MATH 21200 section 8.7

Trapezoidal Rule

$$T_n = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$$

Simpson's Rule a.k.a. Parabolic Rule

$$S_n = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$$

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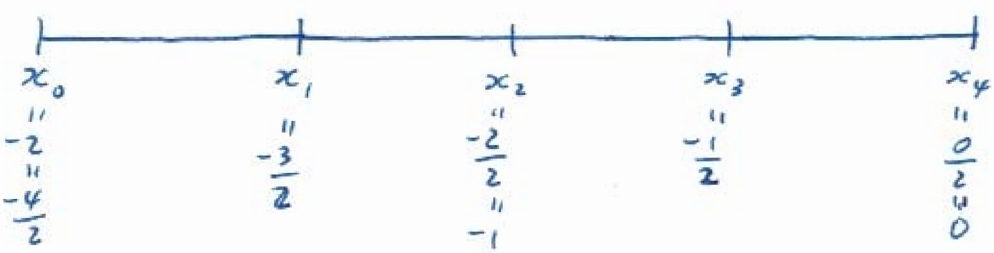
4) $\int_{-2}^{0}(x^2-1)\,dx \qquad f(x)=x^2-1, \quad a=-2,\ b=0, \quad n=4$

$$\Delta x = \frac{b-a}{n} = \frac{(0)-(-2)}{4} = \frac{2}{4} = \frac{1}{2}$$

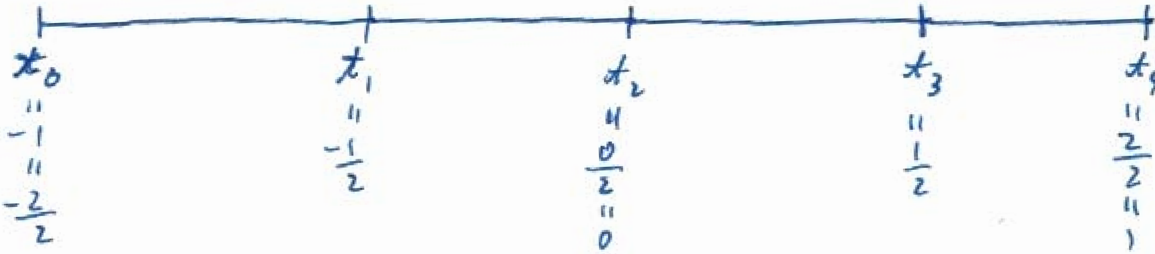


$$T_4 = \frac{\left(\frac{1}{2}\right)}{2}\left[\left((-2)^2-1\right)+2\left(\left(\tfrac{-3}{2}\right)^2-1\right)+2\left((-1)^2-1\right)+2\left(\left(\tfrac{-1}{2}\right)^2-1\right)+\left((0)^2-1\right)\right]$$

$$S_4 = \frac{\left(\frac{1}{2}\right)}{3}\left[\left((-2)^2-1\right)+4\left(\left(\tfrac{-3}{2}\right)^2-1\right)+2\left((-1)^2-1\right)+4\left(\left(\tfrac{-1}{2}\right)^2-1\right)+\left((0)^2-1\right)\right]$$

6) $\int_{-1}^{1} (t^3+1)\,dt \quad f(t)=t^3+1,\ a=-1,\ b=1,\ n=4$

$$\Delta t = \frac{b-a}{n} = \frac{(1)-(-1)}{4} = \frac{2}{4} = \frac{1}{2}$$

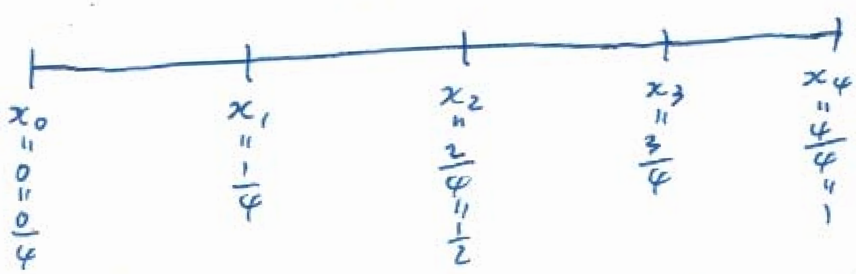


$$T_4 = \frac{\left(\frac{1}{2}\right)}{2}\left[\left((-1)^3+1\right) + 2\left(\left(\tfrac{-1}{2}\right)^3+1\right) + 2\left((0)^3+1\right) + 2\left(\left(\tfrac{1}{2}\right)^3+1\right) + \left((1)^3+1\right)\right]$$

$$S_4 = \frac{\left(\frac{1}{2}\right)}{3}\left[\left((-1)^3-1\right) + 4\left(\left(\tfrac{-1}{2}\right)^3+1\right) + 2\left((0)^3+1\right) + 4\left(\left(\tfrac{1}{2}\right)^3+1\right) + \left((1)^3+1\right)\right]$$

) $\int_{0}^{1} \sin \pi t\,dt\ ,\ f(t)=\sin(\pi t),\ a=0,\ b=1,\ n=4$

$$\Delta t = \frac{b-a}{n} = \frac{(1)-(0)}{4} = \frac{1}{4}$$



$$T_4 = \frac{\left(\frac{1}{4}\right)}{2}\left[\sin(\pi(0)) + 2\sin\left(\pi\left(\tfrac{1}{4}\right)\right) + 2\sin\left(\pi\left(\tfrac{1}{2}\right)\right) + 2\sin\left(\pi\left(\tfrac{3}{4}\right)\right) + \sin(\pi(1))\right]$$

$$S_4 = \frac{\left(\frac{1}{4}\right)}{3}\left[\sin(\pi(0)) + 4\sin\left(\pi\left(\tfrac{1}{4}\right)\right) + 2\sin\left(\pi\left(\tfrac{1}{2}\right)\right) + 4\sin\left(\pi\left(\tfrac{3}{4}\right)\right) + \sin(\pi(1))\right]$$

24) since this is a calculation exercise with heavy numerical computation, MS Excel was used.

We need to convert mph to ft/sec.

$$x \text{ mph} = x\left(\frac{\text{miles}}{\text{hour}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ miles}}\right)\left(\frac{1 \text{ hour}}{3600 \text{ sec}}\right) = \frac{528}{360}x \text{ ft/sec}$$

here $\Delta t_i = t_i - t_{i-1}$; each step of Trapezoidal rule is $\frac{\Delta t_i}{2}(v_i + v_{i-1})$

i	t_i	v_i (mph)	Δt_i	$\frac{\Delta t_i}{2}$	v_i (ft/sec)	$v_i + v_{i-1}$	$\frac{\Delta t_i}{2}(v_i + v_{i-1})$
0	0	0	0.00	0.00	0.00	0.00	0
1	2.2	30	2.20	1.10	44.00	44.00	48.4
2	3.2	40	1.00	0.50	58.67	102.67	51.33333
3	4.5	50	1.30	0.65	73.33	132.00	85.8
4	5.9	60	1.40	0.70	88.00	161.33	112.9333
5	7.8	70	1.90	0.95	102.67	190.67	181.1333
6	10.2	80	2.40	1.20	117.33	220.00	264
7	12.7	90	2.50	1.25	132.00	249.33	311.6667
8	16	100	3.30	1.65	146.67	278.67	459.8
9	20.6	110	4.60	2.30	161.33	308.00	708.4
10	26.2	120	5.60	2.80	176.00	337.33	944.5333
11	37.1	130	10.90	5.45	190.67	366.67	1998.333

$\sum \frac{\Delta t_i}{2}(v_i + v_{i-1}) =$ 5166.333

$$T = \frac{2.20}{2}(44-0) + \frac{1.00}{2}(58.67+44) + \frac{1.30}{2}(73.33+58.67) + \frac{1.40}{2}(88+73.33)$$

$$+ \frac{1.90}{2}(102.67+88) + \frac{2.40}{2}(117.33+102.67) + \frac{2.50}{2}(132+117.33)$$

$$+ \frac{3.30}{2}(146.67+132) + \frac{4.60}{2}(161.33+146.67) + \frac{5.60}{2}(176+161.33)$$

$$+ \frac{10.90}{2}(190.67+176) \approx \underline{5166.333 \text{ ft}}$$

$$T \approx 5166.333 \text{ ft}\left(\frac{1 \text{ mile}}{5280 \text{ ft}}\right) \approx \underline{0.978472159 \text{ miles}}$$