See L'Hospital's Rule on page 306. To use rule the expression must be $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.

$$\lim_{x\to a}\frac{f(x)}{g(x)}\overset{L}{=}\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

L'Hospital Rule only works for the fractional expression; therefore, the other indeterminate forms must be converted into fractional expression before applying L'Hospital Rule.
Other indeterminate forms: $(0)(\infty)$, $\infty-\infty$, 0^0, ∞^0, 1^∞

Additional examples:

2) $$\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}\frac{\overset{0}{x^2+x-6}}{\underset{0}{x-2}}\overset{L}{=}\lim_{x\to 2}\frac{2x+1}{1}=\frac{2(2)+1}{1}=\frac{5}{1}=5$$

6) $$\lim_{x\to 0}\frac{x^2}{1-\cos x}=\lim_{x\to 0}\frac{\overset{0}{x^2}}{\underset{0}{1-\cos x}}\overset{L}{=}\lim_{x\to 0}\frac{\overset{0}{2x}}{\underset{0}{\sin x}}\overset{L}{=}\lim_{x\to 0}\frac{2}{\cos x}=\frac{2}{\cos(0)}=\frac{2}{1}=2\,;$$

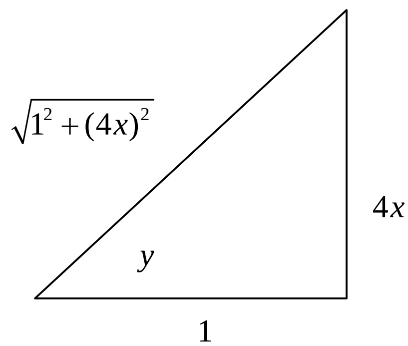
N.B.: you could stop at 2x/sin x since limit of sin x/x = limit of reciprocal of x/sin x = 1.

12) $$\lim_{t\to 0}\frac{8^t-5^t}{t}=\lim_{t\to 0}\frac{\overset{0}{8^t-5^t}}{\underset{0}{t}}\overset{L}{=}\lim_{t\to 0}\frac{(\ln 8)8^t-(\ln 5)5^t}{1}=\frac{(\ln 8)8^{(0)}-(\ln 5)5^{(0)}}{1}=\ln 8-\ln 5=\ln\left(\frac{8}{5}\right)$$

16) $$\lim_{x\to 0}\frac{\cos mx-\cos nx}{x^2}=\lim_{x\to 0}\frac{\overset{0}{\cos mx-\cos nx}}{\underset{0}{x^2}}\overset{L}{=}\lim_{x\to 0}\frac{\overset{0}{-m\sin(mx)+n\sin(nx)}}{\underset{0}{2x}}\overset{L}{=}\lim_{x\to 0}\frac{-m^2\cos(mx)+n^2\cos(nx)}{2}$$
$$=\frac{-m^2\cos(m(0))+n^2\cos(n(0))}{2}=\frac{-m^2(1)+n^2(1)}{2}=\frac{n^2-m^2}{2}$$

18) $$\lim_{x\to 0}\frac{\overset{0}{x}}{\underset{0}{\tan^{-1}(4x)}}\overset{L}{=}\lim_{x\to 0}\frac{1}{\frac{4}{\sqrt{1+16x^2}}}=\lim_{x\to 0}\frac{\sqrt{1+16x^2}}{4}=\frac{\sqrt{1+16(0)^2}}{4}=\frac{\sqrt{1}}{4}=\frac{1}{4}$$

$$y = \tan^{-1}(4x)$$
$$\tan y = 4x$$
$$\sec^2 y \frac{dy}{dx} = 4$$
$$\frac{dy}{dx} = \frac{4}{\sec^2 y} = \frac{4}{\left(\sqrt{1^2 + (4x)^2}\right)^2} = \frac{4}{\sqrt{1+16x^2}}$$



22) $$\lim_{x\to a^+}\frac{\cos x\ln(x-a)}{\ln(e^x-e^a)}=\left(\lim_{x\to a^+}\cos x\right)\left(\lim_{x\to a^+}\frac{\ln(x-a)}{\ln(e^x-e^a)}\right)=\cos(a^+)(1)=\cos a$$

$$\lim_{x\to a^+}\frac{\ln(x-a)}{\ln(e^x-e^a)}=\lim_{x\to a^+}\frac{\overset{-\infty}{\ln(x-a)}}{\underset{-\infty}{\ln(e^x-e^a)}}\overset{L}{=}\lim_{x\to a^+}\frac{\frac{1}{x-a}(1)}{\frac{1}{e^x-e^a}(e^x)(1)}=\lim_{x\to a^+}\frac{\overset{0}{e^x-e^a}}{\underset{0}{(x-a)}}\overset{L}{=}\lim_{x\to a^+}\frac{e^x}{[e^x(1)](x-a)+(e^x)[1]}$$

$$=\lim_{x\to a^+}\frac{e^x}{[e^x(1)](x-a)+(e^x)[1]}=\lim_{x\to a^+}\frac{e^x}{e^x\{(x-a)+1\}}=\lim_{x\to a^+}\frac{1}{(x-a)+1}=\frac{1}{((a^+)-a)+1}=1$$

24) $$\lim_{x\to\infty}\sqrt{x}e^{\frac{-x}{2}}=\lim_{x\to\infty}\overset{\infty}{\sqrt{x}}\,\overset{0}{e^{\frac{-x}{2}}}=\lim_{x\to\infty}\frac{\overset{\infty}{\sqrt{x}}}{\underset{\infty}{e^{\frac{x}{2}}}}\overset{L}{=}\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{\left[e^{\frac{x}{2}}\left(\frac{1}{2}\right)\right]}=\lim_{x\to\infty}\frac{1}{(\sqrt{x})(e^{\frac{x}{2}})}=0$$

28) $$\lim_{x\to\infty}x\tan\left(\frac{1}{x}\right)=\lim_{x\to\infty}\overset{\infty}{x}\,\overset{0}{\tan\left(\frac{1}{x}\right)}=\lim_{x\to\infty}\frac{\overset{0}{\tan\left(\frac{1}{x}\right)}}{\underset{0}{\frac{1}{x}}}\overset{L}{=}\lim_{x\to\infty}\frac{\sec^2\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)}{\frac{-1}{x^2}}=\lim_{x\to\infty}\sec^2\left(\frac{1}{x}\right)=\sec^2(0)=(1)^2=1$$

32) $$\lim_{x\to1^+}[\ln(x^7-1)-\ln(x^5-1)]=\lim_{x\to1^+}[\overset{\infty}{\ln(x^7-1)}-\overset{\infty}{\ln(x^5-1)}]=\lim_{x\to1^+}\ln\left(\frac{(x^7-1)}{(x^5-1)}\right)=\ln\left(\frac{7}{5}\right)$$

$$\lim_{x\to1^+}\frac{(x^7-1)}{(x^5-1)}=\lim_{x\to1^+}\frac{\overset{0}{(x^7-1)}}{\underset{0}{(x^5-1)}}\overset{L}{=}\lim_{x\to1^+}\frac{7x^6}{5x^4}=\lim_{x\to1^+}\frac{7}{5}x^2=\frac{7}{5}(1^+)^2=\frac{7}{5}$$

36) $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}$

$$y=\left(1+\frac{a}{x}\right)^{bx}$$

$$\ln y=\ln\left(1+\frac{a}{x}\right)^{bx}$$

$$\ln y=bx\ln\left(1+\frac{a}{x}\right)=\frac{b\ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\frac{b\ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\overset{0}{b\ln\left(1+\frac{a}{x}\right)}}{\underset{0}{\frac{1}{x}}}\overset{L}{=}\lim_{x\to\infty}\frac{b\left[\frac{1}{\left(1+\frac{a}{x}\right)}(a)\left(\frac{-1}{x^2}\right)\right]}{\frac{-1}{x^2}}=\lim_{x\to\infty}\frac{ab}{1+\frac{a}{x}}=\frac{ab}{1+0}=ab$$

$$\ln y=ab$$
$$\Downarrow$$
$$y=e^{ab}$$

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{bx}=e^{ab}$$