The Cross Product of Two Vectors in Space

Definition

The **cross product** $\mathbf{u}\times\mathbf{v}=\vec{u}\times\vec{v}$ ("$\mathbf{u}=\vec{u}$ **cross** $\mathbf{v}=\vec{v}$") is the vector

$$\mathbf{u}\times\mathbf{v}=(|\mathbf{u}||\mathbf{v}|\sin\theta)\mathbf{n} \quad\text{or}\quad \vec{u}\times\vec{v}=(|\vec{u}||\vec{v}|\sin\theta)\vec{n}$$

where $\mathbf{u}=\vec{u}$ and $\mathbf{v}=\vec{v}$ are not parallel vectors and $\mathbf{n}=\vec{n}$ is the unit vector perpendicular to the plane (generated by $\mathbf{u}=\vec{u}$ and $\mathbf{v}=\vec{v}$) by the **right-hand rule**.

Parallel Vectors

Nonzero vectors $\mathbf{u}=\vec{u}$ and $\mathbf{v}=\vec{v}$ are parallel if and only if $\mathbf{u}\times\mathbf{v}=\mathbf{0}$ or $\vec{u}\times\vec{v}=0$

Properties of the Cross Product

If $\mathbf{u}=\vec{u}$, $\mathbf{v}=\vec{v}$, $\mathbf{w}=\vec{w}$ are any vectors and r , s are scalars, then

1.	$(r\mathbf{u})\times(s\mathbf{v})=(rs)(\mathbf{u}\times\mathbf{v})$	$(r\vec{u})\times(s\vec{v})=(rs)(\vec{u}\times\vec{v})$
2.	$\mathbf{u}\times(\mathbf{v}+\mathbf{w})=\mathbf{u}\times\mathbf{v}+\mathbf{u}\times\mathbf{w}$	$\vec{u}\times(\vec{v}+\vec{w})=\vec{u}\times\vec{v}+\vec{u}\times\vec{w}$
3.	$\mathbf{v}\times\mathbf{u}=-(\mathbf{u}\times\mathbf{v})$	$\vec{v}\times\vec{u}=-(\vec{u}\times\vec{v})$
4.	$(\mathbf{v}+\mathbf{w})\times\mathbf{u}=\mathbf{v}\times\mathbf{u}+\mathbf{w}\times\mathbf{u}$	$(\vec{v}+\vec{w})\times\vec{u}=\vec{v}\times\vec{u}+\vec{w}\times\vec{u}$
5.	$\mathbf{0}\times\mathbf{u}=\mathbf{0}$	$\vec{0}\times\vec{u}=\vec{0}$
6.	$\mathbf{u}\times(\mathbf{v}\times\mathbf{w})=(\mathbf{u}\bullet\mathbf{w})\mathbf{v}-(\mathbf{u}\bullet\mathbf{v})\mathbf{w}$	$\vec{u}\times(\vec{v}\times\vec{w})=(\vec{u}\bullet\vec{w})\vec{v}-(\vec{u}\bullet\vec{v})\vec{w}$

$|\mathbf{u}\times\mathbf{v}|=|\vec{u}\times\vec{v}|$ Is the Area of a Parallelogram

Because $\mathbf{n}=\vec{n}$ is a unit vector, the magnitude of $\mathbf{u}\times\mathbf{v}=\vec{u}\times\vec{v}$ is

$$|\mathbf{u}\times\mathbf{v}|=|\mathbf{u}||\mathbf{v}||\sin\theta||\mathbf{n}|=|\mathbf{u}||\mathbf{v}|\sin\theta \quad\text{or}\quad |\vec{u}\times\vec{v}|=|\vec{u}||\vec{v}||\sin\theta||\vec{n}|=|\vec{u}||\vec{v}|\sin\theta$$

Determinant Formula for $\mathbf{u}\times\mathbf{v}=\vec{u}\times\vec{v}$

Calculating the Cross Product as a Determinant

If $\mathbf{u}=u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k}$ and $\mathbf{v}=v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k}$, then $\mathbf{u}\times\mathbf{v}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\u_1&u_2&u_3\\v_1&v_2&v_3\end{vmatrix}=\begin{vmatrix}u_2&u_3\\v_2&v_3\end{vmatrix}\mathbf{i}-\begin{vmatrix}u_1&u_3\\v_1&v_3\end{vmatrix}\mathbf{j}+\begin{vmatrix}u_1&u_2\\v_1&v_2\end{vmatrix}\mathbf{k}$

If $\vec{u}=u_1\vec{i}+u_2\vec{j}+u_3\vec{k}$ and $\vec{v}=v_1\vec{i}+v_2\vec{j}+v_3\vec{k}$, then $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\u_1&u_2&u_3\\v_1&v_2&v_3\end{vmatrix}=\begin{vmatrix}u_2&u_3\\v_2&v_3\end{vmatrix}\vec{i}-\begin{vmatrix}u_1&u_3\\v_1&v_3\end{vmatrix}\vec{j}+\begin{vmatrix}u_1&u_2\\v_1&v_2\end{vmatrix}\vec{k}$

Triple Scalar or Box Product

The product $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = (\vec{u}\times\vec{v})\cdot\vec{w}$ is called **triple scalar product** (**box product**)of $\mathbf{u} = \vec{u}$, $\mathbf{v} = \vec{v}$, and $\mathbf{w} = \vec{w}$ (in that order).

$$\left|(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}\right| = \left|\mathbf{u}\times\mathbf{v}\right|\left|\mathbf{w}\right|\left|\cos\theta\right| \quad or \quad \left|(\vec{u}\times\vec{v})\cdot\vec{w}\right| = \left|\vec{u}\times\vec{v}\right|\left|\vec{w}\right|\left|\cos\theta\right|$$

Calculating the Triple Scalar Product as a Determinant

$$(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = (\vec{u}\times\vec{v})\cdot\vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

2) $\vec{u} = 2\vec{i} + 3\vec{j} \quad \vec{v} = -\vec{i} + \vec{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} \vec{k}$$

$$= \{(3)(0) - (0)(1)\}\vec{i} - \{(2)(0) - (0)(-1)\}\vec{j} + \{(2)(1) - (3)(-1)\}\vec{k}$$

$$= \{0\}\vec{i} - \{0\}\vec{j} + \{2+3\}\vec{k} = 0\vec{i} + 0\vec{j} + 5\vec{k} = 5\vec{k}$$

length: $|\vec{u} \times \vec{v}| = \sqrt{(0)^2 + (0)^2 + (5)^2} = \sqrt{(5)^2} = 5$

direction: $\dfrac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} = \dfrac{5\vec{k}}{5} = \vec{k}$

$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v}) = -(5\vec{k}) = -5\vec{k}$

length: $|\vec{v} \times \vec{u}| = \sqrt{(0)^2 + (0)^2 + (-5)^2} = \sqrt{(-5)^2} = 5$

direction: $\dfrac{\vec{v} \times \vec{u}}{|\vec{v} \times \vec{u}|} = \dfrac{-5\vec{k}}{5} = -\vec{k}$

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4) $\vec{u} = \vec{i} + \vec{j} - \vec{k} \quad \vec{v} = \vec{0} = 0\vec{i} + 0\vec{j} + 0\vec{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \vec{k}$$

$$= \{(1)(0) - (-1)(0)\}\vec{i} - \{(1)(0) - (-1)(0)\}\vec{j} + \{(1)(0) - (1)(0)\}\vec{k}$$

$$= \{0\}\vec{i} - \{0\}\vec{j} + \{0\}\vec{k} = \vec{0}$$

length: $|\vec{u} \times \vec{v}| = \sqrt{(0)^2 + (0)^2 + (0)^2} = 0$ direction: $\dfrac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} = \dfrac{\vec{0}}{0}$ none

$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v}) = -(\vec{0}) = \vec{0}$

length: $|\vec{v} \times \vec{u}| = 0$ direction: none

6) $\vec{u} = \vec{i} \times \vec{j}$, $\vec{v} = \vec{j} \times \vec{k}$

$\vec{u} = \vec{k}$ $\quad$ $\vec{v} = \vec{i}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \vec{k}$$

$$= \{(0)(0)-(1)(0)\}\vec{i} - \{(0)(0)-(1)(1)\}\vec{j} + \{(0)(0)-(0)(1)\}\vec{k}$$

$$= \{0\}\vec{i} - \{-1\}\vec{j} + \{0\}\vec{k} = 1\vec{j} = \vec{j}$$

length: $|\vec{u} \times \vec{v}| = 1$ $\quad$ direction: $\dfrac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} = \dfrac{1\vec{j}}{1} = \vec{j}$

$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v}) = -(1\vec{j}) = -\vec{j}$

length: $|\vec{v} \times \vec{u}| = 1$ $\quad$ direction: $\dfrac{\vec{v} \times \vec{u}}{|\vec{v} \times \vec{u}|} = \dfrac{-\vec{j}}{1} = -\vec{j}$

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8) $\vec{u} = \frac{3}{2}\vec{i} - \frac{1}{2}\vec{j} + \vec{k}$, $\vec{v} = \vec{i} + \vec{j} + 2\vec{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{3}{2} & \frac{-1}{2} & 1 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} \frac{-1}{2} & 1 \\ 1 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} \frac{3}{2} & 1 \\ 1 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} \frac{3}{2} & \frac{-1}{2} \\ 1 & 1 \end{vmatrix} \vec{k}$$

$$= \left\{\left(\tfrac{-1}{2}\right)(2)-(1)(1)\right\}\vec{i} - \left\{\left(\tfrac{3}{2}\right)(2)-(1)(1)\right\}\vec{j} + \left\{\left(\tfrac{3}{2}\right)(1)-\left(\tfrac{-1}{2}\right)(1)\right\}\vec{k}$$

$$= \{-1-1\}\vec{i} - \{3-1\}\vec{j} + \left\{\tfrac{3}{2}+\tfrac{1}{2}\right\}\vec{k} = -2\vec{i} - 2\vec{j} + 2\vec{k}$$

length: $|\vec{u} \times \vec{v}| = \sqrt{(-2)^2+(-2)^2+(2)^2} = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

direction: $\dfrac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} = \dfrac{-2\vec{i} - 2\vec{j} + 2\vec{k}}{2\sqrt{3}} = \dfrac{-1}{\sqrt{3}}\vec{i} - \dfrac{1}{\sqrt{3}}\vec{j} + \dfrac{1}{\sqrt{3}}\vec{k}$

$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v}) = -(-2\vec{i} - 2\vec{j} + 2\vec{k}) = 2\vec{i} + 2\vec{j} - 2\vec{k}$

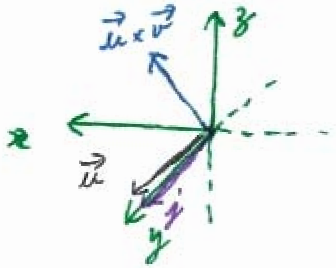
length: $|\vec{v} \times \vec{u}| = \sqrt{(2)^2+(2)^2+(-2)^2} = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

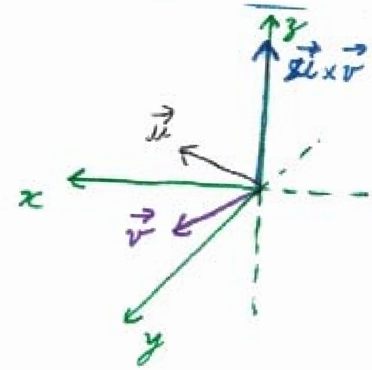
direction: $\dfrac{\vec{v} \times \vec{u}}{|\vec{v} \times \vec{u}|} = \dfrac{2\vec{i} + 2\vec{j} - 2\vec{k}}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}\vec{i} + \dfrac{1}{\sqrt{3}}\vec{j} - \dfrac{1}{\sqrt{3}}\vec{k}$

10) $\vec{u} = \vec{i} - \vec{k}$ $\quad \vec{v} = \vec{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\vec{k}$$

$$= \{(0)(0)-(-1)(1)\}\vec{i} - \{(1)(0)-(-1)(0)\}\vec{j} + \{(1)(1)-(0)(0)\}\vec{k} = \vec{i} + \vec{k}$$





12) $\vec{u} = 2\vec{i} - \vec{j}$ $\quad \vec{v} = \vec{i} + 2\vec{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}\vec{k}$$

$$= \{(-1)(0)-(0)(2)\}\vec{i} - \{(2)(0)-(0)(1)\}\vec{j} + \{(2)(2)-(-1)(1)\}\vec{k}$$

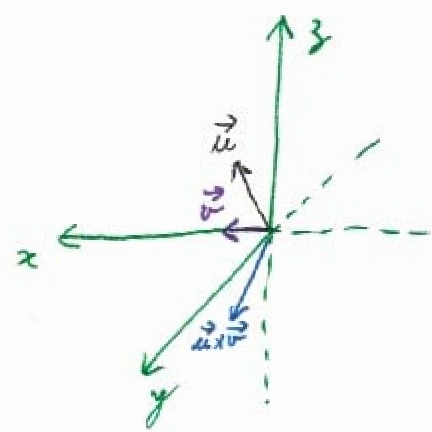
$$= \{0\}\vec{i} - \{0\}\vec{j} + \{5\}\vec{k} = 5\vec{k}$$

14) $\vec{u} = \vec{j} + 2\vec{k}$ $\quad v = \vec{i}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\vec{k}$$

$$= \{(1)(0)-(2)(0)\}\vec{i} - \{(0)(0)-(2)(1)\}\vec{j} + \{(0)(0)-(1)(1)\}\vec{k}$$

$$= \{0\}\vec{i} - \{-2\}\vec{j} + \{-1\}\vec{k} = 2\vec{j} - \vec{k}$$



16) $P(1,1,1)$, $Q(2,1,3)$, $R(3,-1,1)$

$$\overrightarrow{PQ} = \{(2)-(1)\}\vec{i} + \{(1)-(1)\}\vec{j} + \{(3)-(1)\}\vec{k} = \vec{i} + 2\vec{k}$$

$$\overrightarrow{PR} = \{(3)-(1)\}\vec{i} + \{(-1)-(1)\}\vec{j} + \{(1)-(1)\}\vec{k} = 2\vec{i} - 2\vec{j}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ -2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix}\vec{k}$$

$$= \{(0)(0)-(2)(-2)\}\vec{i} - \{(1)(0)-(2)(2)\}\vec{j} + \{(1)(-2)-(0)(2)\}\vec{k}$$

$$= \{+4\}\vec{i} - \{-4\}\vec{j} + \{-2\}\vec{k} = 4\vec{i} + 4\vec{j} - 2\vec{k}$$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{(4)^2+(4)^2+(-2)^2} = \sqrt{(2)^2\{2^2+2^2+1^2\}} = \sqrt{(2)^2\{9\}} = (2)(3) = 6$$

a) Area $= \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}(6) = 3$ b) $\dfrac{\overrightarrow{PQ} \times \overrightarrow{PR}}{|\overrightarrow{PQ} \times \overrightarrow{PR}|} = \dfrac{4\vec{i}+4\vec{j}-2\vec{k}}{6} = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$

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18) $P(-2,2,0)$, $Q(0,1,-1)$, $R(-1,2,-2)$

$$\overrightarrow{PQ} = \{(0)-(-2)\}\vec{i} + \{(1)-(2)\}\vec{j} + \{(-1)-(0)\}\vec{k} = 2\vec{i} - \vec{j} - \vec{k}$$

$$\overrightarrow{PR} = \{(-1)-(-2)\}\vec{i} + \{(2)-(2)\}\vec{j} + \{(-2)-(0)\}\vec{k} = \vec{i} - 2\vec{k}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 0 & -2 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}\vec{k}$$

$$= \{(-1)(-2)-(-1)(0)\}\vec{i} - \{(2)(-2)-(-1)(1)\}\vec{j} + \{(2)(0)-(-1)(1)\}\vec{k}$$

$$= \{2\}\vec{i} - \{-3\}\vec{j} + \{+1\}\vec{k} = 2\vec{i} + 3\vec{j} + \vec{k}$$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{(2)^2+(3)^2+(1)^2} = \sqrt{4+9+1} = \sqrt{14}$$

a) Area $= \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}(\sqrt{14}) = \frac{\sqrt{14}}{2}$ b) $\dfrac{\overrightarrow{PQ} \times \overrightarrow{PR}}{|\overrightarrow{PQ} \times \overrightarrow{PR}|} = \dfrac{2\vec{i}+3\vec{j}+\vec{k}}{\sqrt{14}} = \frac{2}{\sqrt{14}}\vec{i} + \frac{3}{\sqrt{14}}\vec{j} + \frac{1}{\sqrt{14}}\vec{k}$

20) $\vec{u} = \vec{i} - \vec{j} + \vec{k}$, $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$, $\vec{w} = -\vec{i} + 2\vec{j} - \vec{k}$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}(1) - \begin{vmatrix} 2 & -2 \\ -1 & -1 \end{vmatrix}(-1) + \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}(1)$$

$$= \{(1)(-1) - (-2)(2)\}(1) - \{(2)(-1) - (-2)(-1)\}(-1) + \{(2)(2) - (1)(-1)\}(1)$$

$$= \{3\}(1) - \{-4\}(-1) + \{5\}(1) = 3 - 4 + 5 = 4$$

$$(\vec{v} \times \vec{w}) \cdot \vec{u} = \begin{vmatrix} 2 & 1 & -2 \\ -1 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}(2) - \begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix}(1) + \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}(-2)$$

$$= \{(2)(1) - (-1)(-1)\}(2) - \{(-1)(1) - (-1)(1)\}(1) + \{(-1)(-1) - (2)(1)\}(-2)$$

$$= \{1\}(2) - \{0\}(1) + \{-1\}(-2) = 2 - 0 + 2 = 4$$

$$(\vec{w} \times \vec{u}) \cdot \vec{v} = \begin{vmatrix} -1 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix}(-1) - \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix}(2) + \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}(-1)$$

$$= \{(-1)(-2) - (1)(1)\}(-1) - \{(1)(-2) - (1)(2)\}(2) + \{(1)(1) - (-1)(2)\}(-1)$$

$$= \{1\}(-1) - \{-4\}(2) + \{3\}(-1) = -1 + 8 - 3 = 4$$

volume: $|(\vec{u} \times \vec{v}) \cdot \vec{w}| = |4| = 4$ units^3

$|(\vec{v} \times \vec{w}) \cdot \vec{u}| = |4| = 4$ units^3

$|(\vec{w} \times \vec{u}) \cdot \vec{v}| = |4| = 4$ units^3

22) $\vec{u} = \vec{i} + \vec{j} - 2\vec{k} \qquad \vec{v} = -\vec{i} - \vec{k} \qquad \vec{w} = 2\vec{i} + 4\vec{j} - 2\vec{k}$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} 1 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 4 & -2 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 4 & -2 \end{vmatrix}(1) - \begin{vmatrix} -1 & -1 \\ 2 & -2 \end{vmatrix}(1) + \begin{vmatrix} -1 & 0 \\ 2 & 4 \end{vmatrix}(-2)$$

$$= \{(0)(-2) - (-1)(4)\}(1) - \{(-1)(-2) - (-1)(2)\}(1) + \{(-1)(4) - (0)(2)\}(-2)$$

$$= \{4\}(1) - \{4\}(1) + \{-4\}(-2) = 4 - 4 + 8 = 8$$

$$(\vec{v} \times \vec{w}) \cdot \vec{u} = \begin{vmatrix} -1 & 0 & -1 \\ 2 & 4 & -2 \\ 1 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 1 & -2 \end{vmatrix}(-1) - \begin{vmatrix} 2 & -2 \\ 1 & -2 \end{vmatrix}(0) + \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix}(-1)$$

$$= \{(4)(-2) - (-2)(1)\}(-1) - \{(2)(-2) - (-2)(1)\}(0) + \{(2)(1) - (4)(1)\}(-1)$$

$$= \{-6\}(-1) - \{-4\}(0) + \{-2\}(-1) = 6 - 0 + 2 = 8$$

$$(\vec{w} \times \vec{u}) \cdot \vec{v} = \begin{vmatrix} 2 & 4 & -2 \\ 1 & 1 & -2 \\ -1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix}(2) - \begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix}(4) + \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix}(-2)$$

$$= \{(1)(-1) - (-2)(0)\}(2) - \{(1)(-1) - (-2)(-1)\}(4) + \{(1)(0) - (1)(-1)\}(-2)$$

$$= \{-1\}(2) - \{-3\}(4) + \{1\}(-2) = -2 + 12 - 2 = 8$$

volume: $|(\vec{u} \times \vec{v}) \cdot \vec{w}| = |8| = 8 \text{ units}^3$

$|(\vec{v} \times \vec{w}) \cdot \vec{u}| = |8| = 8 \text{ units}^3$

$|(\vec{w} \times \vec{u}) \cdot \vec{v}| = |8| = 8 \text{ units}^3$

28) let $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$ $\quad \vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$

a) $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3 = v_1u_1 + v_2u_2 + v_3u_3 = \vec{v}\times\vec{u}$ true

b) $$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\vec{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\vec{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\vec{k}$$

$$= \{u_2v_3 - u_3v_2\}\vec{i} - \{u_1v_3 - u_3v_1\}\vec{j} + \{u_1v_2 - u_2v_1\}\vec{k}$$

$$= \{u_2v_3 - u_3v_2\}\vec{i} + \{u_3v_1 - u_1v_3\}\vec{j} + \{u_1v_2 - u_2v_1\}\vec{k}$$

$$\vec{v}\times\vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = \begin{vmatrix} v_2 & v_3 \\ u_2 & u_3 \end{vmatrix}\vec{i} - \begin{vmatrix} v_1 & v_3 \\ u_1 & u_3 \end{vmatrix}\vec{j} + \begin{vmatrix} v_1 & v_2 \\ u_1 & u_2 \end{vmatrix}\vec{k}$$

$$= \{v_2u_3 - v_3u_2\}\vec{i} - \{v_1u_3 - v_3u_1\}\vec{j} + \{v_1u_2 - v_2u_1\}\vec{k}$$

$$= \{v_2u_3 - v_3u_2\}\vec{i} + \{v_3u_1 - v_1u_3\}\vec{j} + \{v_1u_2 - v_2u_1\}\vec{k}$$

$$-(\vec{v}\times\vec{u}) = -\left(\{v_2u_3 - v_3u_2\}\vec{i} + \{v_3u_1 - v_1u_3\}\vec{j} + \{v_1u_2 - v_2u_1\}\vec{k}\right)$$

$$= \{v_3u_2 - v_2u_3\}\vec{i} + \{v_1u_3 - v_3u_1\}\vec{j} + \{v_2u_1 - v_1u_2\}\vec{k}$$

$$= \{u_2v_3 - u_3v_2\}\vec{i} + \{u_3v_1 - u_1v_3\}\vec{j} + \{u_1v_2 - u_2v_1\}\vec{k}$$

$$= \vec{u}\times\vec{v}$$

true

28) continued

c) $-\vec{u} = -u_1\vec{i} - u_2\vec{j} - u_3\vec{k}$

$$(-\vec{u})\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -u_1 & -u_2 & -u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} -u_2 & -u_3 \\ v_2 & v_3 \end{vmatrix}\vec{i} - \begin{vmatrix} -u_1 & -u_3 \\ v_1 & v_3 \end{vmatrix}\vec{j} + \begin{vmatrix} -u_1 & -u_2 \\ v_1 & v_2 \end{vmatrix}\vec{k}$$

$$= \{(-u_2)v_3 - (-u_3)v_2\}\vec{i} - \{(-u_1)v_3 - (-u_3v_1)\}\vec{j} + \{(-u_1)v_2 - (-u_2)v_1\}\vec{k}$$

$$= \{-u_2v_3 + u_3v_2\}\vec{i} - \{-u_1v_3 + u_3v_1\}\vec{j} + \{-u_1v_2 + u_2v_1\}\vec{k}$$

$$= \{u_3v_2 - u_2v_3\}\vec{i} + \{u_1v_3 - u_3v_1\}\vec{j} + \{u_2v_1 - u_1v_2\}\vec{k}$$

using result of part b,

$$-(\vec{u}\times\vec{v}) = -\left(\{u_2v_3 - u_3v_2\}\vec{i} + \{u_3v_1 - u_1v_3\}\vec{j} + \{u_1v_2 - u_2v_1\}\vec{k}\right)$$

$$= \{u_3v_2 - u_2v_3\}\vec{i} + \{u_1v_3 - u_3v_1\}\vec{j} + \{u_2v_1 - u_1v_2\}\vec{k}$$

<u>true</u>

d) $c\vec{u} = cu_1\vec{i} + cu_2\vec{j} + cu_3\vec{k}$ $\qquad$ $c\vec{v} = cv_1\vec{i} + cv_2\vec{j} + cv_3\vec{k}$

$$(c\vec{u})\cdot\vec{v} = (cu_1)v_1 + (cu_2)v_2 + (cu_3)v_3 = cu_1v_1 + cu_2v_2 + cu_3v_3$$

$$\vec{u}\cdot(c\vec{v}) = u_1(cv_1) + u_2(cv_2) + u_3(cv_3) = cu_1v_1 + cu_2v_2 + cu_3v_3$$

using result of part a,

$$c(\vec{u}\cdot\vec{v}) = c(u_1v_1 + u_2v_2 + u_3v_3) = cu_1v_1 + cu_2v_2 + cu_3v_3$$

<u>true</u>

28) continued

e) using result of part b,

$$c\left(\vec{u}\times\vec{v}\right) = c\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = c\left(\{u_2v_3 - u_3v_2\}\vec{i} + \{u_3v_1 - u_1v_3\}\vec{j} + \{u_1v_2 - u_2v_1\}\vec{k}\right)$$

$$= \{cu_2v_3 - cu_3v_2\}\vec{i} + \{cu_3v_1 - cu_1v_3\}\vec{j} + \{cu_1v_2 - cu_2v_1\}\vec{k}$$

using vectors defined in part d,

$$(c\vec{u})\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ cu_1 & cu_2 & cu_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} cu_2 & cu_3 \\ v_2 & v_3 \end{vmatrix}\vec{i} - \begin{vmatrix} cu_1 & cu_3 \\ v_1 & v_3 \end{vmatrix}\vec{j} + \begin{vmatrix} cu_1 & cu_2 \\ v_1 & v_2 \end{vmatrix}\vec{k}$$

$$= \{cu_2v_3 - cu_3v_2\}\vec{i} - \{cu_1v_3 - cu_3v_1\}\vec{j} + \{cu_1v_2 - cu_2v_1\}\vec{k}$$

$$= \{cu_2v_3 - cu_1v_2\}\vec{i} + \{cu_3v_1 - cu_1v_3\}\vec{j} + \{cu_1v_2 - cu_2v_1\}\vec{k}$$

$$\vec{u}\times(c\vec{v}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ cv_1 & cv_2 & cv_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ cv_2 & cv_3 \end{vmatrix}\vec{i} - \begin{vmatrix} u_1 & u_3 \\ cv_1 & cv_3 \end{vmatrix}\vec{j} + \begin{vmatrix} u_1 & u_2 \\ cv_1 & cv_2 \end{vmatrix}\vec{k}$$

$$= \{u_2cv_3 - u_3cv_2\}\vec{i} - \{u_1cv_3 - u_3cv_1\}\vec{j} + \{u_1cv_2 - u_2cv_1\}\vec{k}$$

$$= \{cu_2v_3 - cu_3v_2\}\vec{i} + \{cu_3v_1 - cu_1v_3\}\vec{j} + \{cu_1v_2 - cu_2v_1\}\vec{k}$$

<u>true</u>

f) $|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$$|\vec{u}|^2 = \left(\sqrt{u_1^2 + u_2^2 + u_3^2}\right)^2 = u_1^2 + u_2^2 + u_3^2 = u_1u_1 + u_2u_2 + u_3u_3 = \vec{u}\cdot\vec{u}$$ <u>true</u>

28) continued

g) $\vec{u} \times \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ u_2 & u_3 \end{vmatrix} \vec{i} - \begin{vmatrix} u_1 & u_3 \\ u_1 & u_3 \end{vmatrix} \vec{j} + \begin{vmatrix} u_1 & u_2 \\ u_1 & u_2 \end{vmatrix} \vec{k}$

$= \{u_2 u_3 - u_3 u_2\} \vec{i} - \{u_1 u_3 - u_3 u_1\} \vec{j} + \{u_1 u_2 - u_2 u_1\} \vec{k}$

$= 0\vec{i} + 0\vec{j} + 0\vec{k} = \vec{0}$

$(\vec{u} \times \vec{u}) \cdot \vec{u} = \vec{0} \cdot \vec{u} = (0)u_1 + (0)u_2 + (0)u_3 = 0 + 0 + 0 = 0$ <u>true</u>

h) using result of part b

$\vec{u} \times \vec{v} = \{u_2 v_3 - u_3 v_2\} \vec{i} + \{u_3 v_1 - u_1 v_3\} \vec{j} + \{u_1 v_2 - u_2 v_1\} \vec{k}$

$(\vec{u} \times \vec{v}) \cdot \vec{u} = \{u_2 v_3 - u_3 v_2\} u_1 + \{u_3 v_1 - u_1 v_3\} u_2 + \{u_1 v_2 - u_2 v_1\} u_3$

$= u_1 u_2 v_3 - u_1 u_3 v_2 + u_2 u_3 v_1 - u_1 u_2 v_3 + u_1 u_3 v_2 - u_2 u_3 v_1$

$= 0$

$\vec{v} \cdot (\vec{u} \times \vec{v}) = v_1 \{u_2 v_3 - u_3 v_2\} + v_2 \{u_3 v_1 - u_1 v_3\} + v_3 \{u_1 v_2 - u_2 v_1\}$

$= u_2 v_1 v_3 - u_3 v_1 v_2 + u_3 v_1 v_2 - u_1 v_2 v_3 + u_1 v_2 v_3 - u_2 v_1 v_3$

$= 0$

<u>true</u>

30)

$$(\vec{i}\times\vec{j})\times\vec{j} = (\vec{k})\times\vec{j} = -\vec{i}$$

$$\vec{i}\times(\vec{j}\times\vec{j}) = \vec{i}\times(\vec{0}) = \vec{0}$$

The cross product of vectors is not associative.

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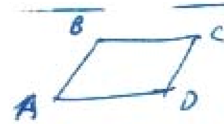
36) $A(0,0)$ $B(7,3)$ $C(9,8)$ $D(2,5)$



$\vec{AB} = \{(7)-(0)\}\vec{i} + \{(3)-(0)\}\vec{j} = 7\vec{i}+3\vec{j}$, $\vec{DC} = \{(9)-(2)\}\vec{i} + \{(8)-(5)\}\vec{j} = 7\vec{i}+3\vec{j}$

$\vec{AB}$ and $\vec{DC}$ are parallel

$\vec{BC} = \{(9)-(7)\}\vec{i} + \{(8)-(3)\}\vec{j} = 2\vec{i}+5\vec{j}$, $\vec{AD} = \{(2)-(0)\}\vec{i} + \{(5)-(0)\}\vec{j} = 2\vec{i}+5\vec{j}$

$\vec{BC}$ and $\vec{AD}$ are parallel

$$\vec{AB}\times\vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 3 & 0 \\ 2 & 5 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 7 & 0 \\ 2 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 7 & 3 \\ 2 & 5 \end{vmatrix}\vec{k}$$

$$= \{(3)(0)-(0)(5)\}\vec{i} - \{(7)(0)-(0)(2)\}\vec{j} + \{(7)(5)-(3)(2)\}\vec{k}$$

$$= \{0\}\vec{i} - \{0\}\vec{j} + \{35-6\}\vec{k} = 29\vec{k}$$

$$\text{Area} = |\vec{AB}\times\vec{AD}| = \sqrt{(0)^2+(0)^2+(29)^2} = \sqrt{(29)^2} = 29 \text{ unit}^2$$

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38) $A(-6,0)$ $B(1,-4)$ $C(3,1)$ $D(-4,5)$

$\vec{AB} = \{(1)-(-6)\}\vec{i} + \{(-4)-(0)\}\vec{j} = 7\vec{i}-4\vec{j}$ $\vec{AB}$ and $\vec{DC}$ are

$\vec{DC} = \{(-4)-(3)\}\vec{i} + \{(5)-(1)\}\vec{j} = -7\vec{i}+4\vec{j}$ parallel

38) continued

$\overrightarrow{BC} = \{(3)-(1)\}\vec{i} + \{(1)-(-4)\}\vec{j} = 2\vec{i}+5\vec{j}$ $\overrightarrow{BC}$ and $\overrightarrow{AD}$ are

$\overrightarrow{AD} = \{(-4)-(-6)\}\vec{i} + \{(5)-(0)\}\vec{j} = 2\vec{i}+5\vec{j}$ parallel

$$\overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & -4 & 0 \\ 2 & 5 & 0 \end{vmatrix} = \begin{vmatrix} -4 & 0 \\ 5 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 7 & 0 \\ 2 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 7 & -4 \\ 2 & 5 \end{vmatrix}\vec{k}$$

$$= \{(-4)(0)-(0)(5)\}\vec{i} - \{(7)(0)-(0)(2)\}\vec{j} + \{(7)(5)-(-4)(2)\}\vec{k}$$

$$= \{0\}\vec{i} - \{0\}\vec{j} + \{35+8\}\vec{k} = 43\vec{k}$$

Area $= |\overrightarrow{AB}\times\overrightarrow{AD}| = \sqrt{(0)^2+(0)^2+(43)^2} = \sqrt{(43)^2} = 43$ units^2

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40) $A(1,0,-1)$ $B(1,7,2)$ $C(2,4,-1)$ $D(0,3,2)$

$\overrightarrow{AB} = \{(1)-(1)\}\vec{i} + \{(7)-(0)\}\vec{j} + \{(2)-(-1)\}\vec{k} = 7\vec{j}+3\vec{k}$ $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are

$\overrightarrow{DC} = \{(2)-(0)\}\vec{i} + \{(4)-(3)\}\vec{j} + \{(-1)-(2)\}\vec{k} = 2\vec{i}+\vec{j}-3\vec{k}$ not parallel

$\overrightarrow{AC} = \{(2)-(1)\}\vec{i} + \{(4)-(0)\}\vec{j} + \{(-1)-(-1)\}\vec{k} = \vec{i}+4\vec{j}$ $\overrightarrow{AC}$ and $\overrightarrow{DB}$ are

$\overrightarrow{DB} = \{(1)-(0)\}\vec{i} + \{(7)-(3)\}\vec{j} + \{(2)-(2)\}\vec{k} = \vec{i}+4\vec{j}$ parallel

$\overrightarrow{AD} = \{(0)-(1)\}\vec{i} + \{(3)-(0)\}\vec{j} + \{(2)-(-1)\}\vec{k} = -\vec{i}+3\vec{j}+3\vec{k}$ $\overrightarrow{AD}$ and $\overrightarrow{CB}$ are

$\overrightarrow{CB} = \{(1)-(2)\}\vec{i} + \{(7)-(4)\}\vec{j} + \{(2)-(-1)\}\vec{k} = -\vec{i}+3\vec{j}+3\vec{k}$ parallel

$$\overrightarrow{AC}\times\overrightarrow{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 0 \\ -1 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 3 & 3 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix}\vec{k}$$

$$= \{(4)(3)-(0)(3)\}\vec{i} - \{(1)(3)-(0)(-1)\}\vec{j} + \{(1)(3)-(4)(-1)\}\vec{k}$$

$$= \{12\}\vec{i} - \{3\}\vec{j} + \{7\}\vec{k} = 12\vec{i}-3\vec{j}+7\vec{k}$$

Area $= |\overrightarrow{AC}\times\overrightarrow{AD}| = \sqrt{(12)^2+(-3)^2+(7)^2} = \sqrt{144+9+49} = \sqrt{202}$ units^2

42) $A(-1,-1)$ $B(3,3)$ $C(2,1)$

$\vec{AB} = \{(3)-(-1)\}\vec{i} + \{(3)-(-1)\}\vec{j} = 4\vec{i}+4\vec{j}$ $\vec{AC} = \{(2)-(-1)\}\vec{i} + \{(1)-(-1)\}\vec{j} = 3\vec{i}+2\vec{j}$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 4 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 2 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 4 & 0 \\ 3 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 4 & 4 \\ 3 & 2 \end{vmatrix}\vec{k}$$

$$= \{(4)(0)-(0)(2)\}\vec{i} - \{(4)(0)-(0)(3)\}\vec{j} + \{(4)(2)-(4)(3)\}\vec{k} = \{0\}\vec{i} - \{0\}\vec{j} + \{8-12\}\vec{k} = -4\vec{k}$$

$$\text{Area} = \tfrac{1}{2}\left|\vec{AB}\times\vec{AC}\right| = \tfrac{1}{2}\sqrt{(0)^2+(0)^2+(-4)^2} = \tfrac{1}{2}(4) = 2 \text{ units}^2$$

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44) $A(-6,0)$ $B(10,-5)$ $C(-2,4)$

$\vec{AB} = \{(10)-(-6)\}\vec{i} + \{(-5)-(0)\}\vec{j} = 16\vec{i}-5\vec{j}$ $\vec{AC} = \{(-2)-(-6)\}\vec{i} + \{(4)-(0)\}\vec{j} = 4\vec{i}+4\vec{j}$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 16 & -5 & 0 \\ 4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} -5 & 0 \\ 4 & 0 \end{vmatrix}\vec{i} - \begin{vmatrix} 16 & 0 \\ 4 & 0 \end{vmatrix}\vec{j} + \begin{vmatrix} 16 & -5 \\ 4 & 4 \end{vmatrix}\vec{k}$$

$$= \{(-5)(0)-(0)(4)\}\vec{i} - \{(16)(0)-(0)(4)\}\vec{j} + \{(16)(4)-(-5)(4)\}\vec{k} = \{0\}\vec{i} - \{0\}\vec{j} + \{64+20\}\vec{k}$$

$$= 84\vec{k}$$ $$\text{Area} = \tfrac{1}{2}\left|\vec{AB}\times\vec{AC}\right| = \tfrac{1}{2}\sqrt{(0)^2+(0)^2+(84)^2} = \tfrac{1}{2}(84) = 42 \text{ units}^2$$

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46) $A(0,0,0)$ $B(-1,1,-1)$ $C(3,0,3)$

$\vec{AB} = \{(-1)-(0)\}\vec{i} + \{(1)-(0)\}\vec{j} + \{(-1)-(0)\}\vec{k} = -\vec{i}+\vec{j}-\vec{k}$ $\vec{AC} = \{(3)-(0)\}\vec{i} + \{(0)-(0)\}\vec{j} + \{(3)-(0)\}\vec{k} = 3\vec{i}+3\vec{k}$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & -1 \\ 3 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix}\vec{i} - \begin{vmatrix} -1 & -1 \\ 3 & 3 \end{vmatrix}\vec{j} + \begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}\vec{k}$$

$$= \{(1)(3)-(-1)(0)\}\vec{i} - \{(-1)(3)-(-1)(3)\}\vec{j} + \{(-1)(0)-(1)(3)\}\vec{k} = \{3\}\vec{i} - \{0\}\vec{j} + \{-3\}\vec{k} = 3\vec{i} - 3\vec{k}$$

$$\text{Area} = \tfrac{1}{2}\left|\vec{AB}\times\vec{AC}\right| = \tfrac{1}{2}\sqrt{(3)^2+(0)^2+(-3)^2} = \tfrac{1}{2}\sqrt{9+9} = \tfrac{1}{2}\sqrt{9(1+1)} = \frac{3\sqrt{2}}{2} \text{ units}^2$$