

See L'Hospital's Rule on page 306. To use rule the expression must be  $\frac{0}{0}$  or  $\frac{\pm\infty}{\pm\infty}$ .

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \stackrel{L}{=} \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

L'Hospital Rule only works for the fractional expression; therefore, the other indeterminate forms must be converted into fractional expression before applying L'Hospital Rule.

Other indeterminate forms:  $(0)(\infty)$ ,  $\infty - \infty$ ,  $0^0$ ,  $\infty^0$ ,  $1^\infty$

Additional examples:

$$2) \quad \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2} \stackrel{L}{=} \lim_{x \rightarrow 2} \frac{2x + 1}{1} = \frac{2(2) + 1}{1} = \frac{5}{1} = 5$$

$$6) \quad \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} \stackrel{L}{=} \lim_{x \rightarrow 0} \frac{2x}{\sin x} = \lim_{x \rightarrow 0} \frac{2}{\cos x} = \frac{2}{\cos(0)} = \frac{2}{1} = 2$$

$$12) \quad \lim_{t \rightarrow 0} \frac{8^t - 5^t}{t} = \lim_{t \rightarrow 0} \frac{8^t - 5^t}{t} \stackrel{L}{=} \lim_{t \rightarrow 0} \frac{(\ln 8)8^t - (\ln 5)5^t}{1} = \frac{(\ln 8)8^{(0)} - (\ln 5)5^{(0)}}{1} = \ln 8 - \ln 5 = \ln\left(\frac{8}{5}\right)$$

$$16) \quad \lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2} = \lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2} \stackrel{L}{=} \lim_{x \rightarrow 0} \frac{-m \sin(mx) + n \sin(nx)}{2x} = \lim_{x \rightarrow 0} \frac{-m^2 \cos(mx) + n^2 \cos(nx)}{2} \\ = \frac{-m^2 \cos(m(0)) + n^2 \cos(n(0))}{2} = \frac{-m^2(1) + n^2(1)}{2} = \frac{n^2 - m^2}{2}$$

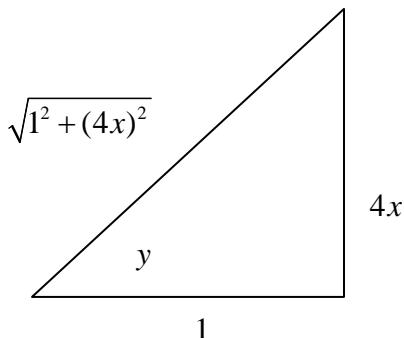
$$18) \quad \lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)} = \lim_{x \rightarrow 0} \frac{1}{\frac{4}{1+16x^2}} = \lim_{x \rightarrow 0} \frac{1+16x^2}{4} = \frac{1+16(0)^2}{4} = \frac{1+0}{4} = \frac{1}{4}$$

$$y = \tan^{-1}(4x)$$

$$\tan y = 4x$$

$$\sec^2 y \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{\sec^2 y} = \frac{4}{\left(\sqrt{1^2 + (4x)^2}\right)^2} = \frac{4}{1+16x^2}$$



$$22) \quad \lim_{x \rightarrow a^+} \frac{\cos x \ln(x-a)}{\ln(e^x - e^a)} = \left( \lim_{x \rightarrow a^+} \cos x \right) \left( \lim_{x \rightarrow a^+} \frac{\ln(x-a)}{\ln(e^x - e^a)} \right) = \cos(a^+)(1) = \cos a$$

$$\begin{aligned} \lim_{x \rightarrow a^+} \frac{\ln(x-a)}{\ln(e^x - e^a)} &= \lim_{x \rightarrow a^+} \frac{\ln(x-a)}{\ln(e^x - e^a)} \stackrel{L}{=} \lim_{x \rightarrow a^+} \frac{\frac{1}{x-a}(1)}{\frac{1}{e^x - e^a}(e^x)(1)} = \lim_{x \rightarrow a^+} \frac{e^x - e^a}{(x-a)} \stackrel{0}{=} \lim_{x \rightarrow a^+} \frac{e^x}{[e^x(1)](x-a) + (e^x)[1]} \\ &= \lim_{x \rightarrow a^+} \frac{e^x}{[e^x(1)](x-a) + (e^x)[1]} = \lim_{x \rightarrow a^+} \frac{e^x}{e^x\{(x-a)+1\}} = \lim_{x \rightarrow a^+} \frac{1}{(x-a)+1} = \frac{1}{((a^+)-a)+1} = 1 \end{aligned}$$

$$24) \quad \lim_{x \rightarrow \infty} \sqrt{x} e^{\frac{-x}{2}} = \lim_{x \rightarrow \infty} \sqrt{x} e^{\frac{0}{2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{e^{\frac{x}{2}}} \stackrel{L}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{x}}}{\left[ e^{\frac{x}{2}} \left( \frac{1}{2} \right) \right]} = \lim_{x \rightarrow \infty} \frac{1}{(\sqrt{x})(e^{\frac{x}{2}})} = 0$$

$$28) \quad \lim_{x \rightarrow \infty} x \tan\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} x \tan\left(\frac{0}{x}\right) = \lim_{x \rightarrow \infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} \stackrel{L}{=} \lim_{x \rightarrow \infty} \frac{\sec^2\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x \rightarrow \infty} \sec^2\left(\frac{1}{x}\right) = \sec^2(0) = (1)^2 = 1$$

$$32) \quad \lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)] = \lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)] = \lim_{x \rightarrow 1^+} \ln\left(\frac{(x^7 - 1)}{(x^5 - 1)}\right) = \ln\left(\frac{7}{5}\right)$$

$$\lim_{x \rightarrow 1^+} \frac{(x^7 - 1)}{(x^5 - 1)} = \lim_{x \rightarrow 1^+} \frac{(x^7 - 1)}{(x^5 - 1)} \stackrel{0}{=} \lim_{x \rightarrow 1^+} \frac{7x^6}{5x^4} = \lim_{x \rightarrow 1^+} \frac{7}{5} x^2 = \frac{7}{5} (1^+)^2 = \frac{7}{5}$$

$$36) \quad \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$$

$$y = \left(1 + \frac{a}{x}\right)^{bx}$$

$$\ln y = \ln \left(1 + \frac{a}{x}\right)^{bx}$$

$$\ln y = bx \ln \left(1 + \frac{a}{x}\right) = \frac{b \ln \left(1 + \frac{a}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{b \ln \left(1 + \frac{a}{x}\right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{b \ln \left(1 + \frac{a}{x}\right)}{\frac{1}{x}} \stackrel{L}{=} \lim_{x \rightarrow \infty} \frac{b \left[ \frac{1}{\left(1 + \frac{a}{x}\right)} (a) \left(\frac{-1}{x^2}\right) \right]}{\frac{-1}{x^2}} = \lim_{x \rightarrow \infty} \frac{ab}{1 + \frac{a}{x}} = \frac{ab}{1 + 0} = ab$$

$$\ln y = ab$$

↓

$$y = e^{ab}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$$