Corollary of Theorem 6

A series $\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if its partial sums are bounded from above.

$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ this is harmonic series and this series diverges.

Theorem 9 - The Integral Test

Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a continuous, positive, decreasing function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)\, dx$ both converge or both diverge.

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ converges if $p > 1$, diverges if $p \leq 1$.

Bounds for the Remainder in the Integral Test

Suppose $\{a_n\}$ is a sequence of positive terms with $a_k = f(k)$, where f is a continuous positive decreasing function of x for all $x \geq n$, and that $\sum a_n$ converges to S. Then the remainder $R_n = S - s_n$ satisfies the inequalities

$$\int_{n+1}^{\infty} f(x)\, dx \leq R_n \leq \int_n^{\infty} f(x)\, dx$$

4) $\sum_{n=1}^{\infty} \frac{1}{n+4}$

$f(x) = \frac{1}{x+4}$ is positive, continuous, and decreasing for $x \geq 1$

$$\int_1^{\infty} \frac{1}{x+4}\,dx = \lim_{U\to\infty} \int_1^U \frac{1}{x+4}\,dx = \lim_{U\to\infty} \Big[\ln|x+4| + C\Big]_1^U$$

$$= \lim_{U\to\infty} \Big\{ \Big[\underbrace{\ln|U+4|}_{+\infty} + C\Big] - \Big[\ln|(1)+4| + C\Big] \Big\} = \infty$$

since $\int_1^{\infty} \frac{1}{x+4}\,dx$ diverges, $\sum_{n=1}^{\infty} \frac{1}{n+4}$ diverges

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8) $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$

$f(x) = \frac{\ln(x^2)}{x}$ is positive and continuous for $x \geq 2$

$$\frac{df}{dx} = \frac{(x)\left[\frac{1}{x^2}(2x)\right] - (\ln(x^2))[1]}{(x)^2} = \frac{2 - \ln(x^2)}{x^2}$$

$\frac{df}{dx} < 0$ for $x > e$ so $f(x)$ is decreasing for $x \geq 3$

$p = \ln(x^2)$

$dp = \frac{1}{x^2}(2x)\,dx \Rightarrow 2\,dp = \frac{1}{x}\,dx$

$$\int \frac{\ln(x^2)}{x}\,dx = \int p\,(2\,dp) = p + C = \ln(x^2) + C$$

$$\int_3^{\infty} \frac{\ln(x^2)}{x}\,dx = \lim_{U\to\infty} \int_3^U \frac{\ln(x^2)}{x}\,dx = \lim_{U\to\infty} \Big[\ln(x^2) + C\Big]_3^U$$

$$= \lim_{U\to\infty} \Big\{ \Big[\underbrace{\ln(U^2)}_{+\infty} + C\Big] - \Big[\ln((3)^2) + C\Big] \Big\} = +\infty$$

since $\int_3^{\infty} \frac{\ln(x^2)}{x}\,dx$ diverges, $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n} = \frac{\ln((2)^2)}{(2)} + \underbrace{\sum_{n=3}^{\infty} \frac{\ln(n^2)}{n}}_{\text{diverges}}$ and $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$ diverges

10) $\displaystyle\sum_{n=2}^{\infty} \frac{n-4}{n^2-2n+1} = \sum_{n=2}^{\infty} \frac{n-4}{(n-1)^2}$

$f(x) = \frac{x-4}{x^2-2x+1} = \frac{x-4}{(x-1)^2}$ is continuous for $x \geq 2$ and $f(x) > 0$ for $x > 4$

$$\frac{df}{dx} = \frac{((x-1)^2)[1] - (x-4)[2(x-1)'(1)]}{((x-1)^2)^2} = \frac{(x-1)\{(x-1)[1] - (x-4)[2]\}}{(x-1)^4} = \frac{x-1-2x+8}{(x-1)^3} = \frac{7-x}{(x-1)^3}$$

$\frac{df}{dx} < 0$ for $x < 7$ and $f(x)$ is decreasing for $x \geq 8$.

$$\int \frac{x-4}{(x-1)^2}\,dx = \int \frac{x-1-3}{(x-1)^2}\,dx = \int \left(\frac{x-1}{(x-1)^2} - \frac{3}{(x-1)^2}\right)dx = \int \left(\frac{1}{x-1} - \frac{3}{(x-1)^2}\right)dx$$

$$= \ln|x-1| + \frac{3}{x-1} + C$$

$$\int_8^{\infty} \frac{x-4}{(x-1)^2}\,dx = \lim_{U\to\infty} \int_8^{U} \frac{x-4}{(x-1)^2}\,dx = \lim_{U\to\infty} \left[\ln|x-1| + \frac{3}{x-1} + C\right]_8^U$$

$$= \lim_{U\to\infty} \left\{\left[\underbrace{\ln|U-1|}_{+\infty} + \frac{3}{U-1} + C\right] - \left[\ln|(8)-1| + \frac{3}{(8)-1} + C\right]\right\} = +\infty$$

since $\int_8^{\infty} \frac{x-4}{(x-1)^2}\,dx$ diverges,

$$\sum_{n=2}^{\infty} \frac{n-4}{n^2-2n+1} = \left(\frac{(2)-4}{(2)^2-2(2)+1}\right) + \left(\frac{(3)-4}{(3)^2-2(3)+1}\right) + \left(\frac{(4)-4}{(4)^2-2(4)+1}\right) + \left(\frac{(5)-4}{(5)^2-2(5)+1}\right)$$

$$+ \left(\frac{(6)-4}{(6)^2-2(6)+1}\right) + \left(\frac{(7)-4}{(7)^2-2(7)+1}\right) + \underbrace{\sum_{n=8}^{\infty} \frac{n-4}{n^2-2n+1}}_{\text{diverges}}$$

and $\displaystyle\sum_{n=2}^{\infty} \frac{n-4}{n^2-2n+1}$ diverges

14) $$\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{1}{e^1} + \frac{1}{e^2} + \frac{1}{e^3} + \cdots + \frac{1}{e^n} + \cdots$$

$$= \frac{1}{e}(1) + \frac{1}{e}\left(\frac{1}{e}\right)^1 + \frac{1}{e}\left(\frac{1}{e}\right)^2 + \cdots + \frac{1}{e}\left(\frac{1}{e}\right)^{n-1} + \cdots$$

this is a geometric series with $a = \frac{1}{e}$ and $r = \frac{1}{e}$

since $|r| = |\frac{1}{e}| < 1$, the series converges

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16) $$\sum_{n=1}^{\infty} \frac{5}{n+1}$$

use the integral test, $f(x) = \frac{5}{x+1}$

$$\int_1^{\infty} \frac{5}{x+1}\,dx = \lim_{U\to\infty} \int_1^{U} \frac{5}{x+1}\,dx = \lim_{U\to\infty} \Big[5\ln|x+1| + C\Big]_1^U$$

$$= \lim_{U\to\infty} \left\{ \Big[\underbrace{5\ln|U+1|}_{+\infty} + C\Big] - \Big[5\ln|(1)+1| + C\Big] \right\} = +\infty$$

since $\int_1^{\infty} \frac{5}{x+1}\,dx$ diverges, $\sum_{n=1}^{\infty} \frac{5}{n+1}$ diverges

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18) $$\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{-2}{n^{3/2}} = -2\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$ this is a p-series

$$= -2\left\{ \frac{1}{(1)^{3/2}} + \frac{1}{(2)^{3/2}} + \frac{1}{(3)^{3/2}} + \cdots + \frac{1}{n^{3/2}} + \cdots \right\}$$

since $p = \frac{3}{2} > 1$, the series converges

20) $\sum_{n=1}^{\infty} \frac{-8}{n} = -8 \sum_{n=1}^{\infty} \frac{1}{n} = -8 \left\{ \frac{1}{(1)^1} + \frac{1}{(2)^1} + \frac{1}{(3)^1} + \cdots + \frac{1}{n^1} + \cdots \right\}$

this is a p-series

since $p = 1 \leq 1$, the series diverges

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22) $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$

use the integral test, $f(x) = \frac{\ln x}{\sqrt{x}}$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = (\ln x)(2\sqrt{x}) - \int (2\sqrt{x})\left(\tfrac{1}{x}\,dx\right)$$

$u_1 = \ln x \quad dv_1 = \frac{1}{\sqrt{x}}\,dx \quad = 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx$

$du_1 = \frac{1}{x}\,dx \quad v_1 = 2\sqrt{x} \quad = 2\sqrt{x}\ln x - 2[2\sqrt{x}] + C$

$$= 2\sqrt{x}\ln x - 4\sqrt{x} + C = 2\sqrt{x}(\ln x - 2) + C$$

$$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}\,dx = \lim_{U\to\infty} \int_2^{U} \frac{\ln x}{\sqrt{x}}\,dx = \lim_{U\to\infty} \Big[2\sqrt{x}(\ln x - 2) + C\Big]_2^{U}$$

$$= \lim_{U\to\infty} \Big\{ \big[\underbrace{2\sqrt{U}(\ln U - 2)}_{+\infty} + C\big] - \big[2\sqrt{(2)}(\ln(2) - 2) + C\big] \Big\} = +\infty$$

since $\int_2^{\infty} \frac{\ln x}{\sqrt{x}}\,dx$ diverges, $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ diverges

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24) $\sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$

use the nth-term test for divergence

$$\lim_{n\to\infty} \frac{\overset{+\infty}{5^n}}{\underset{+\infty}{4^n + 3}} \overset{L}{=} \lim_{n\to\infty} \frac{(\ln 5)5^n}{(\ln 4)4^n} = \lim_{n\to\infty} \left(\frac{\ln 5}{\ln 4}\right)\left(\frac{5}{4}\right)^n = +\infty \neq 0 \quad \text{because } \frac{5}{4} > 1$$

the series diverges

26) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

use the integral test, $f(x) = \frac{1}{2x-1} = \frac{1}{2(x-\frac{1}{2})}$

$$\int_1^{\infty} \frac{1}{2(x-\frac{1}{2})}\,dx = \lim_{U\to\infty} \int_1^U \frac{1}{2(x-\frac{1}{2})}\,dx = \lim_{U\to\infty} \left[\tfrac{1}{2}\ln|x-\tfrac{1}{2}|+C\right]_1^U$$

$$= \lim_{U\to\infty} \left\{ \left[\tfrac{1}{2}\underbrace{\ln|U-\tfrac{1}{2}|}_{+\infty}+C\right] - \left[\tfrac{1}{2}\ln|(1)-\tfrac{1}{2}|+C\right] \right\} = +\infty$$

since $\int_1^{\infty} \frac{1}{2x-1}\,dx$ diverges, $\sum_{n=1} \frac{1}{2n-1}$ diverges

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30) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

use the integral test, $f(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$

$p = \sqrt{x}+1$

$dp = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,dp = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx = \int \frac{1}{p}(2\,dp) = 2\ln|p|+C$$
$$= 2\ln|\sqrt{x}+1|+C$$

$$\int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx = \lim_{U\to\infty} \int_1^U \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx = \lim_{U\to\infty} \left[2\ln|\sqrt{x}+1|+C\right]_1^U$$

$$= \lim_{U\to\infty} \left\{ \left[2\underbrace{\ln|\sqrt{U}+1|}_{+\infty}+C\right] - \left[2\ln|\sqrt{(1)}+1|+C\right] \right\} = +\infty$$

since $\int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$ diverges, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ diverges

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32) $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n} = \frac{1}{(\ln 3)^1} + \frac{1}{(\ln 3)^2} + \frac{1}{(\ln 3)^3} + \cdots + \frac{1}{(\ln 3)^n} + \cdots$

$$= \frac{1}{\ln 3}(1) + \frac{1}{\ln 3}\left(\frac{1}{\ln 3}\right)^1 + \frac{1}{\ln 3}\left(\frac{1}{\ln 3}\right)^2 + \cdots + \frac{1}{\ln 3}\left(\frac{1}{\ln 3}\right)^{n-1} + \cdots$$

this is a geometric series with $a = \frac{1}{\ln 3}$ and $r = \frac{1}{\ln 3}$

since $|r| = \left|\frac{1}{\ln 3}\right| < 1$, the series converges

34) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$

use the integral test, $f(x) = \frac{1}{x(1+\ln^2 x)} = \frac{1}{(1+(\ln x)^2)\,x}$

$p = \ln x$ $\quad \int \frac{1}{(1+(\ln x)^2)x}\,dx = \int \frac{1}{1+p^2}\,dp = \int \frac{1}{(1)^2+p^2}\,dp$

$dp = \frac{1}{x}\,dx$

$$= \frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right) + C = \tan^{-1}(\ln x) + C$$

$$\int_1^{\infty} \frac{1}{(1+(\ln x)^2)x}\,dx = \lim_{U\to\infty} \int_1^{U} \frac{1}{(1+(\ln x)^2)x}\,dx = \lim_{U\to\infty} \left[\tan^{-1}(\ln x) + C\right]_1^U$$

$$= \lim_{U\to\infty} \left\{ \left[\tan^{-1}(\ln U) + C\right] - \left[\tan^{-1}(\ln(1)) + C\right] \right\}$$

$$= \left[\tan^{-1}(+\infty)\right] - \left[\tan^{-1}(0)\right] = \left[\frac{\pi}{2}\right] - [0] = \frac{\pi}{2}$$

the series converges

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36) $\sum_{n=1}^{\infty} n \tan \frac{1}{n}$

use the n th-term test for divergence

$$\lim_{n\to\infty} \overset{(+\infty)}{n}\, \overset{(0)}{\tan\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{\overset{0}{\tan\left(\frac{1}{n}\right)}}{\underset{0}{\left(\frac{1}{n}\right)}} \overset{L}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\left(\frac{-1}{n^2}\right)}{\left(\frac{-1}{n^2}\right)} = \lim_{n\to\infty} \sec^2\left(\frac{1}{n}\right)$$

$$= \sec^2(0) = (1)^2 = 1 \neq 0$$

the series diverges

38) $\sum_{x=1}^{\infty} \frac{2}{1+e^x}$

use the integral test, $f(x) = \frac{2}{1+e^x}$

$p = e^x \Rightarrow \ln p = x$

$\frac{1}{p} dp = dx$

$\frac{2}{(p)^1(p+1)^1} = \frac{A}{(p)^1} + \frac{B}{(p+1)^1}$

$2 = A(p+1) + B(p)$

constant term: $2 = A$

p-term: $0 = A + B$

$B = -A = -2$

$$\int \frac{2}{1+e^x} dx = \int \frac{2}{1+p}\left(\frac{1}{p} dp\right)$$

$$= \int \frac{2}{p(p+1)} dp$$

$$= \int \left(\frac{(2)}{(p)^1} + \frac{(-2)}{(p+1)^1}\right) dp$$

$$= 2\ln|p| - 2\ln|p+1| + C$$

$$= 2\ln|e^x| - 2\ln|e^x+1| + C$$

$$= 2\ln\left(\frac{e^x}{e^x+1}\right) + C$$

$$\int_1^{\infty} \frac{2}{1+e^x} dx = \lim_{U\to\infty} \int_1^U \frac{2}{1+e^x} dx = \lim_{U\to\infty} \left[2\ln\left(\frac{e^x}{e^x+1}\right) + C\right]_1^U$$

$$= \lim_{U\to\infty} \left\{\left[2\ln\left(\frac{e^U}{e^U+1}\right) + C\right] - \left[2\ln\left(\frac{e^{(1)}}{e^{(1)}+1}\right) + C\right]\right\}$$

$$= [0] - \left[2\ln\left(\frac{e}{e+1}\right)\right] = -2\ln\left(\frac{e}{e+1}\right)$$

because $\lim_{U\to\infty}\left\{2\ln\left(\frac{e^U}{e^U+1}\right)\right\} = 2\ln\left\{\lim_{U\to\infty}\left(\frac{\overset{+\infty}{e^U}}{\underset{+\infty}{e^U+1}}\right)\right\} \overset{L}{=} 2\ln\left\{\lim_{U\to\infty}\frac{e^U}{e^U}\right\}$

$= 2\ln\left\{\lim_{U\to\infty} 1\right\} = 2\ln\{1\} = 2(0) = 0$

the series converges

42) $\sum_{n=3}^{\infty} \frac{7}{\sqrt{n+1}\,\ln\sqrt{n+1}}$

use the integral test, $f(x) = \frac{7}{\sqrt{x+1}\,\ln\sqrt{x+1}}$

$\int \frac{7}{\sqrt{x+1}\,\ln\sqrt{x+1}}\,dx$ is not an easy integral, so we should use the Direct Comparison Test (of integrals)

note that $\ln y < y$

$$\Downarrow$$

$$\ln\sqrt{x+1} < \sqrt{x+1}$$

$$\Downarrow$$

$$\sqrt{x+1}\,\ln\sqrt{x+1} < \left(\sqrt{x+1}\right)\left(\sqrt{x+1}\right) = (x+1)$$

$$\Downarrow$$

$$\frac{1}{x+1} < \frac{1}{\sqrt{x+1}\,\ln\sqrt{x+1}}$$

$$\Downarrow$$

$$\int_3^{\infty} \frac{1}{x+1}\,dx < \int_3^{\infty} \frac{1}{\sqrt{x+1}\,\ln\sqrt{x+1}}\,dx$$

and

$$\int_3^{\infty} \frac{7}{x+1}\,dx < \int_3^{\infty} \frac{7}{\sqrt{x+1}\,\ln\sqrt{x+1}}\,dx$$

$$\int_3^{\infty} \frac{7}{x+1}\,dx = \lim_{u\to\infty} \int_3^{u} \frac{7}{x+1}\,dx$$

$$= \lim_{u\to\infty} \Big[7\ln|x+1| + C\Big]_3^u$$

$$= \lim_{u\to\infty} \Big\{\Big[\underbrace{7\ln|u+1|}_{+\infty} + C\Big] - \Big[7\ln|(3)+1| + C\Big]\Big\}$$

$$= +\infty$$

by the Direct Comparison Test,

$$+\infty = \int_3^{\infty} \frac{7}{x+1}\,dx < \int_3^{\infty} \frac{7}{\sqrt{x+1}\,\ln\sqrt{x+1}}\,dx = \infty$$

since $\int_3^{\infty} \frac{7}{\sqrt{x+1}\,\ln\sqrt{x+1}}\,dx$ diverges, $\sum_{n=3}^{\infty} \frac{7}{\sqrt{n+1}\,\ln\sqrt{n+1}}$ diverges

44) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

use the integral test, $f(x) = \frac{x}{x^2+1}$

$p = x^2+1$

$dp = 2x\,dx \Rightarrow \frac{1}{2}dp = x\,dx$

$$\int \frac{x}{x^2+1}\,dx = \int \frac{1}{p}\left(\frac{1}{2}dp\right) = \frac{1}{2}\ln|p| + C$$

$$= \frac{1}{2}\ln|x^2+1| + C$$

$$\int_1^{\infty} \frac{x}{x^2+1}\,dx = \lim_{U\to\infty}\int_1^U \frac{x}{x^2+1}\,dx = \lim_{U\to\infty}\left[\frac{1}{2}\ln|x^2+1| + C\right]_1^U$$

$$= \lim_{U\to\infty}\left\{\left[\frac{1}{2}\underbrace{\ln|U^2+1|}_{+\infty} + C\right] - \left[\frac{1}{2}\ln|(1)^2+1| + C\right]\right\} = +\infty$$

since $\int_1^{\infty}\frac{x}{x^2+1}\,dx$ diverges, $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ diverges

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46) $\sum_{n=1}^{\infty} \operatorname{sech}^2 n$

use the integral test, $f(x) = \operatorname{sech}^2 x$

$$\int_1^{\infty} \operatorname{sech}^2 x\,dx = \lim_{U\to\infty}\int_1^U \operatorname{sech}^2 x\,dx = \lim_{U\to\infty}\left[\tanh x + C\right]_1^U$$

$$= \lim_{U\to\infty}\left\{\left[\underbrace{\tanh U}_{+1} + C\right] - \left[\tanh(1) + C\right]\right\} = [1] - [\tanh 1] = 1 - \tanh 1$$

the series converges

48) $\sum_{n=3}^{\infty}\left(\frac{1}{n-1}-\frac{2a}{n+1}\right)$

use the integral test to get started $f(x)=\frac{1}{x-1}-\frac{2a}{x+1}$

$$\int_3^{\infty}\left(\frac{1}{x-1}-\frac{2a}{x+1}\right)dx=\lim_{U\to\infty}\int_3^{U}\left(\frac{1}{x-1}-\frac{2a}{x+1}\right)dx=\lim_{U\to\infty}\Big[\ln|x-1|-2a\ln|x+1|+C\Big]_3^U$$

$$=\lim_{U\to\infty}\Big[\ln|x-1|-\ln|(x+1)^{2a}|+C\Big]_3^U=\lim_{U\to\infty}\Big[\ln\left|\frac{x-1}{(x+1)^{2a}}\right|+C\Big]_3^U$$

$$=\lim_{U\to\infty}\left\{\Big[\ln\left|\frac{U-1}{(U+1)^{2a}}\right|+C\Big]-\Big[\ln\left|\frac{(3)-1}{((3)+1)^{2a}}\right|+C\Big]\right\}$$

$$=\lim_{U\to\infty}\Big[\ln\left|\frac{U-1}{(U+1)^{2a}}\right|\Big]-\Big[\ln\left|\frac{(2)}{(4)^{2a}}\right|\Big]$$

$$\lim_{U\to\infty}\frac{\overset{+\infty}{U-1}}{\underset{+\infty}{(U+1)^{2a}}}\overset{L}{=}\lim_{U\to\infty}\frac{1}{2a(U+1)^{2a-1}}$$

$2a-1=0$

$a=\frac{1}{2}$

if $a=\frac{1}{2}$, then $\lim_{U\to\infty}\frac{1}{2(U+1)^{2a}}$

$=\lim_{U\to\infty}\frac{1}{2(U+1)^{2(\frac{1}{2})}}=\lim_{U\to\infty}\frac{1}{2(U+1)}$

$=0$

so $\int_3^{\infty}\left(\frac{1}{x-1}-\frac{2a}{x+1}\right)dx=\Big[0\Big]-\Big[\ln\left|\frac{2}{4^{2(\frac{1}{2})}}\right|\Big]$

$=-\ln\left(\frac{2}{4}\right)=-\ln\left(\frac{1}{2}\right)=-\{\ln(1)-\ln(2)\}$

$=\ln 2$ the series converges.

if $a<\frac{1}{2}$, then $\lim_{U\to\infty}\frac{1}{2a(U+1)^{2a-1}}=+\infty$ and the series diverges.

if $a>\frac{1}{2}$, the terms of the series eventually become negative and the Integral Test does not apply. From that point on, however, the series behaves like a negative multiple of the harmonic series, and so the series diverges.