2) $\int \frac{x^2}{x^2+1}\,dx = \int \left(1 + \frac{(-1)}{x^2+1}\right) dx = \int 1\,dx - \int \frac{1}{x^2+(1)^2}\,dx$

$$\begin{array}{r|l} & \ \ 1 \\ \hline x^2+0x+1 & x^2+0x+0 \\ & -(x^2+0x+1) \\ \hline & \quad -1 \end{array}$$

$= [x] - \left[\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right] + C$

$= x - \tan^{-1} x + C$

---

4) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\cos^2 x \cdot \tan x}$

$\int \frac{1}{\cos^2 x \tan x}\,dx = \int \left(\frac{1}{\cos^2 x}\right)\left(\frac{1}{\tan x}\right) dx = \int \frac{1}{\tan x}(\sec^2 x\,dx)$

$p = \tan x$

$dp = \sec^2 x\,dx$

$= \int \frac{1}{p}\,dp = \ln|p| + C = \ln|\tan x| + C$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\cos^2 x \tan x} = \Big[\ln|\tan x| + C\Big]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left[\ln\left|\tan\left(\frac{\pi}{3}\right)\right| + C\right] - \left[\ln\left|\tan\left(\frac{\pi}{4}\right) + C\right]$

$= \left[\ln\left|\left(\frac{\sqrt{3}}{1}\right)\right|\right] - [\ln(1)]$

$= \ln\sqrt{3} = \ln(3^{\frac{1}{2}}) = \frac{1}{2}\ln 3$

6) $\int \frac{1}{x-\sqrt{x}}\,dx = \int \frac{1}{(\sqrt{x})^2-\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}-1}\left(\frac{1}{\sqrt{x}}\,dx\right)$

$p = \sqrt{x}-1 = x^{\frac{1}{2}}-1$

$dp = \frac{1}{2}x^{-\frac{1}{2}}dx$

$dp = \frac{1}{2\sqrt{x}}\,dx$

$2\,dp = \frac{1}{\sqrt{x}}dx$

$= \int \frac{1}{p}(2\,dp) = 2\ln|p| + C$

$= 2\ln|\sqrt{x}-1| + C$

---

8) $\int \frac{2^{\ln z^3}}{16z}\,dz = \int \frac{1}{16}\,2^{\ln z^3}\left(\frac{1}{z}\,dz\right)$

$p = \ln z^3 = 3\ln z$

$dp = 3\left[\frac{1}{z}\right]dz$

$\frac{1}{3}\,dp = \frac{1}{z}\,dz$

$= \int \frac{1}{16}\,2^{p}\left(\frac{1}{3}\,dp\right)$

$= \left(\frac{1}{16}\right)\left(\frac{1}{3}\right)\left[\frac{2^{p}}{\ln 2}\right] + C$

$= \frac{1}{48\ln 2}\,2^{\ln z^3} + C$

---

10) $\int_1^2 \frac{8\,dx}{x^2-2x+2}$

$\int \frac{8\,dx}{x^2-2x+2} = \int \frac{8}{(x^2-2x+1)+1}\,dx = \int \frac{8}{(x-1)^2+(1)^2}\,dx$

$= 8\left[\frac{1}{1}\tan^{-1}\left(\frac{x-1}{1}\right)\right] + C$

$= 8\tan^{-1}(x-1) + C$

10) continued

$$\int_1^2 \frac{8\,dx}{x^2-2x+2} = \left[8\tan^{-1}(x-1)+C\right]_1^2$$

$$= \left[8\tan^{-1}((2)-1)+C\right] - \left[8\tan^{-1}((1)-1)+C\right]$$

$$= \left[8\tan^{-1}(1)\right] - \left[8\tan^{-1}(0)\right] = \left[8\left(\frac{\pi}{4}\right)\right] - \left[8(0)\right] = 2\pi$$

---

16) $$\int \frac{d\theta}{\sqrt{2\theta-\theta^2}} = \int \frac{1}{\sqrt{+1-1+2\theta-\theta^2}}\,d\theta = \int \frac{1}{\sqrt{1-(\theta^2-2\theta+1)}}\,d\theta$$

$$= \int \frac{1}{\sqrt{(1)^2-(\theta-1)^2}}\,d\theta = \int \frac{1}{\sqrt{(1)^2-p^2}}\,dp = \sin^{-1}\left(\frac{p}{1}\right)+C$$

$p = \theta - 1$

$dp = d\theta$

$$= \sin^{-1} p + C = \sin^{-1}(\theta-1)+C$$

---

22) $$\int \frac{x+2\sqrt{x-1}}{2x\sqrt{x-1}}\,dx = \int\left(\frac{x}{2x\sqrt{x-1}} + \frac{2\sqrt{x-1}}{2x\sqrt{x-1}}\right)dx$$

$$\int \frac{1}{2\sqrt{x-1}}\,dx = \int \frac{1}{2\sqrt{p}}\,dp$$

$p = x-1$

$dp = dx$

$$= \int \frac{1}{2} p^{-\frac{1}{2}}\,dp$$

$$= \frac{1}{2}\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + D$$

$$= p^{\frac{1}{2}} + D$$

$$= \sqrt{p} + D$$

$$= \sqrt{x-1} + D$$

$$= \int\left(\frac{1}{2\sqrt{x-1}} + \frac{1}{x}\right)dx$$

$$= \int \frac{1}{2\sqrt{x-1}}\,dx + \int \frac{1}{x}\,dx$$

$$= \left[\sqrt{x-1}\right] + \ln|x| + C$$

$$= \sqrt{x-1} + \ln|x| + C$$

28) $\int \frac{dx}{(x-2)\sqrt{x^2-4x+3}} = \int \frac{dx}{(x-2)\sqrt{x^2-4x+3+1-1}}$

$p = x-2$

$dp = dx$

$= \int \frac{1}{(x-2)\sqrt{(x^2-4x+4)-1}}\,dx$

formula used

$= \int \frac{1}{(x-2)\sqrt{(x-2)^2-(1)^2}}\,dx$

we will learn how to integrate without formula in section 8.4

$= \int \frac{1}{p\sqrt{p^2-(1)^2}}\,dp$

$= \frac{1}{1}\sec^{-1}\left|\frac{p}{1}\right| + C = \sec^{-1} p + C$

$= \sec^{-1}(x-2) + C$

---

30) $\int 3\sinh\left(\frac{x}{2}+\ln 5\right)dx = \int 3\sinh(p)(2\,dp)$

$p = \frac{x}{2} + \ln 5$

$dp = \frac{1}{2}\,dx$

$2\,dp = dx$

$= 6\cosh p + C$

$= 6\cosh\left(\frac{x}{2}+\ln 5\right) + C$

---

34) $\int e^{z+e^z}\,dz = \int (e^z)(e^{e^z})\,dz = \int (e^{e^z})(e^z\,dz)$

$p = e^z$

$dp = e^z\,dz$

$= \int e^p\,dp = e^p + C$

$= e^{e^z} + C$