MATH 21200   section 8.2

6) $\int_1^e x^3 \ln x \, dx$

$$\int x^3 \ln x \, dx = (\ln x)\left(\frac{x^4}{4}\right) - \int \left(\frac{x^4}{4}\right)\left(\frac{1}{x}dx\right)$$

$u_1 = \ln x \quad dv_1 = x^3 dx$ $\quad = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 \, dx$

$du_1 = \frac{1}{x}dx \quad v_1 = \frac{x^4}{4}$ $\quad = \frac{1}{4}x^4 \ln x - \frac{1}{4}\left[\frac{x^4}{4}\right] + C$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$

$$\int_1^e x^3 \ln x \, dx = \left[\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C\right]_1^e$$

$$= \left[\frac{1}{4}(e)^4 \ln(e) - \frac{1}{16}(e)^4 + C\right] - \left[\frac{1}{4}(1)^4 \ln(1) - \frac{1}{16}(1)^4 + C\right]$$

$$= \left[\frac{1}{4}e^4(1) - \frac{1}{16}e^4\right] - \left[\frac{1}{4}(1)(0) - \frac{1}{16}(1)\right]$$

$$= \left[\frac{4e^4}{16} - \frac{e^4}{16}\right] - \left[\frac{-1}{16}\right] = \frac{3e^4}{16} + \frac{1}{16} = \frac{1+3e^4}{16}$$

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8) $\int x e^{3x} dx = (x)\left(\frac{1}{3}e^{3x}\right) - \int \left(\frac{1}{3}e^{3x}\right)(dx)$

$u_1 = x \quad dv_1 = e^{3x}dx$ $\quad = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x} dx$

$du_1 = dx \quad v_1 = \frac{1}{3}e^{3x}$ $\quad = \frac{1}{3}xe^{3x} - \frac{1}{3}\left[\frac{1}{3}e^{3x}\right] + C$

$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

10) $$\int (x^2-2x+1)e^{2x}\,dx = (x^2-2x+1)\left(\frac{e^{2x}}{2}\right) - \int \left(\frac{e^{2x}}{2}\right)(2x-2)\,dx$$

$u_1 = x^2-2x+1 \quad dv_1 = e^{2x}dx$

$du_1 = (2x-2)dx \quad v_1 = \frac{e^{2x}}{2}$

$u_2 = x-1 \quad dv_2 = e^{2x}dx$

$du_2 = dx \quad v_2 = \frac{e^{2x}}{2}$

$$= \frac{1}{2}(x^2-2x+1)e^{2x} - \int (x-1)e^{2x}\,dx$$

$$= \frac{1}{2}(x^2-2x+1)e^{2x} - \left\{(x-1)\left(\frac{e^{2x}}{2}\right) - \int\left(\frac{e^{2x}}{2}\right)(dx)\right\}$$

$$= \frac{1}{2}(x^2-2x+1)e^{2x} - \frac{1}{2}(x-1)e^{2x} + \frac{1}{2}\int e^{2x}dx$$

$$= \frac{1}{2}(x^2-2x+1)e^{2x} - \frac{1}{2}(x-1)e^{2x} + \frac{1}{2}\left[\frac{e^{2x}}{2}\right] + C$$

$$= \frac{1}{2}(x^2-2x+1)e^{2x} - \frac{1}{2}(x-1)e^{2x} + \frac{1}{4}e^{2x} + C$$

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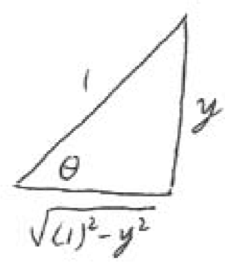
12) $\theta = \sin^{-1} y$

$\Downarrow$

$\sin\theta = y = \frac{y}{1}$

$\cos\theta \frac{d\theta}{dy} = 1$

$$\frac{d\theta}{dy} = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{\sqrt{(1)^2-y^2}}{1}\right)} = \frac{1}{\sqrt{1-y^2}}$$



$$\int \sin^{-1} y\,dy = (\sin^{-1}y)(y) - \int (y)\left(\frac{1}{\sqrt{1-y^2}}dy\right)$$

$u_1 = \sin^{-1} y \quad dv_1 = dy$

$du_1 = \frac{1}{\sqrt{1-y^2}}dy \quad v_1 = y$

$$= y\sin^{-1}y - \int \frac{y}{\sqrt{1-y^2}}dy$$

$$= y\sin^{-1}y - \left[-\sqrt{1-y^2}\right] + C$$

$$\int \frac{y}{\sqrt{1-y^2}}dy = \int \frac{1}{\sqrt{1-y^2}}(y\,dy)$$

$p = 1-y^2$

$dp = -2y\,dy$

$-\frac{1}{2}dp = y\,dy$

$$= \int \frac{1}{\sqrt{y}}\left(\frac{-1}{2}dy\right)$$

$$= \frac{-1}{2}\int y^{-\frac{1}{2}}dy$$

$$= \frac{-1}{2}\left[\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right] + D = -\sqrt{1-y^2} + D$$

$$= y\sin^{-1}y + \sqrt{1-y^2} + C$$

14) $\int 4x \sec^2 2x \, dx = \int (2x)\sec^2(2x)(2\,dx)$

$p = 2x \quad dp = 2\,dx \qquad = \int p \sec^2 p \, dp$

$u_1 = p \quad dv_1 = \sec^2 p \, dp \qquad = (p)(\tan p) - \int (\tan p)(dp)$

$du_1 = dp \quad v_1 = \tan p \qquad = p \tan p - \int \tan p \, dp$

$= p \tan p - \ln|\sec p| + C$

$= 2x \tan(2x) - \ln|\sec(2x)| + C$

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22) $u_1 = e^{-y} \quad dv_1 = \cos y \, dy \qquad u_2 = e^{-y} \quad dv_2 = \sin y \, dy$

$du_1 = -e^{-y} dy \quad v_1 = \sin y \qquad du_2 = -e^{-y} dy \quad v_2 = -\cos y$

$\int e^{-y} \cos y \, dy = (e^{-y})(\sin y) - \int (\sin y)(-e^{-y} dy)$

$\int e^{-y} \cos y \, dy = e^{-y} \sin y + \int e^{-y} \sin y \, dy$

$\int e^{-y} \cos y \, dy = e^{-y} \sin y + \{(e^{-y})(-\cos y) - \int (-\cos y)(-e^{-y} dy)\}$

$\int e^{-y} \cos y \, dy = e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y \, dy$

$2\int e^{-y} \cos y \, dy = e^{-y} \sin y - e^{-y} \cos y$

$\int e^{-y} \cos y \, dy = \frac{1}{2}(e^{-y} \sin y - e^{-y} \cos y) + C$

$= \frac{1}{2} e^{-y} \sin y - \frac{1}{2} e^{-y} \cos y + C$

24)
$u_1 = \sin(2x)$ $\quad dv_1 = e^{-2x}dx$ $\quad u_2 = \cos(2x)$ $\quad dv_2 = e^{-2x}dx$

$du_1 = 2\cos(2x)dx$ $\quad v_1 = \frac{-1}{2}e^{-2x}$ $\quad du_2 = -2\sin(2x)dx$ $\quad v_2 = \frac{-1}{2}e^{-2x}$

$$\int e^{-2x}\sin(2x)dx = (\sin(2x))\left(\frac{-1}{2}e^{-2x}\right) - \int\left(\frac{-1}{2}e^{-2x}\right)(2\cos(2x)dx)$$

$$\int e^{-2x}\sin(2x)dx = \frac{-1}{2}e^{-2x}\sin(2x) + \int e^{-2x}\cos(2x)dx$$

$$\int e^{-2x}\sin(2x)dx = \frac{-1}{2}e^{-2x}\sin(2x) + \left\{(\cos(2x))\left(\frac{-1}{2}e^{-2x}\right) - \int\left(\frac{-1}{2}e^{-2x}\right)(-2\sin(2x)dx)\right\}$$

$$\int e^{-2x}\sin(2x)dx = \frac{-1}{2}e^{-2x}\sin(2x) - \frac{1}{2}e^{-2x}\cos(2x) - \int e^{-2x}\sin(2x)dx$$

$$2\int e^{-2x}\sin(2x)dx = \frac{-1}{2}e^{-2x}\sin(2x) - \frac{1}{2}e^{-2x}\cos(2x)$$

$$\int e^{-2x}\sin(2x)dx = \frac{1}{2}\left(\frac{-1}{2}e^{-2x}\sin(2x) - \frac{1}{2}e^{-2x}\cos(2x)\right) + C$$

$$= \frac{-1}{4}e^{-2x}\sin(2x) - \frac{1}{4}e^{-2x}\cos(2x) + C$$

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26) $\int_0^1 x\sqrt{1-x}\,dx$ it is more effiuent to use only substitution technique

$$\int x\sqrt{1-x}\,dx = \int(1-p)\sqrt{p}\,(-1\,dp) = \int(p-1)\left(p^{\frac{1}{2}}\right)dp$$

$p = 1-x \Rightarrow x = 1-p$

$$= \int\left(p^{\frac{3}{2}} - p^{\frac{1}{2}}\right)dp = \left[\frac{p^{\frac{5}{2}}}{\frac{5}{2}}\right] - \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$dp = -dx$

$-1\,dp = dx$

$$= \frac{2}{5}(\sqrt{p})^5 - \frac{2}{3}(\sqrt{p})^3 + C$$

$$= \frac{2}{5}(\sqrt{1-x})^5 - \frac{2}{3}(\sqrt{1-x})^3 + C$$

26) continued

$$\int_0^1 x\sqrt{1-x}\,dx = \left[\frac{2}{5}\left(\sqrt{1-x}\right)^5 - \frac{2}{3}\left(\sqrt{1-x}\right)^3 + C\right]_0^1$$

$$= \left[\frac{2}{5}\left(\sqrt{1-(1)}\right)^5 - \frac{2}{3}\left(\sqrt{1-(1)}\right)^3 + C\right] - \left[\frac{2}{5}\left(\sqrt{1-(0)}\right)^5 - \frac{2}{3}\left(\sqrt{1-(0)}\right)^3 + C\right]$$

$$= \left[\frac{2}{5}(0)^5 - \frac{2}{3}(0)^3\right] - \left[\frac{2}{5}(1)^5 - \frac{2}{3}(1)^3\right]$$

$$= [0] - \left[\frac{2}{5} - \frac{2}{3}\right] = -\left[\frac{6}{15} - \frac{10}{15}\right] = -\left[\frac{-4}{15}\right] = \frac{4}{15}$$

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28) $$\int \ln(x+x^2)\,dx = (\ln(x+x^2))(x) - \int (x)\left(\frac{2x+1}{x^2+x}\,dx\right)$$

$u_1 = \ln(x+x^2)$ $\quad dv_1 = dx$

$du_1 = \frac{1}{x+x^2}(1+2x)\,dx$ $\quad v_1 = x$

$du_1 = \frac{2x+1}{x^2+x}\,dx$

$$= x\ln(x+x^2) - \int \frac{x(2x+1)}{x(x+1)}\,dx$$

$$= x\ln(x+x^2) - \int \frac{2x+1}{x+1}\,dx$$

$$= x\ln(x+x^2) - \int \left(2 + \frac{(-1)}{x+1}\right)dx$$

$$= x\ln(x+x^2) - \int 2\,dx + \int \frac{1}{x+1}\,dx$$

$$= x\ln(x+x^2) - 2x + \ln|x+1| + C$$

$$\begin{array}{r|l} & \;2 \\ x+1 & \;2x+1 \\ & -(2x+2) \\ \hline & \quad -1 \end{array}$$

$$\int \frac{1}{x+1}\,dx = \int \frac{1}{p}\,dp$$

$p = x+1$ $\quad = \ln|p| + C$

$dp = dx$ $\quad = \ln|x+1| + C$

30) $\int z(\ln z)^2\,dz = \int (e^p)(p^2)(e^p\,dp)$

$p = \ln z$

$\Downarrow$

$e^p = z$

$dz = e^p\,dp$

$= \int p^2 e^{2p}\,dp$

$= (p^2)\left(\frac{e^{2p}}{2}\right) - \int \left(\frac{e^{2p}}{2}\right)(2p\,dp)$

$= \frac{1}{2}p^2e^{2p} - \int p e^{2p}\,dp$

$u_1 = p^2 \qquad dv_1 = e^{2p}\,dp$

$du_1 = 2p\,dp \qquad v_1 = \frac{e^{2p}}{2}$

$= \frac{1}{2}p^2e^{2p} - \left\{(p)\left(\frac{e^{2p}}{2}\right) - \int\left(\frac{e^{2p}}{2}\right)(dp)\right\}$

$= \frac{1}{2}p^2e^{2p} - \frac{1}{2}pe^{2p} - \frac{1}{2}\int e^{2p}\,dp$

$u_2 = p \qquad v_2 = e^{2p}\,dp$

$du_2 = dp \qquad v_2 = \frac{e^{2p}}{2}$

$= \frac{1}{2}p^2e^{2p} - \frac{1}{2}pe^{2p} - \frac{1}{2}\left[\frac{e^{2p}}{2}\right] + C$

$= \frac{1}{2}p^2e^{2p} - \frac{1}{2}pe^{2p} - \frac{1}{4}e^{2p} + C$

$= \frac{1}{2}p^2(e^p)^2 - \frac{1}{2}p(e^p)^2 - \frac{1}{4}(e^p)^2 + C$

$= \frac{1}{2}(\ln z)^2(z)^2 - \frac{1}{2}(\ln z)(z)^2 - \frac{1}{4}(z)^2 + C$

$= \frac{1}{2}z^2(\ln z)^2 - \frac{1}{2}z^2\ln z - \frac{1}{4}z^2 + C$

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32) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = \int \cos\sqrt{x}\left(\frac{1}{\sqrt{x}}\,dx\right)$

$p = \sqrt{x} = x^{\frac{1}{2}}$

$dp = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$dp = \frac{1}{2\sqrt{x}}\,dx$

$2\,dp = \frac{1}{\sqrt{x}}\,dx$

$= \int \cos p\,(2\,dp)$

$= 2\sin p + C$

$= 2\sin\sqrt{x} + C$

34) $\int \frac{1}{x(\ln x)^2}\,dx = \int \frac{1}{(\ln x)^2}\left(\frac{1}{x}dx\right) = \int \frac{1}{p^2}\,dp = \int p^{-2}\,dp$

$p = \ln x$ $\quad = \left[\frac{p^{-1}}{-1}\right] + C = \frac{-1}{p} + C = \frac{-1}{\ln x} + C$

$dp = \frac{1}{x}dx$

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40) $\int x^2 \sin x^3\,dx = \int \sin x^3 \,(x^2 dx) = \int \sin p \left(\frac{1}{3}dp\right)$

$p = x^3$ $\quad = \frac{1}{3}[-\cos p] + C$

$dp = 3x^2 dx$

$\frac{1}{3}dp = x^2 dx$ $\quad = \frac{-1}{3}\cos x^3 + C$

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42) $u_1 = \sin 2x$ $\quad dv_1 = \cos 4x\,dx$ $\quad u_2 = \cos 2x$ $\quad dv_2 = \sin 4x\,dx$

$du_1 = 2\cos 2x\,dx$ $\quad v_1 = \frac{\sin 4x}{4}$ $\quad du_2 = -2\sin 2x\,dx$ $\quad v_2 = \frac{-\cos 4x}{4}$

$$\int \sin 2x \cos 4x\,dx = (\sin 2x)\left(\frac{\sin 4x}{4}\right) - \int \left(\frac{\sin 4x}{4}\right)(2\cos 2x\,dx)$$

$$\int \sin 2x \cos 4x\,dx = \frac{1}{4}\sin 2x \sin 4x - \frac{1}{2}\int \cos 2x \sin 4x\,dx$$

$$\int \sin 2x \cos 4x\,dx = \frac{1}{4}\sin 2x \sin 4x - \frac{1}{2}\left\{(\cos 2x)\left(\frac{-\cos 4x}{4}\right) - \int\left(\frac{-\cos 4x}{4}\right)(-2\sin 2x\,dx)\right\}$$

$$\int \sin 2x \cos 4x\,dx = \frac{1}{4}\sin 2x \sin 4x + \frac{1}{8}\cos 2x \cos 4x + \frac{1}{4}\int \sin 2x \cos 4x\,dx$$

$$\frac{3}{4}\int \sin 2x \cos 4x\,dx = \frac{1}{4}\sin 2x \sin 4x + \frac{1}{8}\cos 2x \cos 4x$$

$$\int \sin 2x \cos 4x\,dx = \frac{4}{3}\left(\frac{1}{4}\sin 2x \sin 4x + \frac{1}{8}\cos 2x \cos 4x\right) + C$$

$$= \frac{1}{3}\sin 2x \sin 4x + \frac{1}{6}\cos 2x \cos 4x + C$$

48) $\int_0^{\frac{\pi}{2}} x^3 \cos 2x \, dx$

$$\int x^3 \cos 2x \, dx = (x^3)\left(\frac{\sin 2x}{2}\right) - \int \left(\frac{\sin 2x}{2}\right)(3x^2 dx)$$

$u_1 = x^3 \quad dv_1 = \cos 2x \, dx$

$du_1 = 3x^2 dx \quad v_1 = \frac{\sin 2x}{2}$

$u_2 = x^2 \quad v_2 = \sin 2x \, dx$

$du_2 = 2x \, dx \quad v_2 = \frac{-\cos 2x}{2}$

$u_3 = x \quad v_3 = \cos 2x \, dx$

$du_3 = dx \quad v_3 = \frac{\sin 2x}{2}$

$$= \frac{1}{2}x^3 \sin 2x - \frac{3}{2}\int x^2 \sin 2x \, dx$$

$$= \frac{1}{2}x^3 \sin 2x - \frac{3}{2}\left\{(x^2)\left(\frac{-\cos 2x}{2}\right) - \int\left(\frac{-\cos 2x}{2}\right)(2x \, dx)\right\}$$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{2}\int x \cos 2x \, dx$$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{2}\left\{(x)\left(\frac{\sin 2x}{2}\right) - \int\left(\frac{\sin 2x}{2}\right)(dx)\right\}$$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x + \frac{3}{4}\int \sin 2x \, dx$$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x + \frac{3}{4}\left[\frac{-\cos 2x}{2}\right] + C$$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x + C$$

$$\int_0^{\frac{\pi}{2}} x^3 \cos 2x \, dx = \left[\frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x + C\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{1}{2}\left(\frac{\pi}{2}\right)^3 \sin 2\left(\frac{\pi}{2}\right) + \frac{3}{4}\left(\frac{\pi}{2}\right)^2 \cos 2\left(\frac{\pi}{2}\right) - \frac{3}{4}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right) - \frac{3}{8}\cos 2\left(\frac{\pi}{2}\right) + C\right]$$

$$- \left[\frac{1}{2}(0)^3 \sin 2(0) + \frac{3}{4}(0)^2 \cos 2(0) - \frac{3}{4}(0)\sin 2(0) - \frac{3}{8}\cos 2(0) + C\right]$$

$$= \left[\left(\frac{\pi^3}{16}\right)\sin(\pi) + \left(\frac{3\pi^2}{16}\right)\cos(\pi) - \left(\frac{3\pi}{8}\right)\sin(\pi) - \left(\frac{3}{8}\right)\cos(\pi)\right] - \left[0 + 0 - 0 - \frac{3}{8}(1)\right]$$

$$= \left[\left(\frac{\pi^3}{16}\right)(0) + \left(\frac{3\pi^2}{16}\right)(-1) - \left(\frac{3\pi}{8}\right)(0) - \left(\frac{3}{8}\right)(-1)\right] - \left[\frac{-3}{8}\right]$$

$$= \left[0 - \frac{3\pi^2}{16} - 0 + \frac{3}{8}\right] - \left[\frac{-3}{8}\right] = \frac{6}{8} - \frac{3\pi^2}{16} = \underline{\underline{\frac{3}{4} - \frac{3\pi^2}{16}}}$$

50) $\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)\,dx$

$$\int 2x \sin^{-1}(x^2)\,dx = \int \sin^{-1}(x^2)(2x\,dx) = \int \sin^{-1}p\,dp$$

$p = x^2$

$dp = 2x\,dx$

$u_1 = \sin^{-1}p \qquad dv_1 = dp$

$du_1 = \frac{1}{\sqrt{1-p^2}}\,dp \qquad v_1 = p$

[see example #2]

$$= (\sin^{-1}p)(p) - \int (p)\left(\frac{1}{\sqrt{1-p^2}}\,dp\right)$$

$$= p\sin^{-1}p - \int \frac{p}{\sqrt{1-p^2}}\,dp$$

$$= p\sin^{-1}p - \left[-\sqrt{1-p^2}\right] + C$$

$$= p\sin^{-1}p + \sqrt{1-p^2} + C$$

$$\int \frac{p}{\sqrt{1-p^2}}\,dp = \int \frac{1}{\sqrt{1-p^2}}(p\,dp)$$

$q = 1-p^2$

$dq = -2p\,dp$

$-\frac{1}{2}dq = p\,dp$

$$= \int \frac{1}{\sqrt{q}}\left(\frac{-1}{2}dq\right)$$

$$= \frac{-1}{2}\left[\frac{q^{\frac{1}{2}}}{\frac{1}{2}}\right] + D$$

$$= -\sqrt{q} + D$$

$$= -\sqrt{1-p^2} + D$$

$$= x^2\sin^{-1}x^2 + \sqrt{1-(x^2)^2} + C$$

$$= x^2\sin^{-1}x^2 + \sqrt{1-x^4} + C$$

$$\int_0^{\frac{1}{\sqrt{2}}} 2x\sin^{-1}(x^2)\,dx = \left[x^2\sin^{-1}x^2 + \sqrt{1-x^4} + C\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \left[\left(\tfrac{1}{\sqrt{2}}\right)^2 \sin^{-1}\left(\tfrac{1}{\sqrt{2}}\right)^2 + \sqrt{1-\left(\tfrac{1}{\sqrt{2}}\right)^4} + C\right] - \left[(0)^2\sin^{-1}(0)^2 + \sqrt{1-(0)^4} + C\right]$$

$$= \left[\left(\tfrac{1}{2}\right)\sin^{-1}\left(\tfrac{1}{2}\right) + \sqrt{1-\tfrac{1}{4}}\right] - \left[0 + \sqrt{1}\right]$$

$$= \left[\left(\tfrac{1}{2}\right)\left(\tfrac{\pi}{6}\right) + \sqrt{\tfrac{3}{4}}\right] - [1] = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

52) $\theta = \tan^{-1}\frac{x}{2}$

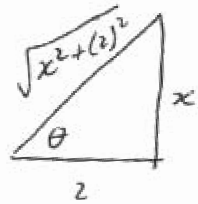
$\Downarrow$

$\tan\theta = \frac{x}{2}$

$\sec^2\theta \frac{d\theta}{dx} = \frac{1}{2}$

$\frac{d\theta}{dx} = \frac{1}{2\sec^2\theta} = \frac{1}{2\left(\frac{\sqrt{x^2+(2)^2}}{2}\right)^2}$

$= \frac{1}{\frac{x^2+4}{2}} = \frac{2}{x^2+4}$



$$\int x^2 \tan^{-1}\frac{x}{2}\,dx = \left(\tan^{-1}\frac{x}{2}\right)\left(\frac{x^3}{3}\right) - \int\left(\frac{x^3}{3}\right)\left(\frac{2}{x^2+4}\,dx\right)$$

$u_1 = \tan^{-1}\frac{x}{2}$ $\quad dv_1 = x^2\,dx$

$du_1 = \frac{2}{x^2+4}\,dx$ $\quad v_1 = \frac{x^3}{3}$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{2}{3}\int\frac{x^3}{x^2+4}\,dx$$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{2}{3}\int\left(x + \frac{(-4x)}{x^2+4}\right)dx$$

$$\begin{array}{r} x \\ x^2+0x+4\,\overline{\big)\,x^3+0x^2+0x+0} \\ -(x^3+0x^2+4x) \\ \hline -4x \end{array}$$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{2}{3}\int x\,dx + \frac{4}{3}\int\frac{2x}{x^2+4}\,dx$$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{2}{3}\left[\frac{x^2}{2}\right] + \frac{4}{3}\left[\ln|x^2+4|\right] + C$$

$$\int\frac{2x}{x^2+4}\,dx = \int\frac{1}{x^2+4}(2x\,dx)$$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{1}{3}x^2 + \frac{4}{3}\ln|x^2+4| + C$$

$p = x^2+4$

$dp = 2x\,dx$

$= \int\frac{1}{p}\,dp$

$= \ln|p| + D$

$= \ln|x^2+4| + D$

$$= \frac{1}{3}x^3\tan^{-1}\frac{x}{2} - \frac{1}{3}x^2 + \frac{4}{3}\ln(x^2+4) + C$$