MATH 21200 section 7.3

2) $\sinh x = \frac{4}{3}$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - \left(\frac{4}{3}\right)^2 = 1$

$\cosh^2 x - \frac{16}{9} = 1$

$\cosh^2 x = 1 + \frac{16}{9} = \frac{25}{9}$

$\cosh x = \sqrt{\frac{25}{9}} = \frac{5}{3}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}$

$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

$\coth x = \frac{1}{\tanh x} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

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4) $\cosh x = \frac{13}{5}, \ x > 0$

$\cosh^2 x - \sinh^2 x = 1$

$\left(\frac{13}{5}\right)^2 - \sinh^2 x = 1$

$\frac{169}{25} - 1 = \sinh^2 x$

$\frac{144}{25} = \sinh^2 x$

$\sinh x = \sqrt{\frac{144}{25}} = \frac{12}{5}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\frac{12}{5}}{\frac{13}{5}} = \frac{12}{13}$

$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{1}{\frac{13}{5}} = \frac{5}{13}$

$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

$\coth x = \frac{1}{\tanh x} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

6) $\sinh(2\ln x) = \sinh(\ln(x^2)) = \dfrac{e^{(\ln(x^2))} - e^{-(\ln(x^2))}}{2}$

$= \dfrac{e^{\ln(x^2)} - e^{\ln(x^{-2})}}{2} = \dfrac{x^2 - x^{-2}}{2} = \dfrac{x^2 - \frac{1}{x^2}}{2}$

$= \dfrac{\frac{x^4}{x^2} - \frac{1}{x^2}}{2} = \dfrac{x^4 - 1}{2x^2}$

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8) $\cosh(3x) - \sinh(3x) = \left(\dfrac{e^{(3x)} + e^{-(3x)}}{2}\right) - \left(\dfrac{e^{(3x)} - e^{-(3x)}}{2}\right)$

$= \frac{1}{2}e^{3x} + \frac{1}{2}e^{-3x} - \frac{1}{2}e^{3x} + \frac{1}{2}e^{-3x} = e^{-3x} = \dfrac{1}{e^{3x}}$

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14) $y = \frac{1}{2}\sinh(2x+1)$

$\dfrac{dy}{dx} = \frac{1}{2}\left[\cosh(2x+1)(2)\right] = \cosh(2x+1)$

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16) $y = t^2 \tanh\left(\frac{1}{t}\right)$

$\dfrac{dy}{dt} = (t^2)\left[\operatorname{sech}^2\left(\frac{1}{t}\right)\left(-1t^{-2}\right)\right] + \left(\tanh\left(\frac{1}{t}\right)\right)[2t]$

$= (t^2)\left[\frac{-1}{t^2}\operatorname{sech}^2\left(\frac{1}{t}\right)\right] + \left(\tanh\left(\frac{1}{t}\right)\right)[2t]$

$= 2t\tanh\left(\frac{1}{t}\right) - \operatorname{sech}^2\left(\frac{1}{t}\right)$

18) $y = \ln(\cosh z)$

$$\frac{dy}{dz} = \frac{1}{\cosh z}(\sinh z)(1) = \frac{\sinh z}{\cosh z} = \tanh z$$

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24) $$\operatorname{csch}(\ln 2x) = \frac{1}{\sinh(\ln 2x)} = \frac{1}{\dfrac{e^{(\ln 2x)} - e^{-(\ln 2x)}}{2}}$$

$$= \frac{2}{e^{\ln(2x)} - e^{\ln(2x)^{-1}}} = \frac{2}{(2x)-(2x)^{-1}} = \left(\frac{\frac{2}{1}}{\frac{(2x)}{1} - \frac{1}{2x}}\right)\left(\frac{\frac{2x}{1}}{\frac{2x}{1}}\right)$$

$$= \frac{4x}{4x^2-1}$$

$$y = (4x^2-1)\operatorname{csch}(\ln 2x) = (4x^2-1)\left(\frac{4x}{4x^2-1}\right) = 4x$$

$$\frac{dy}{dx} = 4$$

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42) $$\int \sinh\frac{x}{5}\,dx = \int \sinh p\,(5\,dp) = 5\cosh p + C$$

$p = \frac{x}{5} = \frac{1}{5}x$ $\qquad = 5\cosh\left(\frac{x}{5}\right) + C$

$dp = \frac{1}{5}dx$

$5\,dp = dx$

44) $\int 4\cosh(3x - \ln 2)\,dx = \int 4\cosh p\left(\frac{1}{3}\,dp\right)$

$p = 3x - \ln 2$

$dp = 3\,dx$

$\frac{1}{3}\,dp = dx$

$= \frac{4}{3}\sinh p + C$

$= \frac{4}{3}\sinh(3x - \ln 2) + C$

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46) $\int \coth\frac{\theta}{\sqrt{3}}\,d\theta = \int \frac{\cosh\left(\frac{\theta}{\sqrt{3}}\right)}{\sinh\left(\frac{\theta}{\sqrt{3}}\right)}\,d\theta = \int \frac{1}{p}\left(\sqrt{3}\,dp\right)$

$p = \sinh\left(\frac{\theta}{\sqrt{3}}\right)$

$dp = \cosh\left(\frac{\theta}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)d\theta$

$\sqrt{3}\,dp = \cosh\left(\frac{\theta}{\sqrt{3}}\right)d\theta$

$= \sqrt{3}\ln|p| + C$

$= \sqrt{3}\ln\left|\sinh\left(\frac{\theta}{\sqrt{3}}\right)\right| + C$

$= \sqrt{3}\ln\left|\frac{e^{\left(\frac{\theta}{\sqrt{3}}\right)} - e^{-\left(\frac{\theta}{\sqrt{3}}\right)}}{2}\right| + C$

$= \sqrt{3}\ln\left|\frac{e^{\frac{\theta}{\sqrt{3}}} - e^{\frac{-\theta}{\sqrt{3}}}}{2}\right| + C$

$= \sqrt{3}\left\{\ln\left|e^{\frac{\theta}{\sqrt{3}}} - e^{\frac{-\theta}{\sqrt{3}}}\right| - \ln 2\right\} + C$

$D = C - \sqrt{3}\ln 2$

$= \sqrt{3}\ln\left|e^{\frac{\theta}{\sqrt{3}}} - e^{\frac{-\theta}{\sqrt{3}}}\right| - \sqrt{3}\ln 2 + C$

$= \sqrt{3}\ln\left|e^{\frac{\theta}{\sqrt{3}}} - e^{\frac{-\theta}{\sqrt{3}}}\right| + D$

52) $\int_0^{\ln 2} \tanh 2x \, dx$

$$\int \tanh 2x \, dx = \int \frac{\sinh 2x}{\cosh 2x} \, dx = \int \frac{1}{p} \left(\frac{1}{2} dp\right) = \frac{1}{2} \ln|p| + C$$

$p = \cosh 2x$

$dp = (\sinh 2x)(2) dx$

$\frac{1}{2} dp = \sinh 2x \, dx$

$$= \frac{1}{2} \ln|\cosh 2x| + C$$

$$\int_0^{\ln 2} \tanh 2x \, dx = \left[\frac{1}{2} \ln|\cosh 2x| + C\right]_0^{\ln 2}$$

$$= \left[\frac{1}{2} \ln|\cosh 2(\ln 2)| + C\right] - \left[\frac{1}{2} \ln|\cosh 2(0)| + C\right]$$

$$= \left[\frac{1}{2} \ln|\cosh(\ln 4)|\right] - \left[\frac{1}{2} \ln|\cosh(0)|\right]$$

$$= \left[\frac{1}{2} \ln\left|\frac{e^{(\ln 4)} + e^{-(\ln 4)}}{2}\right|\right] - \left[\frac{1}{2} \ln\left|\frac{e^{(0)} + e^{-(0)}}{2}\right|\right]$$

$$= \left[\frac{1}{2} \ln\left|\frac{e^{\ln(4)} + e^{\ln(4^{-1})}}{2}\right|\right] - \left[\frac{1}{2} \ln\left|\frac{1+1}{2}\right|\right]$$

$$= \left[\frac{1}{2} \ln\left|\frac{4 + 4^{-1}}{2}\right|\right] - \left[\frac{1}{2} \ln|1|\right] = \left[\frac{1}{2} \ln\left|\frac{4 + \frac{1}{4}}{2}\right|\right] - \left[\frac{1}{2}(0)\right]$$

$$= \frac{1}{2} \ln\left(2 + \frac{1}{8}\right) = \frac{1}{2} \ln\left(\frac{16}{8} + \frac{1}{8}\right) = \frac{1}{2} \ln\left(\frac{17}{8}\right)$$