MATH 20100 section 5.1

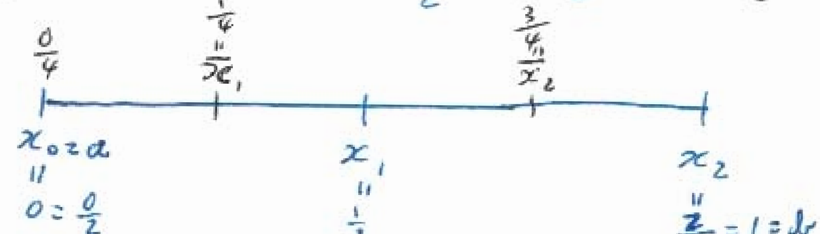
Given $f(x)$ defined on a closed interval $[a, b]$ and if we divide this interval into n subintervals, then the width of the subinterval is $\Delta x = \frac{b-a}{n}$

an arbitrary term is $x_k = a + k\Delta x$. note: $x_0 = a$ and $x_n = b$

---

2 & 6) $f(x) = x^3 \quad [0, 1]$

a & c part $n = 2$ $\quad \Delta x = \frac{(1)-(0)}{2} = \frac{1}{2} = \frac{2}{4} \quad \frac{\Delta x}{2} = \frac{(\frac{1}{2})}{2} = \frac{1}{4}$



$x_k = a + k\Delta x$

$x_k = (0) + k\left(\frac{1}{2}\right) = \frac{k}{2}$

#2 part:

a) $L_2 = \Delta x\{f(x_0) + f(x_1)\} = \left(\frac{1}{2}\right)\left\{\left(\frac{0}{2}\right)^3 + \left(\frac{1}{2}\right)^3\right\} = \sum_{k=0}^{1} f(x_k)\Delta x = \sum_{k=0}^{1} \left(\frac{k}{2}\right)^3\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{8}\right) = \frac{1}{16}$

c) $R_2 = \Delta x\{f(x_1) + f(x_2)\} = \left(\frac{1}{2}\right)\left\{\left(\frac{1}{2}\right)^3 + \left(\frac{2}{2}\right)^3\right\} = \sum_{k=1}^{2} f(x_k)\Delta x = \sum_{k=1}^{2} \left(\frac{k}{2}\right)^3\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)\left\{\frac{1}{8} + 1\right\} = \left(\frac{1}{2}\right)\left\{\frac{9}{8}\right\} = \frac{9}{16}$

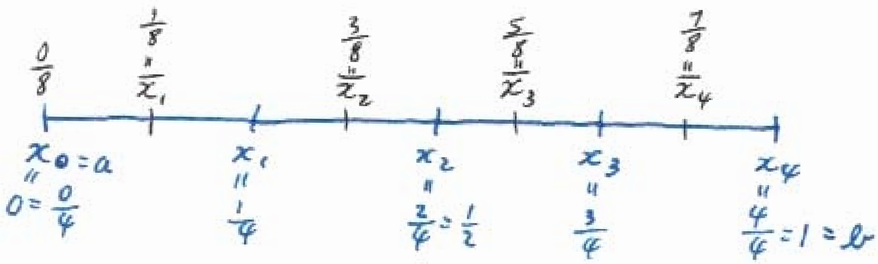
#6 part:

$M_2 = \Delta x\{f(\bar{x}_1) + f(\bar{x}_2)\} = \left(\frac{1}{2}\right)\left\{\left(\frac{1}{4}\right)^3 + \left(\frac{3}{4}\right)^3\right\} = \left(\frac{1}{2}\right)\left\{\frac{1}{64} + \frac{27}{64}\right\}$

$= \left(\frac{1}{2}\right)\left\{\frac{28}{64}\right\} = \frac{14}{64} = \frac{7}{32}$

2 & 6) continued

b & d part $\quad n=4 \quad \Delta x = \frac{(1)-(0)}{4} = \frac{1}{4} = \frac{2}{8} \quad \frac{\Delta x}{2} = \frac{\left(\frac{1}{4}\right)}{2} = \frac{1}{8}$



$x_k = a + k\Delta x$

$x_k = (0) + k\left(\frac{1}{4}\right) = \frac{k}{4}$

#2 part:

b) $L_4 = \Delta x \left\{ f(x_0) + f(x_1) + f(x_2) + f(x_3) \right\} = \sum_{k=0}^{3} f(x_k)\Delta x = \sum_{k=0}^{3} \left(\frac{k}{4}\right)^3 \left(\frac{1}{4}\right)$

$= \left(\frac{1}{4}\right) \left\{ \left(\frac{0}{4}\right)^3 + \left(\frac{1}{4}\right)^3 + \left(\frac{2}{4}\right)^3 + \left(\frac{3}{4}\right)^3 \right\}$

$= \left(\frac{1}{4}\right) \left\{ (0) + \left(\frac{1}{64}\right) + \left(\frac{8}{64}\right) + \left(\frac{27}{64}\right) \right\} = \left(\frac{1}{4}\right) \left\{ \frac{36}{64} \right\} = \frac{9}{64}$

d) $R_4 = \Delta x \left\{ f(x_1) + f(x_2) + f(x_3) + f(x_4) \right\} = \sum_{k=1}^{4} f(x_k)\Delta x = \sum_{k=1}^{4} \left(\frac{k}{4}\right)^3 \left(\frac{1}{4}\right)$

$= \left(\frac{1}{4}\right) \left\{ \left(\frac{1}{4}\right)^3 + \left(\frac{2}{4}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{4}{4}\right)^3 \right\}$

$= \left(\frac{1}{4}\right) \left\{ \left(\frac{1}{64}\right) + \left(\frac{8}{64}\right) + \left(\frac{27}{64}\right) + \left(\frac{64}{64}\right) \right\} = \left(\frac{1}{4}\right) \left\{ \frac{100}{64} \right\} = \frac{25}{64}$

#6 part:

$M_4 = \Delta x \left\{ f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) \right\}$

$= \left(\frac{1}{4}\right) \left\{ \left(\frac{1}{8}\right)^3 + \left(\frac{3}{8}\right)^3 + \left(\frac{5}{8}\right)^3 + \left(\frac{7}{8}\right)^3 \right\}$

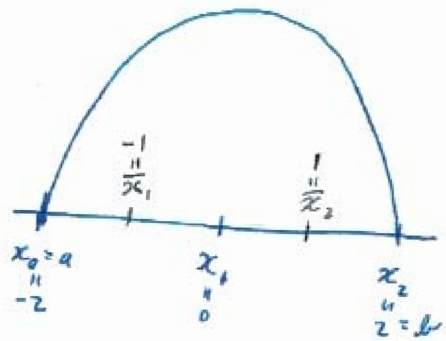
$= \left(\frac{1}{4}\right) \left\{ \frac{1}{512} + \frac{27}{512} + \frac{125}{512} + \frac{343}{512} \right\} = \left(\frac{1}{4}\right) \left\{ \frac{496}{512} \right\} = \frac{124}{512} = \frac{62}{256} = \frac{31}{128}$

4 & 8) $f(x) = 4 - x^2$ $\quad [-2, 2]$

a & c part $\quad n = 2$

$\Delta x = \frac{(2)-(-2)}{2} = \frac{4}{2} = 2$

$\frac{\Delta x}{2} = \frac{2}{2} = 1$



#2 part:

a) $lower_2 = \Delta x \{f(x_0) + f(x_2)\} = (2)\{(4-(-2)^2) + (4-(2)^2)\}$
$= (2)\{(4-4) + (4-4)\} = (2)\{0+0\} = 0$

c) $Upper_2 = \Delta x \{f(x_1) + f(x_1)\} = (2)\{(4-(0)^2) + (4-(0)^2)\}$
$= (2)\{(4) + (4)\} = (2)\{8\} = 16$

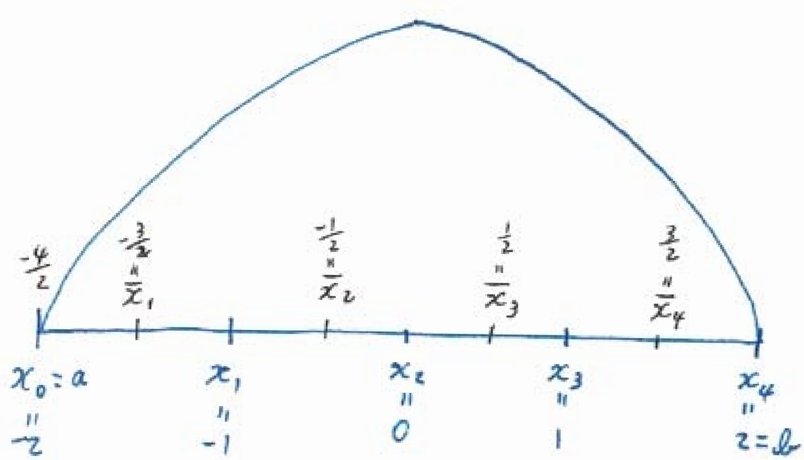
#8 part:

$M_2 = \Delta x \{f(\bar{x}_1) + f(\bar{x}_2)\} = (2)\{(4-(-1)^2) + (4-(1)^2)\} = (2)\{(4-1)+(4-1)\}$
$= (2)\{3+3\} = (2)\{6\} = 12$

b & d part $\quad n = 4$

$\Delta x = \frac{(2)-(-2)}{4} = \frac{4}{4} = 1 = \frac{2}{2}$

$\frac{\Delta x}{2} = \frac{1}{2}$



#2 part:

b) $lower_4 = \Delta x \{f(x_0) + f(x_1) + f(x_3) + f(x_4)\}$
$= (1)\{(4-(-2)^2) + (4-(-1)^2) + (4-(1)^2) + (4-(2)^2)\}$
$= (1)\{(4-4) + (4-1) + (4-1) + (4-4)\} = (1)\{0+3+3+0\} = (1)\{6\} = 6$

4 & 8) continued

d) $\text{Upper}_4 = \Delta x \{ f(x_1) + f(x_2) + f(x_2) + f(x_4) \}$

$= (1)\{(4-(-1)^2) + (4-(0)^2) + (4-(0)^2) + (4-(1)^2)\}$

$= (1)\{(4-1) + (4-0) + (4-0) + (4-1)\}$

$= (1)\{3+4+4+3\} = (1)\{14\} = 14$

#8 part:

$M_4 = \Delta x \{ f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) \}$

$= (1)\{(4-(\frac{-3}{2})^2) + (4-(\frac{-1}{2})^2) + (4-(\frac{1}{2})^2) + (4-(\frac{3}{2})^2)\}$

$= (1)\{(4-\frac{9}{4}) + (4-\frac{1}{4}) + (4-\frac{1}{4}) + (4-\frac{9}{4})\}$

$= (1)\{(\frac{16}{4}-\frac{9}{4}) + (\frac{16}{4}-\frac{1}{4}) + (\frac{16}{4}-\frac{1}{4}) + (\frac{16}{4}-\frac{9}{4})\}$

$= (1)\{\frac{7}{4} + \frac{15}{4} + \frac{15}{4} + \frac{7}{4}\} = (1)\{\frac{44}{4}\} = \frac{44}{4} = 11$

---

10) $\Delta t = 5 \text{ min} = 300 \text{ sec}$ $\quad n = 12$

a) $L = (300)\{(1)+(1.2)+(1.7)+(2.0)+(1.8)+(1.6)+(1.4)+(1.2)+(1.0)+(1.8)+(1.5)+(1.2)\}$

$= (300)\{17.4\} = (30)\{174\} = 5220$ meters.

b) $R = (300)\{(1.2)+(1.7)+(2.0)+(1.8)+(1.6)+(1.4)+(1.2)+(1.0)+(1.8)+(1.5)+(1.2)+(0)\}$

$= (300)\{16.4\} = (30)\{164\} = 4920$ meters