x^2)^2$
 $= \pi(1-x^2) \quad = \pi(x^4)$

$A = A_o - A_i = \pi(1-x^2) - \pi(x^4) = \pi(1-x^2-x^4)$

$\Delta V = A \Delta x = \pi(1-x^2-x^4) \Delta x$



$$\begin{aligned}
 V &= \int_{-\sqrt{\frac{\sqrt{5}-1}{2}}}^{+\sqrt{\frac{\sqrt{5}-1}{2}}} \pi(1-x^2-x^4) dx = \pi \left[x - \frac{1}{3}x^3 - \frac{1}{5}x^5 + C \right]_{-\sqrt{\frac{\sqrt{5}-1}{2}}}^{+\sqrt{\frac{\sqrt{5}-1}{2}}} \\
 &= \pi \left\{ \left[\left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) - \frac{1}{3} \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right)^3 - \frac{1}{5} \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right)^5 + C \right] - \left[\left(-\sqrt{\frac{\sqrt{5}-1}{2}} \right) - \frac{1}{3} \left(-\sqrt{\frac{\sqrt{5}-1}{2}} \right)^3 - \frac{1}{5} \left(-\sqrt{\frac{\sqrt{5}-1}{2}} \right)^5 + C \right] \right\} \\
 &= \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ \left[1 - \frac{1}{3} \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right)^2 - \frac{1}{5} \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right)^4 \right] - \left[-1 + \frac{1}{3} \left(-\sqrt{\frac{\sqrt{5}-1}{2}} \right)^2 + \frac{1}{5} \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right)^4 \right] \right\} \\
 &= \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ 2 - \frac{2}{3} \left(\frac{\sqrt{5}-1}{2} \right) - \frac{2}{5} \left(\frac{\sqrt{5}-1}{2} \right)^2 \right\} = \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ 2 - \frac{\sqrt{5}-1}{3} - \frac{(5-2\sqrt{5}+1)}{10} \right\} \\
 &= \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ 2 - \frac{\sqrt{5}-1}{3} - \frac{3-\sqrt{5}}{5} \right\} = \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ \frac{30}{15} - \frac{5\sqrt{5}-5}{15} - \frac{9-3\sqrt{5}}{15} \right\} = \pi \left(\sqrt{\frac{\sqrt{5}-1}{2}} \right) \left\{ \frac{26-2\sqrt{5}}{15} \right\} \text{units}^3
 \end{aligned}$$

- b) about y-axis
 This volume must be computed in 2 pieces.

$$y = \left(+\sqrt{\frac{\sqrt{5}-1}{2}} \right)^2 = \frac{\sqrt{5}-1}{2}$$

interval 1: $0 \leq y \leq \frac{\sqrt{5}-1}{2}$

$$y = x^2 \quad x^2 + y^2 = 1$$

$$x = \sqrt{y} \quad x = \sqrt{1-y^2} \quad y \geq 0$$

$$r_{1o} = (\sqrt{y}) + 0 \quad r_{1i} = (0) + 0$$

$$= (\sqrt{y}) \quad = (0)$$

$$A_{1o} = \pi(\sqrt{y})^2 = \pi(y) \quad A_{1i} = \pi(0)^2 = \pi(0)$$

$$A_1 = A_{1o} - A_{1i} = \pi(y) - \pi(0) = \pi(y)$$

$$\Delta V_1 = A_1 \Delta y = \pi(y) \Delta y$$

$$V_1 = \int_0^{\frac{\sqrt{5}-1}{2}} \pi(y) dy = \pi \left[\frac{1}{2} y^2 + C \right]_0^{\frac{\sqrt{5}-1}{2}} = \pi \left\{ \left[\frac{1}{2} \left(\frac{\sqrt{5}-1}{2} \right)^2 + C \right] - \left[\frac{1}{2} (0)^2 + C \right] \right\} = \frac{\pi}{2} \left\{ \frac{5-2\sqrt{5}+1}{4} \right\}$$

$$= \frac{\pi}{2} \left\{ \frac{6-2\sqrt{5}}{4} \right\} = \pi \left(\frac{3-\sqrt{5}}{4} \right) \text{units}^3$$

interval 2: $\frac{\sqrt{5}-1}{2} \leq y \leq 1$

$$r_{2o} = (\sqrt{1-y^2}) + 0 = (\sqrt{1-y^2}) \quad r_{2i} = (0) + 0 = (0)$$

$$A_{2o} = \pi(\sqrt{1-y^2})^2 = \pi(1-y^2) \quad A_{2i} = \pi(0)^2 = \pi(0)$$

$$A_2 = A_{2o} - A_{2i} = \pi(1-y^2) - \pi(0) = \pi(1-y^2)$$

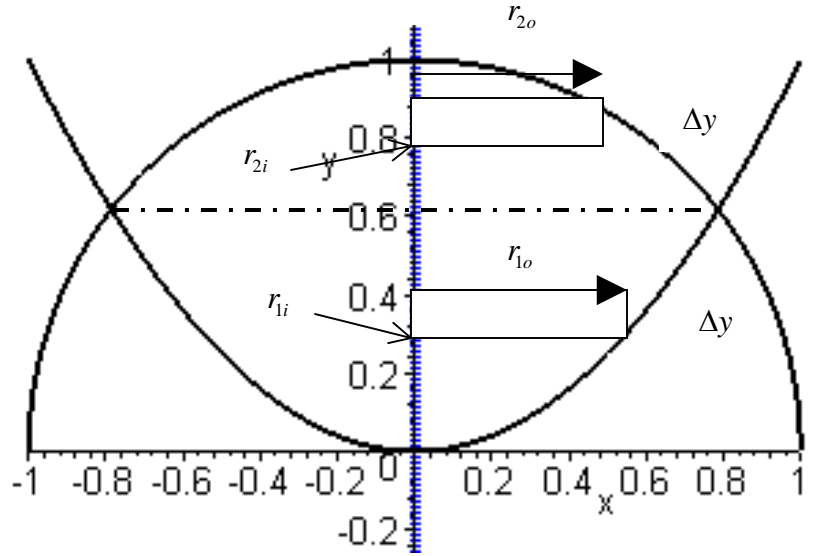
$$\Delta V_2 = A_2 \Delta y = \pi(1-y^2) \Delta y$$

$$V_2 = \int_{\frac{\sqrt{5}-1}{2}}^1 \pi(1-y^2) dy = \pi \left[y - \frac{1}{3} y^3 + C \right]_{\frac{\sqrt{5}-1}{2}}^1 = \pi \left\{ \left[(1) - \frac{1}{3} (1)^3 + C \right] - \left[\left(\frac{\sqrt{5}-1}{2} \right) - \frac{1}{3} \left(\frac{\sqrt{5}-1}{2} \right)^3 + C \right] \right\}$$

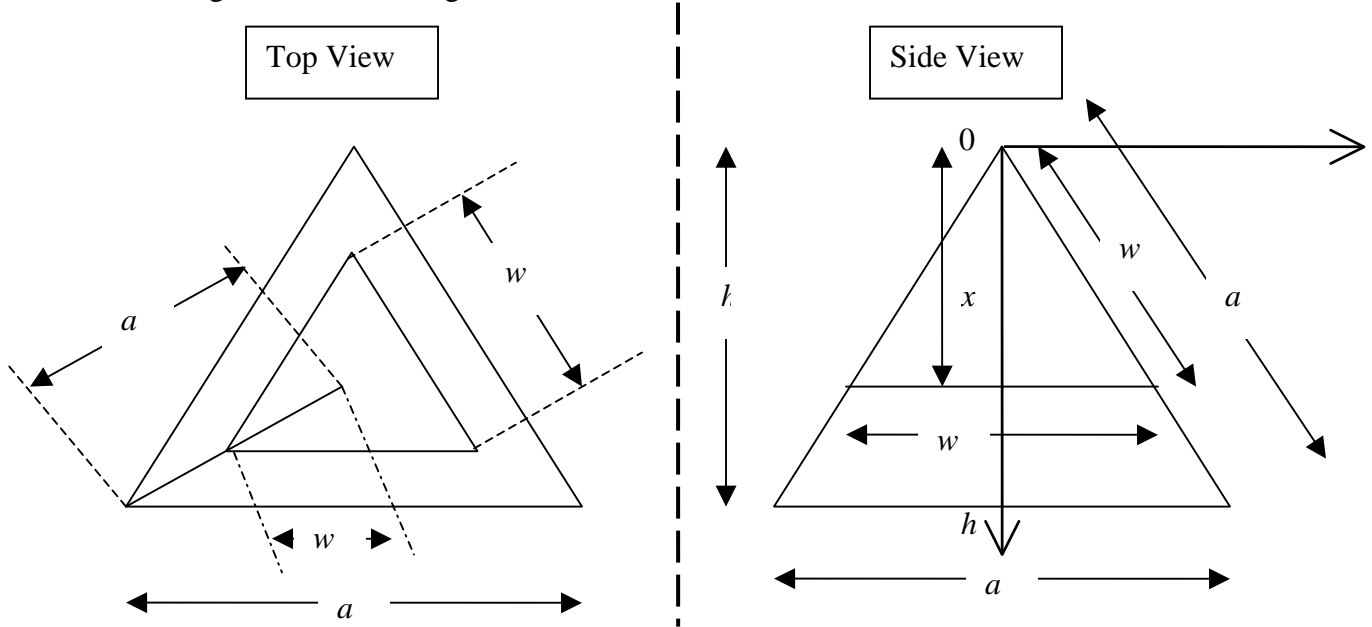
$$= \pi \left\{ \left[1 - \frac{1}{3} \right] - \left(\frac{\sqrt{5}-1}{2} \right) \left[1 - \frac{1}{3} \left(\frac{\sqrt{5}-1}{2} \right)^2 \right] \right\} = \pi \left\{ \left[\frac{2}{3} \right] - \left(\frac{\sqrt{5}-1}{2} \right) \left[1 - \frac{1}{3} \left(\frac{6-2\sqrt{5}}{4} \right) \right] \right\}$$

$$= \pi \left\{ \left[\frac{2}{3} \right] - \left(\frac{\sqrt{5}-1}{2} \right) \left[\frac{6}{6} - \frac{3-\sqrt{5}}{6} \right] \right\} = \pi \left\{ \left[\frac{2}{3} \right] - \left(\frac{\sqrt{5}-1}{2} \right) \left[\frac{3+\sqrt{5}}{6} \right] \right\} = \pi \left\{ \left[\frac{2}{3} \right] - \left(\frac{1+\sqrt{5}}{6} \right) \right\} = \pi \left(\frac{3-\sqrt{5}}{6} \right) \text{units}^3$$

$$V = V_1 + V_2 = \pi \left(\frac{3-\sqrt{5}}{4} \right) + \pi \left(\frac{3-\sqrt{5}}{6} \right) = \pi \left\{ \left(\frac{9-3\sqrt{5}}{12} \right) + \left(\frac{6-2\sqrt{5}}{12} \right) \right\} = \pi \left(\frac{15-5\sqrt{5}}{12} \right) \text{units}^3$$



- 36) For the 3D representation, see the figure in the text. Also, setting the positive x -axis to go down we have the following cross section diagrams.



From the side view we can set up a ratio to obtain:

$$\frac{w}{a} = \frac{x}{h} \Rightarrow w = \frac{a}{h}x$$

From Top View, we can see that we need to set up an area of triangle.

$$\frac{b}{w} = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Rightarrow b = \frac{\sqrt{3}}{2}w$$

$$A_v = \frac{1}{2}bw = \frac{1}{2}\left(\frac{\sqrt{3}}{2}w\right)w = \frac{\sqrt{3}}{4}w^2 = \frac{\sqrt{3}}{4}\left(\frac{a}{h}x\right)^2 = \frac{\sqrt{3}a^2}{4h^2}x^2$$

$$\Delta V = A_v \Delta x = \left(\frac{\sqrt{3}a^2}{4h^2}x^2\right)\Delta x$$

$$V = \int_0^h \left(\frac{\sqrt{3}a^2}{4h^2}x^2\right) dx = \left[\frac{\sqrt{3}a^2}{12h^2}x^3 + C\right]_0^h = \left[\frac{\sqrt{3}a^2}{12h^2}(h)^3 + C\right] - \left[\frac{\sqrt{3}a^2}{12h^2}(0)^3 + C\right]$$

$$= \left[\frac{\sqrt{3}a^2h}{12}\right] - [0] = \frac{a^2h}{4\sqrt{3}} \text{ units}^3$$

