The Distance Between $P_1(x_1, y_1, z_1)$ **and** $P_2(x_2, y_2, z_2)$

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The Standard Equation for the Sphere of Radius a **and Center** (x_0, y_0, z_0)

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

2) $x=-1,\ z=0 \quad (-1, y, 0)$

line through the point $(-1,0,0)$ parallel to y-axis.

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4) $x=1,\ y=0 \quad (1, 0, z)$

line through the point $(1,0,0)$ parallel to z-axis

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6) $x^2+y^2=4\ ,\ z=-2$

circle $x^2+y^2=(2)^2$ in the plane $z=-2$

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8) $y^2+z^2=1\ ,\ x=0$

circle $y^2+z^2=(1)^2$ in the yz plane

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10) $x^2+y^2+z^2=25\ ,\ y=-4$

$x^2+(-4)^2+z^2=25$

$x^2+16+z^2=25$

$x^2+z^2=9$

$x^2+z^2=(3)^2$

circle $x^2+z^2=(3)^2$ in the plane $y=-4$

12) $x^2+(y-1)^2+z^2=4$, $y=0$

$x^2+((0)-1)^2+z^2=4$

$x^2+1+z^2=4$

$x^2+z^2=3$

circle $x^2+z^2=(\sqrt{3})^2$

in the xz plane

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14) $x^2+y^2+z^2=4$, $y=x$

circle formed by the intersection $x^2+y^2+z^2=4$
and the plane $y=x$

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16) $z=y^2$, $x=1$

parabola $z=y^2$ in the plane $x=1$

18-a) $0 \le x \le 1$

stack of yz planes bounded by planes $x=0$ and $x=1$

18-b) $0 \le x \le 1$, $0 \le y \le 1$

rectangular column bounded by the planes $x=0, x=1, y=0, y=1$

18-c) $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

a cube generated by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$
"The unit cube in the first octant having one vertex at the origin"

20-a) $x^2+y^2 \le 1$, $z=0$

filled in circle with circumference $x^2+y^2=1$ in the xy-plane

20-b) $x^2+y^2 \le 1$, $z=3$

filled in circle with circumference $x^2+y^2=1$ in the plane $z=3$

20-c) $x^2+y^2 \le 1$, no restriction on z

solid cylindrical column ["rod"] of radius 1 along z-axis

22-a) $x=y$, $z=0$

the line $y=x$ in the xy plane

22-b) $x=y$, no restriction on z

the plane $y=x$ consisting of all points of the form (x,x,z)

24-a) $z=1-y$, no restriction on x

all points on the plane $z=1-y$

24-b) $z=y^3$, $x=2$

all points on the curve $z=y^3$ in the plane $x=2$

26) $P_1(-1,1,5)$, $P_2(2,5,0)$

$$|P_1P_2| = \sqrt{(2-(-1))^2+(5-1)^2+(0-5)^2} = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$$

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28) $P_1(3,4,5)$, $P_2(2,3,4)$

$$|P_1P_2| = \sqrt{(2-3)^2+(3-4)^2+(4-5)^2} = \sqrt{1+1+1} = \sqrt{3}$$

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30) $P_1(5,3,-2)$, $P_2(0,0,0)$

$$|P_1P_2| = \sqrt{(0-5)^2+(0-3)^2+(0-(-2))^2} = \sqrt{25+9+4} = \sqrt{38}$$

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32) point $(-2,1,4)$

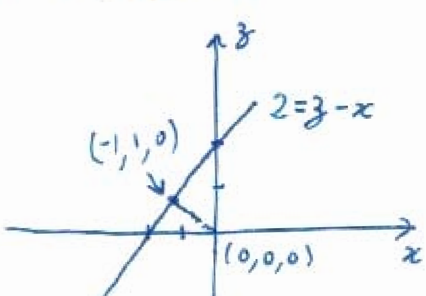
a) plane $x=3$: distance is $|3-(-2)| = |5| = 5$

b) plane $y=-5$: distance is $|-5-1| = |-6| = 6$

c) plane $z=-1$: distance is $|-1-4| = |-5| = 5$

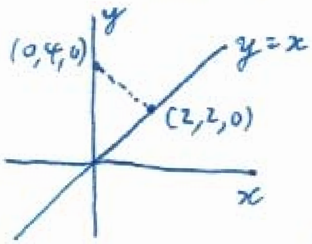
34-a) since $z=3$ plane is parallel to x-axis, distance is $|3-0| = |3| = 3$

34-b) on $y=0$ (xz) plane, the coordinate of $2=z-x$ which has the shortest distance ($\perp$) from the origin is $(-1,0,0)$



distance is $\sqrt{(-1-0)^2+(1-0)^2+(0-0)^2} = \sqrt{1+1+0} = \sqrt{2}$

34-c) on $z=0$ (xy) plane, the coordinate of $y=x$ which has the shortest distance $(\perp)$ from the point $(0,4,0)$ is $(2,2,0)$



distance is $\sqrt{(2-0)^2+(2-4)^2+(0-0)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

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36) plane through the point $(3,1,-2)$ $\perp$ to

a) x-axis : $x=3$ b) y-axis : $y=1$ c) z-axis : $z=-2$

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38) circle of radius 2 centered at $(0,0,0)$

a) xy-plane : $x^2+y^2=(2)^2$, $z=0$

b) yz-plane : $y^2+z^2=(2)^2$, $x=0$

c) xz-plane : $x^2+z^2=(2)^2$, $y=0$

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40) circle of radius 1 centered at $(-3,4,1)$ $\parallel$ to

a) xy-plane : $(x-(-3))^2+(y-4)^2=(1)^2$, $z=1$

b) yz-plane : $(y-4)^2+(z-1)^2=(1)^2$, $x=-3$

c) xz-plane : $(x-(-3))^2+(z-1)^2=(1)^2$, $y=4$

42) distance from origin $(0,0,0)$: $\sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = \sqrt{x^2+y^2+z^2}$

distance from $(0,2,0)$: $\sqrt{(x-0)^2+(y-2)^2+(z-0)^2} = \sqrt{x^2+(y-2)^2+z^2}$

now set these equation equal and solve.

$$\sqrt{x^2+(y-2)^2+z^2} = \sqrt{x^2+y^2+z^2}$$
$$x^2+(y-2)^2+z^2 = x^2+y^2+z^2$$
$$(y-2)^2 = y^2$$
$$y^2-4y+4 = y^2$$
$$0 = 4y-4$$
$$0 = 4(y-1)$$
$$y-1=0$$
$$y=1$$

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44) set of points 2 units from $(0,0,1)$ $(x-0)^2+(y-0)^2+(z-1)^2=(2)^2$

$x^2+y^2+(z-1)^2=4$

set of points 2 units from $(0,0,-1)$ $(x-0)^2+(y-0)^2+(z-(-1))^2=(2)^2$

$x^2+y^2+(z+1)^2=4$

since both equations equal to 4, we can set them equal and solve.

$$x^2+y^2+(z-1)^2 = x^2+y^2+(z+1)^2$$
$$(z-1)^2 = (z+1)^2$$
$$z^2-2z+1 = z^2+2z+1$$
$$0=4z$$
$$0=z$$

this occurs on plane $z=0$ (xy-plane).

$$x^2+y^2+(0-1)^2=4$$
$$x^2+y^2+1=4$$
$$x^2+y^2=3, \quad z=0$$

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52) $(x-1)^2+\left(y+\frac{1}{2}\right)^2+(z+3)^2=25$

$\left(x-(1)\right)^2+\left(y-\left(\frac{-1}{2}\right)\right)^2+\left(z-(-3)\right)^2=(5)^2$

center: $(1,\frac{-1}{2},-3)$; $r=5$

54) $x^2 + \left(y + \frac{1}{3}\right)^2 + \left(z - \frac{1}{3}\right)^2 = \frac{16}{9}$

$(x-(0))^2 + \left(y - \left(\frac{-1}{3}\right)\right)^2 + \left(z - \left(\frac{1}{3}\right)\right)^2 = \left(\frac{4}{3}\right)^2$

center: $\left(0, \frac{-1}{3}, \frac{1}{3}\right)$

$r = \frac{4}{3}$

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56) $x^2 + y^2 + z^2 - 6y + 8z = 0$

$\frac{b_y}{2} = \frac{-6}{2} = -3 \quad \left(\frac{b_y}{2}\right)^2 = (-3)^2 = 9$

$\frac{b_z}{2} = \frac{8}{2} = 4 \quad \left(\frac{b_z}{2}\right)^2 = (4)^2 = 16$

$x^2 + y^2 - 6y + z^2 + 8z = 0$

$x^2 + y^2 - 6y + 9 + z^2 + 8z + 16 = 0 + 9 + 16$

$x^2 + (y-3)^2 + (z+4)^2 = 25$

$(x-(0))^2 + (y-(3))^2 + (z-(-4))^2 = (5)^2$

center: $(0, 3, -4)$

$r = 5$

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58) $3x^2 + 3y^2 + 3z^2 + 2y - 2z = 9$

$\frac{b_y}{2} = \frac{\frac{2}{3}}{2} = \frac{1}{3} \quad \left(\frac{b_y}{2}\right) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\frac{b_z}{2} = \frac{\frac{-2}{3}}{2} = \frac{-1}{3} \quad \left(\frac{b_z}{2}\right) = \left(\frac{-1}{3}\right)^2 = \frac{1}{9}$

$3x^2 + 3y^2 + 2y + 3z^2 - 2z = 9$

$x^2 + y^2 + \frac{2}{3}y + z^2 - \frac{2}{3}z = 3$

$x^2 + y^2 + \frac{2}{3}y + \frac{1}{9} + z^2 - \frac{2}{3}z + \frac{1}{9} = 3 + \frac{1}{9} + \frac{1}{9}$

$x^2 + \left(y + \frac{1}{3}\right)^2 + \left(z - \frac{1}{3}\right)^2 = \frac{27}{9} + \frac{2}{9} = \frac{29}{9}$

$(x-(0))^2 + \left(y - \left(\frac{-1}{3}\right)\right)^2 + \left(z - \left(\frac{1}{3}\right)\right)^2 = \left(\frac{\sqrt{29}}{3}\right)^2$

center: $\left(0, \frac{-1}{3}, \frac{1}{3}\right)$

$r = \frac{\sqrt{29}}{3}$

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60) $(x-1)^2 + (y-2)^2 + (z+1)^2 = 103 + 2x + 4y - 2z$

$\frac{b_x}{2} = \frac{-4}{2} = -2 \quad \left(\frac{b_x}{2}\right)^2 = (-2)^2 = 4$

$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 2z + 1 = 103 + 2x + 4y - 2z$

$\frac{b_y}{2} = \frac{-8}{2} = -4 \quad \left(\frac{b_y}{2}\right)^2 = (-4)^2 = 16$

$x^2 - 4x + y^2 - 8y + z^2 + 4z = 97$

$\frac{b_z}{2} = \frac{4}{2} = 2 \quad \left(\frac{b_z}{2}\right)^2 = (2)^2 = 4$

$x^2 - 4x + 4 + y^2 - 8y + 16 + z^2 + 4z + 4 = 97 + 4 + 16 + 4$

$(x-2)^2 + (y-4)^2 + (z+2)^2 = 121$

$(x-(2))^2 + (y-(4))^2 + (z-(-2))^2 = (11)^2$

center: $(2, 4, -2)$; $r = 11$

62) center: $(0,-1,5)$; $r=2$

$$(x-(0))^2+(y-(-1))^2+(z-(5))^2=(2)^2$$
$$x^2+(y+1)^2+(z-5)^2=4$$

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64) center: $(0,-7,0)$; $r=7$

$$(x-(0))^2+(y-(-7))^2+(z-(0))^2=(7)^2$$
$$x^2+(y+7)^2+z^2=49$$

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66) formula for distance from $P(x,y,z)$ to the

a) xy-plane: $(x,y,0)$ distance is z

b) yz-plane: $(0,y,z)$ distance is x

c) xz-plane: $(x,0,z)$ distance is y

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72) point equidistant from points $(0,0,0)$, $(0,4,0)$, $(3,0,0)$, $(2,2,-3)$

distance from $(0,0,0) \Rightarrow \sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = \sqrt{x^2+y^2+z^2}$

distance from $(0,4,0) \Rightarrow \sqrt{(x-0)^2+(y-4)^2+(z-0)^2} = \sqrt{x^2+(y-4)^2+z^2}$

distance from $(3,0,0) \Rightarrow \sqrt{(x-3)^2+(y-0)^2+(z-0)^2} = \sqrt{(x-3)^2+y^2+z^2}$

distance from $(2,2,-3) \Rightarrow \sqrt{(x-2)^2+(y-2)^2+(z-(-3))^2} = \sqrt{(x-2)^2+(y-2)^2+(z+3)^2}$

point equidistant will satisfy

$$\sqrt{x^2+y^2+z^2} = \sqrt{x^2+(y-4)^2+z^2} = \sqrt{(x-3)^2+y^2+z^2} = \sqrt{(x-2)^2+(y-2)^2+(z+3)^2}$$

now solve the following

$$\sqrt{x^2+y^2+z^2} = \sqrt{x^2+(y-4)^2+z^2}$$
$$x^2+y^2+z^2 = x^2+(y-4)^2+z^2$$
$$y^2=(y-4)^2$$
$$y^2=y^2-8y+16$$

$$8y-16=0$$
$$8(y-2)=0$$
$$y=2$$

72) continued...

$$\sqrt{x^2+y^2+z^2} = \sqrt{(x-3)^2+y^2+z^2}$$
$$x^2+y^2+z^2 = (x-3)^2+y^2+z^2$$
$$x^2 = (x-3)^2$$
$$x^2 = x^2-6x+9$$

$$6x-9=0$$
$$3(2x-3)=0$$
$$2x-3=0$$
$$2x=3$$
$$x=\frac{3}{2}$$

$$\sqrt{x^2+y^2+z^2} = \sqrt{(x-2)^2+(y-2)^2+(z+3)^2}$$
$$x^2+y^2+z^2 = (x-2)^2+(y-2)^2+(z+3)^2$$
$$x^2+y^2+z^2 = (x^2-4x+4)+(y^2-4y+4)+(z^2+6z+9)$$

$$0 = -4x+4-4y+4+6z+9$$

$$4x+4y-17=6z$$

combining with previous results,

$$6z = 4\left(\tfrac{3}{2}\right)+4(2)-17 = 12+8-17 = 3$$

$$z = \frac{3}{6} = \frac{1}{2}$$

so the equidistant point is $\left(\frac{3}{2}, 2, \frac{1}{2}\right)$