MATH 21200 section 7.1

4) $\int \frac{8r\,dr}{4r^2-5} = \int \frac{1}{p}\,dp = \ln|p| + c = \ln|4r^2-5| + C$

$p = 4r^2-5$

$dp = 8r\,dr$

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6) $\int \frac{\sec y \tan y}{2+\sec y}\,dy = \int \frac{1}{p}\,dp = \ln|p| + C$

$p = 2+\sec y$ $= \ln|2+\sec y| + C$

$dp = \sec y \tan y\,dy$

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8) $\int \frac{\sec x\,dx}{\sqrt{\ln(\sec x+\tan x)}} = \int \frac{1}{\sqrt{p}}\,dp = \int p^{-\frac{1}{2}}\,dp = \frac{p^{\frac{1}{2}}}{\frac{1}{2}} + C$

$p = \ln(\sec x+\tan x)$ {see formula in pg 462} $= 2\sqrt{\ln(\sec x+\tan x)} + C$

$dp = \sec x\,dx$

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12) $\int \frac{\ln(\ln x)}{x \ln x}\,dx = \int p\,dp = \frac{p^2}{2} + C$

$p = \ln(\ln x)$ $= \frac{1}{2}(\ln(\ln x))^2 + C$

$dp = \frac{1}{\ln x}\left(\frac{1}{x}\right)dx$

20) $\int \frac{e^{\frac{-1}{x^2}}}{x^3} dx = \int e^p \left(\frac{1}{2} dp\right) = \frac{1}{2} e^p + C$

$p = \frac{-1}{x^2} = -x^{-2}$ $\qquad = \frac{1}{2} e^{\frac{-1}{x^2}} + C$

$dp = -(-2x^{-3})dx$

$dp = \frac{2}{x^3} dx \Rightarrow \frac{1}{2} dp = \frac{1}{x^3} dx$

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24) $\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2}) dx$

$\int 2x e^{x^2} \cos(e^{x^2}) dx = \int \cos p \, dp = \sin p + C$

$p = e^{x^2}$ $\qquad = \sin(e^{x^2}) + C$

$dp = (e^{x^2})(2x) dx$

$\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2}) dx = \left[\sin(e^{x^2}) + C\right]_0^{\sqrt{\ln \pi}}$

$= \left[\sin\left(e^{(\sqrt{\ln \pi})^2}\right) + C\right] - \left[\sin\left(e^{(0)^2}\right) + C\right]$

$= \left[\sin(e^{\ln \pi})\right] - \left[\sin(e^0)\right]$

$= [\sin(\pi)] - [\sin(1)]$

$= [0] - [\sin(1)] = \underline{\underline{-\sin(1)}}$

32) $\int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt$

$$\int \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt = \int \left(\frac{1}{3}\right)^{p} dp = \frac{\left(\frac{1}{3}\right)^{p}}{\ln\left(\frac{1}{3}\right)} + C$$

$p = \tan t$

$dp = \sec^2 t \, dt$

$$= \frac{\left(\frac{1}{3}\right)^{\tan t}}{\ln 1 - \ln 3} + C = \frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{\tan t} + C$$

$$\int_0^{\frac{\pi}{4}} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt = \left[\frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{\tan t} + C\right]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{\tan\left(\frac{\pi}{4}\right)} + C\right] - \left[\frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{\tan(0)} + C\right]$$

$$= \left[\frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{(1)}\right] - \left[\frac{-1}{\ln 3}\left(\frac{1}{3}\right)^{(0)}\right] = \underline{\underline{\frac{1}{\ln 3} - \frac{1}{3 \ln 3}}}$$

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38) $\int_1^4 \frac{\log_2 x}{x} \, dx$

$$\int \frac{\log_2 x}{x} \, dx = \int p \left((\ln 2) \, dp\right) = (\ln 2)\left[\frac{p^2}{2}\right] + C$$

$p = \log_2 x = \frac{\ln x}{\ln 2} = \frac{1}{\ln 2} \ln x$

$$= \frac{1}{2}(\ln 2)\left(\log_2 x\right)^2 + C$$

$dp = \left(\frac{1}{\ln 2}\right)\left(\frac{1}{x}\right) dx$

$(\ln 2) \, dp = \frac{1}{x} \, dx$

38) continued

$$\int_1^4 \frac{\log_2 x}{x}\,dx = \left[\frac{1}{2}(\ln 2)(\log_2 x)^2 + C\right]_1^4$$

$$= \left[\frac{1}{2}(\ln 2)(\log_2(4))^2 + C\right] - \left[\frac{1}{2}(\ln 2)(\log_2(1))^2 + C\right]$$

$$= \left[\frac{1}{2}(\ln 2)(2)^2\right] - \left[\frac{1}{2}(\ln 2)(0)^2\right] = \underline{\underline{2\ln 2}}$$

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42) $\displaystyle\int_{\frac{1}{10}}^{10} \frac{\log_{10}(10x)}{x}\,dx$

$$\int \frac{\log_{10}(10x)}{x}\,dx = \int p\,((\ln 10)\,dp) = (\ln 10)\frac{p^2}{2} + C$$

$$= \frac{1}{2}(\ln 10)(\log_{10}(10x))^2 + C$$

$p = \log_{10}(10x) = \dfrac{\ln(10x)}{\ln 10}$

$dp = \dfrac{1}{\ln 10}\left(\dfrac{1}{10x}(10)\right)dx$

$dp = \dfrac{1}{\ln 10}\left(\dfrac{1}{x}\right)dx \Rightarrow (\ln 10)\,dp = \dfrac{1}{x}\,dx$

$$\int_{\frac{1}{10}}^{10} \frac{\log_{10}(10x)}{x}\,dx = \left[\frac{1}{2}(\ln 10)(\log_{10}(10x))^2 + C\right]_{\frac{1}{10}}^{10}$$

$$= \left[\frac{1}{2}(\ln 10)(\log_{10}(10(10)))^2 + C\right] - \left[\frac{1}{2}(\ln 10)\left(\log_{10}\left(10\left(\tfrac{1}{10}\right)\right)\right)^2 + C\right]$$

$$= \left[\frac{1}{2}(\ln 10)(\log_{10}(100))^2\right] - \left[\frac{1}{2}(\ln 10)(\log_{10}(1))^2\right]$$

$$= \left[\frac{1}{2}(\ln 10)(2)^2\right] - \left[\frac{1}{2}(\ln 10)(0)^2\right]$$

$$= \underline{\underline{2\ln 10}}$$