Additional examples:

Trigonometric Integration:

4) $\int_0^{\frac{\pi}{2}} \sin^5 x\, dx$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sin^5 x\, dx &= \left[-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C\right]_0^{\frac{\pi}{2}} \\
&= \left[-\cos\left(\frac{\pi}{2}\right) + \frac{2}{3}\cos^3\left(\frac{\pi}{2}\right) - \frac{1}{5}\cos^5\left(\frac{\pi}{2}\right) + C\right] - \left[-\cos(0) + \frac{2}{3}\cos^3(0) - \frac{1}{5}\cos^5(0) + C\right] \\
&= \left[-(0) + \frac{2}{3}(0)^3 - \frac{1}{5}(0)^5\right] - \left[-(1) + \frac{2}{3}(1)^3 - \frac{1}{5}(1)^5\right] \\
&= 1 - \frac{2}{3} + \frac{1}{5} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}
\end{aligned}$$

$$\begin{aligned}
\int \sin^5 x\, dx &= \int (\sin^2 x)^2 (\sin x\, dx) = \int (1-\cos^2 x)^2 (\sin x\, dx) \\
&= \int (1 - 2\cos^2 x + \cos^4 x)(\sin x\, dx) \\
&= \int (1 - 2p^2 + p^4)(-1\, dx) \\
&= -\left(p - \frac{2}{3}p^3 + \frac{1}{5}p^5\right) + C \\
&= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C
\end{aligned}$$

$$\begin{aligned}
p &= \cos x \\
dp &= -\sin x\, dx \\
-1\, dp &= \sin x\, dx
\end{aligned}$$

6) $\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$

$$\begin{aligned}
\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta &= \left[\frac{1}{2}\theta - \frac{3}{4}\sin\left(\frac{2}{3}\theta\right) + C\right]_0^{2\pi} \\
&= \left[\frac{1}{2}(2\pi) - \frac{3}{4}\sin\left(\frac{2}{3}(2\pi)\right) + C\right] - \left[\frac{1}{2}(0) - \frac{3}{4}\sin\left(\frac{2}{3}(0)\right) + C\right] \\
&= \left[\pi - \frac{3}{4}\sin\left(\frac{4\pi}{3}\right)\right] - \left[(0) - \frac{3}{4}(0)\right] = \pi - \frac{3}{4}(0) = \pi
\end{aligned}$$

$$\begin{aligned}
\int \sin^2\left(\frac{1}{3}\theta\right) d\theta &= \int \frac{1}{2}\left(1 - \cos 2\left(\frac{1}{3}\theta\right)\right) d\theta = \int \frac{1}{2}\left(1 - \cos\left(\frac{2}{3}\theta\right)\right) d\theta = \frac{1}{2}\left(\theta - \frac{3}{2}\sin\left(\frac{2}{3}\theta\right)\right) + C \\
&= \frac{1}{2}\theta - \frac{3}{4}\sin\left(\frac{2}{3}\theta\right) + C
\end{aligned}$$

12) $\int x\cos^2 x\,dx$

$$\int x\cos^2 x\,dx = \int x\left(\frac{1}{2}(1+\cos(2x))\right)dx = \int\left(\frac{1}{2}x+\frac{1}{2}x\cos(2x)\right)dx = \int\frac{1}{2}x\,dx+\int\frac{1}{2}x\cos(2x)\,dx$$

$$=\frac{1}{2}\left(\frac{x^2}{2}\right)+\left\{\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)+C\right\}=\frac{1}{4}x^2+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)+C$$

$$\int\frac{1}{2}x\cos(2x)\,dx=\left(\frac{1}{2}x\right)\left(\frac{1}{2}\sin(2x)\right)-\int\left(\frac{1}{2}\sin(2x)\right)\left(\frac{1}{2}\,dx\right)$$

$$=\frac{1}{4}x\sin(2x)-\int\frac{1}{4}\sin(2x)\,dx$$

$$=\frac{1}{4}x\sin(2x)-\frac{1}{4}\left(\frac{-1}{2}\cos(2x)\right)+C$$

$$=\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)+C$$

$u_1=\frac{1}{2}x \qquad dv_1=\cos(2x)\,dx$

$du_1=\frac{1}{2}\,dx \qquad v_1=\frac{1}{2}\sin(2x)$

16) $\int\cos^2 x\sin(2x)\,dx$

$$\int\cos^2 x\sin(2x)\,dx=\int\cos^2 x(2\sin x\cos x)\,dx$$

$$=\int 2\cos^3 x\sin x\,dx$$

$$=\int 2p^3(-1\,dp)$$

$$=-2\left(\frac{p^4}{4}\right)+C=\frac{-1}{2}\cos^4 x+C$$

$p=\cos x$

$dp=-\sin x\,dx$

$-1\,dp=\sin x\,dx$

20) $\int(\tan^2 x+\tan^4 x)\,dx$

$$\int(\tan^2 x+\tan^4 x)\,dx=\int\tan^2 x(1+\tan^2 x)\,dx$$

$$=\int\tan^2 x\sec^2 x\,dx$$

$$=\int\tan^2 x(\sec^2 x\,dx)$$

$$=\int p^2\,(dp)$$

$$=\frac{1}{3}p^3+C$$

$$=\frac{1}{3}\tan^3 x+C$$

$p=\tan x$

$dp=\sec^2 x\,dx$

22) $\int_0^{\frac{\pi}{4}} \sec^4\theta \tan^4\theta \, d\theta$

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} \sec^4\theta \tan^4\theta \, d\theta &= \left[\frac{1}{5}\tan^5\theta + \frac{1}{7}\tan^7\theta + C\right]_0^{\frac{\pi}{4}} \\
&= \left[\frac{1}{5}\tan^5\left(\frac{\pi}{4}\right) + \frac{1}{7}\tan^7\left(\frac{\pi}{4}\right) + C\right] - \left[\frac{1}{5}\tan^5(0) + \frac{1}{7}\tan^7(0) + C\right] \\
&= \left[\frac{1}{5}(1)^5 + \frac{1}{7}(1)^7\right] - \left[\frac{1}{5}(0) + \frac{1}{7}(0)\right] = \frac{1}{5} + \frac{1}{7} = \frac{12}{35}
\end{aligned}$$

$$\begin{aligned}
\int \sec^4\theta \tan^4\theta \, d\theta &= \int \sec^2\theta \tan^4\theta(\sec^2\theta \, d\theta) = \int (1+\tan^2\theta)\tan^4\theta(\sec^2\theta \, d\theta) \\
&= \int (\tan^4\theta + \tan^6\theta)(\sec^2\theta \, d\theta) \\
&= \int (p^4 + p^6)(dp) \\
&= \frac{1}{5}p^5 + \frac{1}{7}p^7 + C = \frac{1}{5}\tan^5\theta + \frac{1}{7}\tan^7\theta + C
\end{aligned}$$

$p = \tan\theta$

$dp = \sec^2\theta \, d\theta$

26) $\int_0^{\frac{\pi}{4}} \tan^4 t \, dt$

$$\begin{aligned}
\int_0^{\frac{\pi}{4}} \tan^4 t \, dt &= \left[\frac{1}{3}\tan^3 t - \tan t + t + C\right]_0^{\frac{\pi}{4}} \\
&= \left[\frac{1}{3}\tan^3\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right) + C\right] - \left[\frac{1}{3}\tan^3(0) - \tan(0) + (0) + C\right] \\
&= \left[\frac{1}{3}(1)^3 - (1) + \frac{\pi}{4}\right] - \left[\frac{1}{3}(0) - (0) + (0)\right] = \frac{\pi}{4} - \frac{2}{3} = \frac{3\pi - 8}{12}
\end{aligned}$$

$$\begin{aligned}
\int \tan^4 t \, dt &= \int (\tan^2 t)(\tan^2 t) \, dt \\
&= \int (\tan^2 t)(\sec^2 t - 1) \, dt \\
&= \int (\sec^2 t \tan^2 t - \tan^2 t) \, dt \\
&= \int \tan^2 t(\sec^2 t \, dt) - \int \tan^2 t \, dt \\
&= \int \tan^2 t(\sec^2 t \, dt) - \int (\sec^2 t - 1) \, dt \\
&= \int \tan^2 t(\sec^2 t \, dt) - \int \sec^2 t \, dt + \int 1 \, dt \\
&= \frac{1}{3}\tan^3 t - \tan t + t + C
\end{aligned}$$

$$\begin{aligned}
\int \tan^2 t(\sec^2 t \, dt) &= \int p^2 \, (dp) \\
&= \frac{1}{3}p^3 + C \\
&= \frac{1}{3}\tan^3 t + C
\end{aligned}$$

$p = \tan t$

$dp = \sec^2 t \, dt$

30) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^3 x\,dx$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^3 x\,dx = \left[\frac{-1}{2}\cot^2 x - \ln\left|\sin x\right| + C\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{-1}{2}\cot^2\left(\frac{\pi}{2}\right) - \ln\left|\sin\left(\frac{\pi}{2}\right)\right| + C\right] - \left[\frac{-1}{2}\cot^2\left(\frac{\pi}{4}\right) - \ln\left|\sin\left(\frac{\pi}{4}\right)\right| + C\right]$$

$$= \left[\frac{-1}{2}(0)^2 - \ln\left|(1)\right|\right] - \left[\frac{-1}{2}(1)^2 - \ln\left|\frac{1}{\sqrt{2}}\right|\right] = \frac{1}{2} + \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} + \ln 1 - \ln\sqrt{2} = \frac{1}{2} - \frac{1}{2}\ln 2$$

$$\begin{aligned}\int \cot^3 x\,dx &= \int (\cot^2 x)(\cot x)\,dx\\ &= \int (\csc^2 x - 1)(\cot x)\,dx\\ &= \int (\csc^2 x\cot x - \cot x)\,dx\\ &= \int \cot x(\csc^2 x\,dx) - \int \cot x\,dx\\ &= \frac{-1}{2}\cot^2 x - \ln\left|\sin x\right| + C\end{aligned}$$

$p = \cot x$

$dp = -\csc^2 x\,dx$

$-1\,dp = \csc^2 x\,dx$

$$\begin{aligned}\int \cot x(\csc^2 x\,dx) &= \int p(-1\,dp)\\ &= \frac{-1}{2}p^2 + C\\ &= \frac{-1}{2}\cot^2 x + C\end{aligned}$$

32) $\int \csc^4 x\cot^6 x\,dx$

$$\begin{aligned}\int \csc^4 x\cot^6 x\,dx &= \int \csc^2 x\cot^6 x(\csc^2 x\,dx) = \int (\cot^2 x + 1)\cot^6 x(\csc^2 x\,dx)\\ &= \int (\cot^8 x + \cot^6 x)(\csc^2 x\,dx)\\ &= \int (p^8 + p^6)(-1\,dp)\\ &= -\left(\frac{1}{9}p^9 + \frac{1}{7}p^7\right) + C = \frac{-1}{9}\cot^9 x - \frac{1}{7}\cot^7 x + C\end{aligned}$$

$p = \cot x$

$dp = -\csc^2 x\,dx$

$-1dp = \csc^2 x\,dx$

36) $\int \frac{dx}{\cos x - 1}$

$$\int \frac{dx}{\cos x - 1} = \int \left(\frac{1}{\cos x - 1}\right)\left(\frac{\cos x + 1}{\cos x + 1}\right)dx = \int \frac{\cos x + 1}{\cos^2 x - 1}\,dx = \int \frac{\cos x + 1}{-(1 - \cos^2 x)}\,dx = \int \frac{\cos x + 1}{-\sin^2 x}\,dx$$

$$= -\int \frac{\cos x}{\sin^2 x}\,dx - \int \frac{1}{\sin^2 x}\,dx = -\int \frac{1}{\sin^2 x}(\cos x\,dx) - \int \csc^2 x\,dx = -\left(\frac{-1}{\sin x}\right) - (-\cot x) + C$$

$$= \csc x + \cot x + C$$

$p = \sin x$

$dp = \cos x\,dx \longrightarrow$

$$\int \frac{1}{\sin^2 x}(\cos x\,dx) = \int \frac{1}{p^2}(dp) = \frac{-1}{p} + C = \frac{-1}{\sin x} + C$$

Trigonometric Substitution (triangulation):

44) $\int_0^2 x^3\sqrt{x^2+4}\,dx$

$$\int_0^2 x^3\sqrt{x^2+4}\,dx = \left[\frac{1}{5}\left(\sqrt{x^2+4}\right)^5 - \frac{4}{3}\left(\sqrt{x^2+4}\right)^3 + C\right]_0^2$$

$$= \left[\frac{1}{5}\left(\sqrt{(2)^2+4}\right)^5 - \frac{4}{3}\left(\sqrt{(2)^2+4}\right)^3 + C\right] - \left[\frac{1}{5}\left(\sqrt{(0)^2+4}\right)^5 - \frac{4}{3}\left(\sqrt{(0)^2+4}\right)^3 + C\right]$$

$$= \left[\frac{1}{5}\left(2\sqrt{2}\right)^5 - \frac{4}{3}\left(2\sqrt{2}\right)^3\right] - \left[\frac{1}{5}(2)^5 - \frac{4}{3}(2)^3\right] = \left(2\sqrt{2}\right)^3\left[\frac{1}{5}\left(2\sqrt{2}\right)^2 + \frac{4}{3}\right] - (2)^3\left[\frac{1}{5}(2)^2 + \frac{4}{3}\right]$$

$$= 16\sqrt{2}\left[\frac{8}{5} - \frac{4}{3}\right] - 8\left[\frac{4}{5} - \frac{4}{3}\right] = 16\sqrt{2}\left[\frac{24-20}{15}\right] - 8\left[\frac{12-20}{15}\right]$$

$$= 16\sqrt{2}\left[\frac{4}{15}\right] - 8\left[\frac{-4}{15}\right] = \frac{64\sqrt{2}-64}{15}$$

method 1:

$$\int x^3\sqrt{x^2+4}\,dx = \int x^2\sqrt{x^2+4}(x\,dx) = \int (p-4)\sqrt{p}\left(\frac{1}{2}\,dp\right)$$

$$= \int\left(\frac{1}{2}p^{\frac{3}{2}} - 2\sqrt{p}\right)dp$$

$$= \frac{1}{2}\left(\frac{2}{5}p^{\frac{5}{2}}\right) - 2\left(\frac{2}{3}p^{\frac{3}{2}}\right) + C$$

$$= \frac{1}{5}\left(x^2+4\right)^{\frac{5}{2}} - \frac{4}{3}\left(x^2+4\right)^{\frac{3}{2}} + C$$

$$= \frac{1}{5}\left(\sqrt{x^2+4}\right)^5 - \frac{4}{3}\left(\sqrt{x^2+4}\right)^3 + C$$

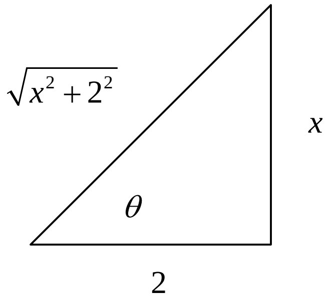
$p = x^2+4$

$dp = 2x\,dx$

$\frac{1}{2}\,dp = x\,dx$

$x^2 = (p-4)$

method 2:



$$\int x^3\sqrt{x^2+4}\,dx = \int (2\tan\theta)^3(2\sec\theta)(2\sec^2\theta\,d\theta)$$

$$= 2^5\int \sec^3\theta\tan^3\theta\,d\theta$$

$$= 2^5\int \sec^2\theta\tan^2\theta(\sec\theta\tan\theta\,d\theta)$$

$$= 2^5\int \sec^2\theta(\sec^2\theta-1)(\sec\theta\tan\theta\,d\theta)$$

$$= 2^5\int (\sec^4\theta-\sec^2\theta)(\sec\theta\tan\theta\,d\theta)$$

$$= 2^5\left(\frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta\right) + C$$

$$= 2^5\left(\frac{1}{5}\left(\frac{\sqrt{x^2+2^2}}{2}\right)^5 - \frac{1}{3}\left(\frac{\sqrt{x^2+2^2}}{2}\right)^3\right) + C$$

$$= \frac{1}{5}\left(\sqrt{x^2+2^2}\right)^5 - \frac{4}{3}\left(\sqrt{x^2+2^2}\right)^3 + C$$

$\frac{x}{2} = \tan\theta$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$\frac{\sqrt{x^2+2^2}}{2} = \sec\theta$

$\sqrt{x^2+2^2} = 2\sec\theta$

48) $\displaystyle\int \frac{t^5}{\sqrt{t^2+2}}\,dt$

method 1:

$$\int \frac{t^5}{\sqrt{t^2+2}}\,dt = \int \frac{(t^2)^2}{\sqrt{t^2+2}}(t\;dt) = \int \frac{(p-2)^2}{\sqrt{p}}\left(\frac{1}{2}\,dp\right) = \int \frac{(p^2-4p+4)}{2\sqrt{p}}\,dp$$

$$= \int \left(\frac{1}{2}p^{\frac{3}{2}} - 2p^{\frac{1}{2}} + 2p^{\frac{-1}{2}}\right)dp$$

$$= \frac{1}{2}\left(\frac{2}{5}p^{\frac{5}{2}}\right) - 2\left(\frac{2}{3}p^{\frac{3}{2}}\right) + 2\left(\frac{2}{1}p^{\frac{1}{2}}\right) + C$$

$$= \frac{1}{5}\left(t^2+2\right)^{\frac{5}{2}} - \frac{4}{3}\left(t^2+2\right)^{\frac{3}{2}} + 4\left(t^2+2\right)^{\frac{1}{2}} + C$$

$$= \frac{1}{5}\left(\sqrt{t^2+2}\right)^5 - \frac{4}{3}\left(\sqrt{t^2+2}\right)^3 + 4\sqrt{t^2+2} + C$$

$$p = t^2+2$$
$$dp = 2t\;dt$$
$$\frac{1}{2}\,dp = t\;dt$$
$$t^2 = p-2$$

method 2:

$$\frac{t}{\sqrt{2}} = \tan\theta$$
$$t = \sqrt{2}\tan\theta$$
$$dt = \sqrt{2}\sec^2\theta\;d\theta$$

$$\frac{\sqrt{t^2+(\sqrt{2})^2}}{\sqrt{2}} = \sec\theta$$
$$\sqrt{t^2+(\sqrt{2})^2} = \sqrt{2}\sec\theta$$

$$\int \frac{t^5}{\sqrt{t^2+2}}\,dt = \int \frac{t^5}{\sqrt{t^2+(\sqrt{2})^2}}\,dt$$

$$= \int \frac{(\sqrt{2}\tan\theta)^5}{(\sqrt{2}\sec\theta)}(\sqrt{2}\sec\theta\;d\theta)$$

$$= (\sqrt{2})^5 \int \sec\theta\tan^5\theta\;d\theta$$

$$= (\sqrt{2})^5 \int (\tan^2\theta)^2(\sec\theta\tan\theta\;d\theta)$$

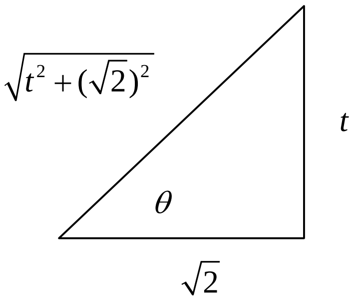
$$= (\sqrt{2})^5 \int (\sec^2\theta-1)^2(\sec\theta\tan\theta\;d\theta)$$

$$= (\sqrt{2})^5 \int (\sec^4\theta - 2\sec^2\theta + 1)(\sec\theta\tan\theta\;d\theta)$$

$$= (\sqrt{2})^5\left(\frac{1}{5}\sec^5\theta - \frac{2}{3}\sec^3\theta + \sec\theta\right) + C$$

$$= (\sqrt{2})^5\left(\frac{1}{5}\left(\frac{\sqrt{t^2+(\sqrt{2})^2}}{\sqrt{2}}\right)^5 - \frac{2}{3}\left(\frac{\sqrt{t^2+(\sqrt{2})^2}}{\sqrt{2}}\right)^3 + \left(\frac{\sqrt{t^2+(\sqrt{2})^2}}{\sqrt{2}}\right)\right) + C$$

$$= \frac{1}{5}\left(\sqrt{t^2+2}\right)^5 - \frac{4}{3}\left(\sqrt{t^2+2}\right)^3 + 4\sqrt{t^2+2} + C$$



52) $\int \frac{x}{\sqrt{1+x^2}}\,dx$

method 1:

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{1+x^2}}(x\,dx) = \int \frac{1}{\sqrt{p}}\left(\frac{1}{2}\,dp\right)$$

$$= \frac{1}{2}\left(\frac{2}{1}\sqrt{p}\right)+C$$

$$= \sqrt{1+x^2}+C$$

$p = 1+x^2$

$dp = 2x\,dx$

$\frac{1}{2}\,dp = x\,dx$

method 2:

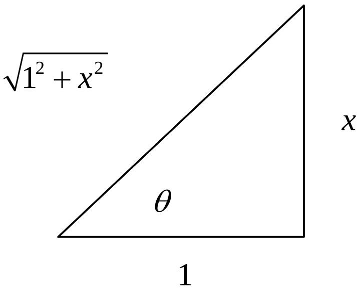
$\frac{x}{1} = \tan\theta$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{1^2+x^2}}{1} = \sec\theta$

$\sqrt{1^2+x^2} = \sec\theta$



$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{x}{\sqrt{1^2+x^2}}\,dx = \int \frac{(\tan\theta)}{(\sec\theta)}(\sec^2\theta\,d\theta)$$

$$= \int \sec\theta\tan\theta\,d\theta = \sec\theta + C = \sqrt{1^2+x^2}+C = \sqrt{1+x^2}+C$$

58) $\int_0^1 \frac{dx}{(x^2+1)^2}$

$$\int_0^1 \frac{dx}{(x^2+1)^2} = \left[\frac{1}{2}\tan^{-1}x + \frac{x}{2(x^2+1)} + C\right]_0^1$$

$$= \left[\frac{1}{2}\tan^{-1}(1) + \frac{(1)}{2((1)^2+1)} + C\right] - \left[\frac{1}{2}\tan^{-1}(0) + \frac{(0)}{2((0)^2+1)} + C\right] = \frac{\pi}{8} + \frac{1}{4} = \frac{\pi+2}{8}$$

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{1}{\left(\sqrt{x^2+1}\right)^4}\,dx = \int \frac{1}{(\sec\theta)^4}(\sec^2\theta\,d\theta)$$

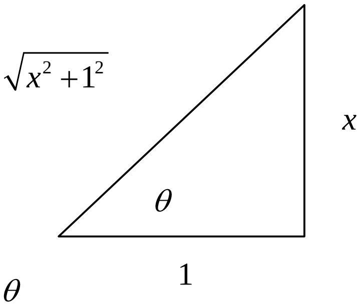
$$= \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta = \int \frac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C = \frac{1}{2}(\theta + \sin\theta\cos\theta) + C$$

$$= \frac{1}{2}\left((\tan^{-1}x) + \left(\frac{x}{\sqrt{x^2+1^2}}\right)\left(\frac{1}{\sqrt{x^2+1^2}}\right)\right) + C$$

$$= \frac{1}{2}\tan^{-1}x + \frac{x}{2\left(\sqrt{x^2+1^2}\right)^2} + C$$

$$= \frac{1}{2}\tan^{-1}x + \frac{x}{2(x^2+1)} + C$$



$\frac{x}{1} = \tan\theta$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{x^2+1^2}}{1} = \sec\theta$

$\sqrt{x^2+1^2} = \sec\theta$

$\theta = \tan^{-1}x$

60) $\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1+\sin^2 t}}\, dt$

$$\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1+\sin^2 t}}\, dt = \left[\ln\left|\frac{1+\sin t}{\sqrt{1+\sin^2 t}}\right| + C \right]_0^{\frac{\pi}{2}}$$

$$= \left[\ln\left|\frac{1+\sin\left(\frac{\pi}{2}\right)}{\sqrt{1+\sin^2\left(\frac{\pi}{2}\right)}}\right| + C \right] - \left[\ln\left|\frac{1+\sin(0)}{\sqrt{1+\sin^2(0)}}\right| + C \right] = \left[\ln\left|\frac{1+(1)}{\sqrt{1+(1)^2}}\right| \right] - \left[\ln\left|\frac{1+(0)}{\sqrt{1+(0)^2}}\right| \right]$$

$$= \left[\ln\left|\frac{2}{\sqrt{2}}\right| \right] - \left[\ln|1| \right] = \left[\ln\left|\sqrt{2}\right| \right] = \ln\sqrt{2} = \frac{1}{2}\ln 2$$

$$\int \frac{\cos t}{\sqrt{1+\sin^2 t}}\, dt = \int \frac{1}{\sqrt{1+\sin^2 t}}(\cos t\; dt)$$

$$= \int \frac{1}{\sqrt{1^2+p^2}}(dp)$$

$$= \int \frac{1}{\sec\theta}(\sec^2\theta\; d\theta)$$

$$= \int \sec\theta\; d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

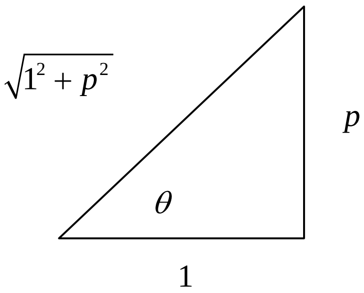
$$= \ln\left|\frac{1}{\sqrt{1^2+p^2}} + \frac{p}{\sqrt{1^2+p^2}}\right| + C$$

$$= \ln\left|\frac{1+p}{\sqrt{1^2+p^2}} =\right| + C$$

$$= \ln\left|\frac{1+\sin t}{\sqrt{1+\sin^2 t}} =\right| + C$$

$p = \sin t$

$dp = \cos t\; dt$



$\frac{p}{1} = \tan\theta$

$p = \tan\theta$

$dp = \sec^2\theta\; d\theta$

$\frac{\sqrt{1^2+p^2}}{1} = \sec\theta$

$\sqrt{1^2+p^2} = \sec\theta$

62) $\int \frac{x^2}{\left(3+4x-4x^2\right)^{\frac{3}{2}}}\, dx$

First we must fix the trinomial in the denominator such that it becomes either sum or difference of squares.

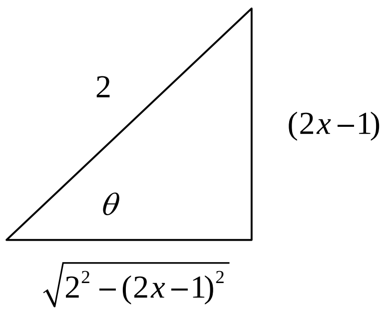
$$3+4x-4x^2 = 3+(1-1)+4x-4x^2 = 3+1-1+4x-4x^2 = 3+1-(1-4x+4x^2) = 4-(4x^2-4x+1)$$

$$= 4-(2x-1)^2 = 2^2-(2x-1)^2$$

$$\frac{(2x-1)}{2} = \sin\theta$$
$$2x-1 = 2\sin\theta$$
$$2x = 2\sin\theta + 1$$
$$x = \sin\theta + \frac{1}{2}$$
$$dx = \cos\theta \, d\theta$$

$$\frac{\sqrt{2^2-(2x-1)^2}}{2} = \cos\theta$$
$$\sqrt{2^2-(2x-1)^2} = 2\cos\theta$$

$$\theta = \sin^{-1}\left(\frac{2x-1}{2}\right)$$



$$\int \frac{x^2}{\left(3+4x-4x^2\right)^{\frac{3}{2}}}\,dx = \int \frac{x^2}{\left(3+4x-4x^2\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{2^2-(2x-1)^2}\right)^3}\,dx = \int \frac{\left(\sin\theta+\frac{1}{2}\right)^2}{(2\cos\theta)^3}(\cos\theta\,d\theta)$$

$$= \int \frac{\left(\sin\theta+\frac{1}{2}\right)^2}{8\cos^2\theta}\,d\theta = \frac{1}{8}\int \frac{\left(\sin^2\theta+\sin\theta+\frac{1}{4}\right)}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{8}\left\{\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta + \int \frac{\sin\theta}{\cos^2\theta}\,d\theta + \frac{1}{4}\int \frac{1}{\cos^2\theta}\,d\theta\right\}$$

$$= \frac{1}{8}\left\{\int \tan^2\theta\,d\theta - \int \frac{(-\sin\theta)}{\cos^2\theta}\,d\theta + \frac{1}{4}\int \sec^2\theta\,d\theta\right\}$$

$$= \frac{1}{8}\left\{\int (\sec^2\theta-1)\,d\theta - \int \frac{(-\sin\theta)}{\cos^2\theta}\,d\theta + \frac{1}{4}\int \sec^2\theta\,d\theta\right\}$$

$$= \frac{1}{8}\left\{\frac{5}{4}\int \sec^2\theta\,d\theta - \int \frac{1}{\cos^2\theta}(-\sin\theta\,d\theta) - \int 1\,d\theta\right\}$$

$$= \frac{1}{8}\left\{\frac{5}{4}\tan\theta - \frac{1}{\cos\theta} - \theta\right\} + C = \frac{1}{8}\left\{\frac{5}{4}\tan\theta - \sec\theta - \theta\right\} + C$$

$$= \frac{1}{8}\left\{\frac{5}{4}\left(\frac{2x-1}{\sqrt{2^2-(2x-1)^2}}\right) - \left(\frac{2}{\sqrt{2^2-(2x-1)^2}}\right) - \left(\sin^{-1}\left(\frac{2x-1}{2}\right)\right)\right\} + C$$

$$= \frac{1}{8}\left\{\frac{5}{4}\left(\frac{2x-1}{\sqrt{3+4x-4x^2}}\right) - \left(\frac{2}{\sqrt{3+4x-4x^2}}\right) - \sin^{-1}\left(\frac{2x-1}{2}\right)\right\} + C$$

$$= \frac{1}{8}\left\{\frac{10x-5}{4\sqrt{3+4x-4x^2}} - \frac{8}{4\sqrt{3+4x-4x^2}} - \sin^{-1}\left(\frac{2x-1}{2}\right)\right\} + C$$

$$= \frac{1}{8}\left\{\frac{10x-13}{4\sqrt{3+4x-4x^2}} - \sin^{-1}\left(\frac{2x-1}{2}\right)\right\} + C$$

$$= \frac{10x-13}{32\sqrt{3+4x-4x^2}} - \frac{1}{8}\sin^{-1}\left(\frac{2x-1}{2}\right) + C$$