MATH 20100 section 5.2

Sigma Notation

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-2} + a_{n-1} + a_n$$

Algebra Rules for Finite Sums

1) Sum Rule: $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

2) Difference Rule: $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

3) Constant Multiple Rule: $\sum_{k=1}^{n} c a_k = c \sum_{k=1}^{n} a_k$

4) Constant Value Rule: $\sum_{k=1}^{n} c = n(c)$

$\left.\begin{array}{l} 3) \\ 4) \end{array}\right\}$ any number c

---

Important formulas for evaluating finite sums:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

2) $\sum_{k=1}^{3} \frac{k-1}{k} = \left(\frac{(1)-1}{(1)}\right) + \left(\frac{(2)-1}{(2)}\right) + \left(\frac{(3)-1}{(3)}\right) = \left(\frac{0}{1}\right) + \left(\frac{1}{2}\right) + \left(\frac{2}{3}\right)$

$= 0 + \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

---

4) $\sum_{k=1}^{5} \sin k\pi = (\sin((1)\pi)) + (\sin((2)\pi)) + (\sin((3)\pi)) + (\sin((4)\pi)) + (\sin((5)\pi))$

$= (0) + (0) + (0) + (0) + (0) = 0$

---

6) $\sum_{k=1}^{4} (-1)^k \cos k\pi = \left((-1)^1 \cos((1)\pi)\right) + \left((-1)^2 \cos((2)\pi)\right) + \left((-1)^3 \cos((3)\pi)\right) + \left((-4)^4 \cos((4)\pi)\right)$

$= \left((-1)(-1)\right) + \left((1)(1)\right) + \left((-1)(-1)\right) + \left((1)(1)\right) = 1 + 1 + 1 + 1 = 4$

8-a) $\sum_{k=1}^{6} (-2)^{k-1} = (-2)^{((1)-1)} + (-2)^{((2)-1)} + (-2)^{((3)-1)} + (-2)^{((4)-1)} + (-2)^{((5)-1)} + (-2)^{((6)-1)}$

$= (-2)^{(0)} + (-2)^{(1)} + (-2)^{(2)} + (-2)^{(3)} + (-2)^{(4)} + (-2)^{(5)}$

$= 1 - 2 + 4 - 8 + 16 - 32$ $\boxed{\text{yes}}$

8-b) $\sum_{k=0}^{5} (-1)^k 2^k = (-1)^{(0)}(2)^{(0)} + (-1)^{(1)}(2)^{(1)} + (-1)^{(2)}(2)^{(2)} + (-1)^{(3)}(2)^{(3)} + (-1)^{(4)}(2)^{(4)} + (-1)^{(5)}(2)^{(5)}$

$= (1)(1) + (-1)(2) + (1)(4) + (-1)(8) + (1)(16) + (-1)(32)$

$= 1 - 2 + 4 - 8 + 16 - 32$ $\boxed{\text{yes}}$

8-c) $\sum_{k=-2}^{3} (-1)^{k+1} 2^{k+2} = (-1)^{((-2)+1)}(2)^{((-2)+2)} + (-1)^{((-1)+1)}(2)^{((-1)+2)} + (-1)^{((0)+1)}(2)^{((0)+2)}$

$+ (-1)^{((1)+1)}(2)^{((1)+2)} + (-1)^{((2)+1)}(2)^{((2)+2)} + (-1)^{((3)+1)}(2)^{((3)+2)}$

$= (-1)^{(-1)}(2)^{(0)} + (-1)^{(0)}(2)^{(1)} + (-1)^{(1)}(2)^{(2)} + (-1)^{(2)}(2)^{(3)} + (-1)^{(3)}(2)^{(4)} + (-1)^{(4)}(2)^{(5)}$

$= -1 + 2 - 4 + 8 - 16 + 32$ $\boxed{\text{no}}$

10-a) $\sum_{k=1}^{4}(k-1)^2 = ((1)-1)^2+((2)-1)^2+((3)-1)^2+((4)-1)^2$

$= (0)^2+(1)^2+(2)^2+(3)^2 = 0+1+4+9$

10-b) $\sum_{k=-1}^{3}(k+1)^2 = ((-1)+1)^2+((0)+1)^2+((1)+1)^2+((2)+1)^2+((3)+1)^2$

$= (0)^2+(1)^2+(2)^2+(3)^2+(4)^2 = 0+1+4+9+16$

10-c) $\sum_{k=-3}^{-1} k^2 = (-3)^2+(-2)^2+(-1)^2 = 9+4+1$

part a and c are equivalent; part b is not equivalent to the other two.

---

12) one choice is $\sum_{k=1}^{4} k^2$

14) one choice is $\sum_{k=1}^{5} 2k$

16) one choice is $\sum_{k=1}^{5} (-1)^k\left(\frac{k}{5}\right)$

---

18) $\sum_{k=1}^{n} a_k = 0$ and $\sum_{k=1}^{n} b_k = 1$

a) $\sum_{k=1}^{n} 8a_k = 8\sum_{k=1}^{n} a_k = 8(0) = 0$

b) $\sum_{k=1}^{n} 250b_k = 250\sum_{k=1}^{n} b_k = 250(1) = 250$

c) $\sum_{k=1}^{n}(a_k+1) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} 1 = (0)+(n)(1) = n$

d) $\sum_{k=1}^{n}(b_k-1) = \sum_{k=1}^{n} b_k - \sum_{k=1}^{n} 1 = (1)-(n)(1) = 1-n$

20-a) $\sum_{k=1}^{13} k = \frac{(13)((13)+1)}{2} = \frac{13(14)}{2} = 13(7) = 91$

20-b) $\sum_{k=1}^{13} k^2 = \frac{(13)((13)+1)(2(13)+1)}{6} = \frac{13(14)(26+1)}{6} = \frac{13(14)(27)}{6}$

$= 13(7)(9) = 819$

20-c) $\sum_{k=1}^{13} k^3 = \left(\frac{(13)((13)+1)}{2}\right)^2 = \left(\frac{13(14)}{2}\right)^2 = \left(13(7)\right)^2 = (91)^2 = 8281$

---

22) $\sum_{k=1}^{5} \frac{\pi k}{15} = \frac{\pi}{15}\sum_{k=1}^{5} k = \frac{\pi}{15}\left(\frac{(5)((5)+1)}{2}\right) = \frac{\pi}{15}\left(\frac{5(6)}{2}\right) = \frac{\pi}{3}(3) = \pi$

---

24) $\sum_{k=1}^{6} (k^2 - 5) = \sum_{k=1}^{6} k^2 - \sum_{k=1}^{6} 5 = \left\{\frac{(6)((6)+1)(2(6)+1)}{6}\right\} - \left\{(6)(5)\right\}$

$= \{(7)(13)\} - \{30\} = \{91\} - \{30\} = 61$

---

26) $\sum_{k=1}^{7} k(2k+1) = \sum_{k=1}^{7} (2k^2 + k) = 2\sum_{k=1}^{7} k^2 + \sum_{k=1}^{7} k$

$= 2\left\{\frac{(7)((7)+1)(2(7)+1)}{6}\right\} + \left\{\frac{(7)((7)+1)}{2}\right\}$

$= 2\left\{\frac{(7)(8)(15)}{6}\right\} + \left\{\frac{(7)(8)}{2}\right\} = 2\{(7)(4)(5)\} + \{(7)(4)\}$

$= 280 + 28 = 308$

28) $\left(\sum_{k=1}^{7} k\right)^2 - \sum_{k=1}^{7} \frac{k^3}{4} = \left(\sum_{k=1}^{7} k\right)^2 - \frac{1}{4}\sum_{k=1}^{7} k^3 = \left(\frac{(7)((7)+1)}{2}\right)^2 - \frac{1}{4}\left(\frac{(7)((7)+1)}{2}\right)^2$

$= \left(\frac{7(8)}{2}\right)^2 - \frac{1}{4}\left(\frac{7(8)}{2}\right)^2 = (7(4))^2 - \frac{1}{4}(7(4))^2 = (7(4))^2 - (7(2))^2$

$= 784 - 196 = 588$

30-a) $\sum_{k=9}^{36} k$ let $l+8=k$, then when $l=1$, $k=9$

also when $k=36$, $l=28$

$= \sum_{l=1}^{28}(l+8) = \sum_{l=1}^{28} l + \sum_{l=1}^{28} 8 = \left\{\frac{(28)((28)+1)}{2}\right\} + \left\{(28)(8)\right\}$

$= \{(14)(29)\} + \{(28)(8)\} = \{406\} + \{224\} = 630$

30-b) $\sum_{k=3}^{17} k^2$ let $i+2=k$, then when $i=1$, $k=3$

also when $k=17$, $i=15$

$= \sum_{i=1}^{15}(i+2)^2 = \sum_{i=1}^{15}(i^2+4i+4) = \sum_{i=1}^{15} i^2 + 4\sum_{i=1}^{15} i + \sum_{i=1}^{15} 4$

$= \left\{\frac{(15)((15)+1)(2(15)+1)}{6}\right\} + 4\left\{\frac{(15)((15)+1)}{2}\right\} + \{(15)(4)\}$

$= \left\{\frac{(5)(16)(31)}{2}\right\} + 2\{(15)(16)\} + \{(15)(4)\}$

$= \{(5)(8)(31)\} + 2\{240\} + \{60\}$

$= \{1240\} + \{480\} + \{60\} = 1780$

30-c) $\sum_{k=18}^{71} k(k-1)$ let $p+17=k$, then when $p=1$, $k=18$
also when $k=71$, $p=54$

$$=\sum_{p=1}^{54}(p+17)((p+17)-1)=\sum_{p=1}^{54}(p+17)(p+16)$$

$$=\sum_{p=1}^{54}(p^2+33p+272)=\sum_{p=1}^{54}p^2+33\sum_{p=1}^{54}p+\sum_{p=1}^{54}272$$

$$=\left\{\frac{(54)((54)+1)(2(54)+1)}{6}\right\}+33\left\{\frac{(54)((54)+1)}{2}\right\}+\left\{(272)(54)\right\}$$

$$=\left\{\frac{(54)(55)(109)}{6}\right\}+33\left\{27(55)\right\}+\left\{(272)(54)\right\}$$

$$=\left\{(9)(55)(109)\right\}+33\left\{27(55)\right\}+\left\{(272)(54)\right\}$$

$$=\left\{53955\right\}+\left\{49005\right\}+\left\{14688\right\}=117648$$

32-a) $\sum_{k=1}^{n}\left(\frac{1}{n}+2n\right)=(n)\left(\frac{1}{n}+2n\right)=1+2n^2$

32-b) $\sum_{k=1}^{n}\frac{c}{n}=(n)\left(\frac{c}{n}\right)=c$

32-c) $\sum_{k=1}^{n}\frac{k}{n^2}=\frac{1}{n^2}\sum_{k=1}^{n}k=\frac{1}{n^2}\left\{\frac{n(n+1)}{2}\right\}=\frac{1}{n}\left\{\frac{n+1}{2}\right\}=\frac{n+1}{2n}$

34) $\sum_{k=2}^{30} [\sin(k-1) - \sin k]$ for this, we should write out some of the term to catch the pattern

$= [\sin((2)-1) - \sin(2)] + [\sin((3)-1) - \sin(3)] + [\sin((4)-1) - \sin(4)] +$
$\quad \cdots + [\sin((28)-1) - \sin(28)] + [\sin((29)-1) - \sin(29)] + [\sin((30)-1) - \sin(30)]$

$= [\sin(1) - \sin(2)] + [\sin(2) - \sin(3)] + [\sin(3) - \sin(4)] +$
$\quad \cdots + [\sin(27) - \sin(28)] + [\sin(28) - \sin(29)] + [\sin(29) - \sin(30)]$

$= \sin(1) - \sin(30)$ because all middle term adds to 0.

---

36) $\sum_{k=1}^{40} \frac{1}{k(k+1)} = \sum_{k=1}^{40} \left[\frac{1}{k} - \frac{1}{k+1}\right]$ for this, we should write out some of the terms to catch the pattern.

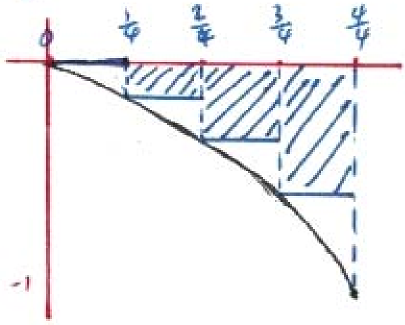
$= \left[\frac{1}{(1)} - \frac{1}{(1)+1}\right] + \left[\frac{1}{(2)} - \frac{1}{(2)+1}\right] + \left[\frac{1}{(3)} - \frac{1}{(3)+1}\right] + \left[\frac{1}{(4)} - \frac{1}{(4)+1}\right] +$
$\quad \cdots + \left[\frac{1}{(37)} - \frac{1}{(37)+1}\right] + \left[\frac{1}{(38)} - \frac{1}{(38)+1}\right] + \left[\frac{1}{(39)} - \frac{1}{(39)+1}\right] + \left[\frac{1}{(40)} - \frac{1}{(40)+1}\right]$

$= \left[1 - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] + \left[\frac{1}{3} - \frac{1}{4}\right] + \left[\frac{1}{4} - \frac{1}{5}\right] +$
$\quad \cdots + \left[\frac{1}{37} - \frac{1}{38}\right] + \left[\frac{1}{38} - \frac{1}{39}\right] + \left[\frac{1}{39} - \frac{1}{40}\right] + \left[\frac{1}{40} - \frac{1}{41}\right]$

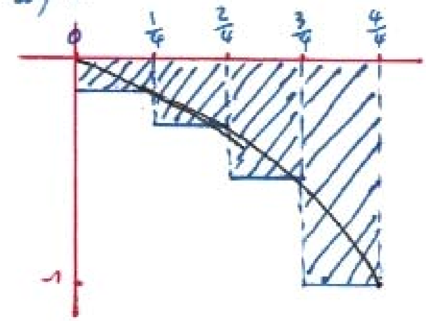
$= 1 - \frac{1}{41} = \frac{41}{41} - \frac{1}{41} = \frac{40}{41}$ because all middle terms adds to 0.

38) $f(x) = -x^2 \quad [0,1] \quad n=4 \quad \Delta x = \frac{(1)-(0)}{4} = \frac{1}{4} \quad \frac{\Delta x}{2} = \frac{\frac{1}{4}}{2} = \frac{1}{8}$

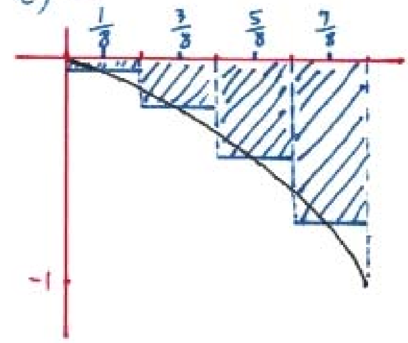
a) L



b) R



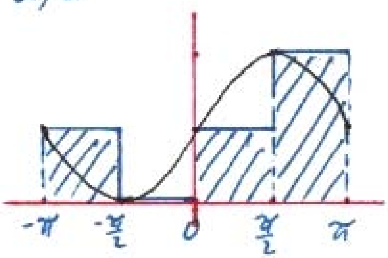
c) M



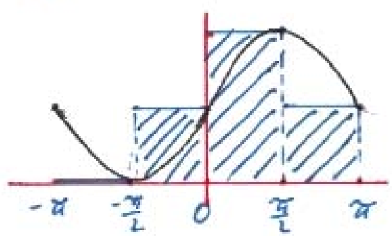
---

40) $f(x) = \sin x + 1 \quad [-\pi, \pi] \quad n=4 \quad \Delta x = \frac{(\pi)-(-\pi)}{4} = \frac{2\pi}{4} = \frac{\pi}{2} \quad \frac{\Delta x}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

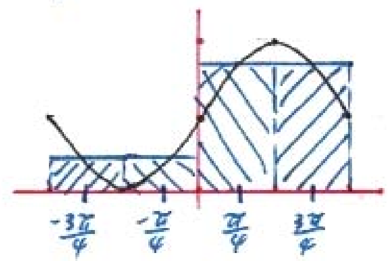
a) L



b) R



c) M



---

44) $f(x) = 2x \quad [0,3] \quad \Delta x = \frac{(3)-(0)}{n} = \frac{3}{n}$

$x_k = a + k\Delta x = (0) + k\left(\frac{3}{n}\right) = \frac{3k}{n} \qquad f(x_k) = 2x_k = 2\left(\frac{3k}{n}\right) = \frac{6k}{n}$

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\frac{6k}{n}\right)\left(\frac{3}{n}\right) = \frac{18}{n^2}\sum_{k=1}^{n} k = \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{18}{2}\left(\frac{n^2+n}{n^2}\right) = 9\left(\frac{n^2}{n^2} + \frac{1}{n^2}\right) = 9\left(1 + \frac{1}{n}\right) = 9 + \frac{9}{n}$$

$$\text{Area} = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \lim_{x\to\infty}\sum_{k=1}^{n}\left(\frac{6k}{n}\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty}\left(9 + \frac{9}{n}\right) = (9+0) = 9 \text{ units}^2$$

46) $f(x)=3x^2 \quad [0,1] \quad \Delta x = \frac{(1)-(0)}{n} = \frac{1}{n}$

$x_k = a + k\Delta x = (0) + k\left(\frac{1}{n}\right) = \frac{k}{n} \quad f(x_k) = 3\left(\frac{k}{n}\right)^2 = \frac{3k^2}{n^2}$

$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\frac{3k^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{3}{n^3}\sum_{k=1}^{n} k^2 = \frac{3}{n^3}\left\{\frac{n(n+1)(2n+1)}{6}\right\}$

$= \frac{3}{6}\left\{\frac{n(n+1)(2n+1)}{n^3}\right\} = \frac{1}{2}\left\{\frac{n(2n^2+3n+1)}{n^3}\right\} = \frac{1}{2}\left\{\frac{2n^3+3n^2+n}{n^3}\right\}$

$= \frac{1}{2}\left\{\frac{2n^3}{n^3} + \frac{3n^2}{n^3} + \frac{n}{n^3}\right\} = \frac{1}{2}\left\{2 + \frac{3}{n} + \frac{1}{n^2}\right\} = 1 + \frac{3}{2n} + \frac{1}{2n^2}$

Area $= \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty}\left(\frac{3k^2}{n^2}\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty}\left(1 + \frac{3}{2n} + \frac{1}{2n^2}\right) = (1+0+0) = 1$ units^2

---

48) $f(x) = 3x + 2x^2 \quad [0,1] \quad \Delta x = \frac{(1)-(0)}{n} = \frac{1}{n}$

$x_k = a + k\Delta x = (0) + k\left(\frac{1}{n}\right) = \frac{k}{n} \quad f(x_k) = 3\left(\frac{k}{n}\right) + 2\left(\frac{k}{n}\right)^2 = \frac{3k}{n} + \frac{2k^2}{n^2}$

$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\frac{3k}{n} + \frac{2k^2}{n^2}\right)\left(\frac{1}{n}\right) = \frac{3}{n^2}\sum_{k=1}^{n} k + \frac{2}{n^3}\sum_{k=1}^{n} k^2$

$= \frac{3}{n^2}\left\{\frac{n(n+1)}{2}\right\} + \frac{2}{n^3}\left\{\frac{n(n+1)(2n+1)}{6}\right\} = \frac{3}{2}\left\{\frac{n^2+n}{n^2}\right\} + \frac{2}{6}\left\{\frac{n(2n^2+3n+1)}{n^3}\right\}$

$= \frac{3}{2}\left\{\frac{n^2+n}{n^2}\right\} + \frac{1}{3}\left\{\frac{2n^3+3n^2+n}{n^3}\right\} = \frac{3}{2}\left\{\frac{n^2}{n^2} + \frac{n}{n^2}\right\} + \frac{1}{3}\left\{\frac{2n^3}{n^3} + \frac{3n^2}{n^3} + \frac{n}{n^3}\right\}$

$= \frac{3}{2}\left\{1 + \frac{1}{n}\right\} + \frac{1}{3}\left\{2 + \frac{3}{n} + \frac{1}{n^2}\right\} = \left\{\frac{3}{2} + \frac{3}{2n}\right\} + \left\{\frac{2}{3} + \frac{1}{n} + \frac{1}{3n^2}\right\}$

Area $= \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty}\left(\frac{3k}{n} + \frac{2k^2}{n^2}\right)\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty}\left(\left\{\frac{3}{2} + \frac{3}{2n}\right\} + \left\{\frac{2}{3} + \frac{1}{n} + \frac{1}{3n^2}\right\}\right) = \left\{\frac{3}{2} + 0\right\} + \left\{\frac{2}{3} + 0 + 0\right\}$

$= \frac{3}{2} + \frac{2}{3} = \frac{9}{6} + \frac{4}{6} = \frac{13}{6}$ units^2

50) $f(x) = x^2 - x^3$ $\quad [-1, 0] \quad \Delta x = \frac{(0)-(-1)}{n} = \frac{1}{n}$

$x_k = a + k\Delta x = (-1) + k\left(\frac{1}{n}\right) = -1 + \frac{k}{n} = \frac{k}{n} - 1$

$$f(x_k) = \left(\frac{k}{n} - 1\right)^2 - \left(\frac{k}{n} - 1\right)^3 = \left(\frac{k^2}{n^2} - \frac{2k}{n} + 1\right) - \left(\frac{k}{n} - 1\right)\left(\frac{k^2}{n^2} - \frac{2k}{n} + 1\right)$$

$$= \left(\frac{k^2}{n^2} - \frac{2k}{n} + 1\right) - \left(\frac{k^3}{n^3} - \frac{2k^2}{n^2} + \frac{k}{n} - \frac{k^2}{n^2} + \frac{2k}{n} - 1\right)$$

$$= \left(\frac{k^2}{n^2} - \frac{2k}{n} + 1\right) - \left(\frac{k^3}{n^3} - \frac{3k^2}{n^2} + \frac{3k}{n} - 1\right) = 2 - \frac{5k}{n} + \frac{4k^2}{n^2} - \frac{k^3}{n^3}$$

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left(2 - \frac{5k}{n} + \frac{4k^2}{n^2} - \frac{k^3}{n^3}\right)\left(\frac{1}{n}\right)$$

$$= \sum_{k=1}^{n} \frac{2}{n} - \frac{5}{n^2}\sum_{k=1}^{n} k + \frac{4}{n^3}\sum_{k=1}^{n} k^2 - \frac{1}{n^4}\sum_{k=1}^{n} k^3$$

$$= \left\{(n)\left(\frac{2}{n}\right)\right\} - \frac{5}{n^2}\left\{\frac{n(n+1)}{2}\right\} + \frac{4}{n^3}\left\{\frac{n(n+1)(2n+1)}{6}\right\} - \frac{1}{n^4}\left\{\left(\frac{n(n+1)}{2}\right)^2\right\}$$

$$= \{2\} - \frac{5}{2}\left\{\frac{n^2+n}{n^2}\right\} + \frac{4}{6}\left\{\frac{n(2n^2+3n+1)}{n^3}\right\} - \frac{1}{(2)^2}\left\{\frac{n^2(n^2+2n+1)}{n^4}\right\}$$

$$= \{2\} - \frac{5}{2}\left\{\frac{n^2}{n^2} + \frac{n}{n^2}\right\} + \frac{2}{3}\left\{\frac{2n^3+3n^2+n}{n^3}\right\} - \frac{1}{4}\left\{\frac{n^4+2n^3+n^2}{n^4}\right\}$$

$$= \{2\} - \frac{5}{2}\left\{1 + \frac{1}{n}\right\} + \frac{2}{3}\left\{\frac{2n^3}{n^3} + \frac{3n^2}{n^3} + \frac{n}{n^3}\right\} - \frac{1}{4}\left\{\frac{n^4}{n^4} + \frac{2n^3}{n^4} + \frac{n^2}{n^4}\right\}$$

$$= \{2\} - \frac{5}{2}\left\{1 + \frac{1}{n}\right\} + \frac{2}{3}\left\{2 + \frac{3}{n} + \frac{1}{n^2}\right\} - \frac{1}{4}\left\{1 + \frac{2}{n} + \frac{1}{n^2}\right\}$$

$$= \{2\} - \left\{\frac{5}{2} + \frac{5}{2n}\right\} + \left\{\frac{4}{3} + \frac{2}{n} + \frac{2}{3n^2}\right\} - \left\{\frac{1}{4} + \frac{1}{2n} + \frac{1}{4n^2}\right\}$$

$$\text{Area} = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty} \left(2 - \frac{5k}{n} + \frac{4k^2}{n^2} - \frac{k^3}{n^3}\right)\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \left(\{2\} - \left\{\frac{5}{2} + \frac{5}{2n}\right\} + \left\{\frac{4}{3} + \frac{2}{n} + \frac{2}{3n^2}\right\} - \left\{\frac{1}{4} + \frac{1}{2n} + \frac{1}{4n^2}\right\}\right)$$

$$= \{2\} - \left\{\frac{5}{2} + 0\right\} + \left\{\frac{4}{3} + 0 + 0\right\} - \left\{\frac{1}{4} + 0 + 0\right\} = 2 - \frac{5}{2} + \frac{4}{3} - \frac{1}{4}$$

$$= \frac{24}{12} - \frac{30}{12} + \frac{16}{12} - \frac{3}{12} = \frac{40-33}{12} = \frac{7}{12} \text{ units}^2$$