Definition

A function F is an **antiderivative** of f on an interval I if $F'(x) = \frac{dF}{dx} = f(x)$ for all x in I.

Theorem 8

If F is an antiderivative of f on and interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Definition

The collection of all antiderivatives of f is called **indefinite integral** of f with respect to x, and is denoted by

$$\int f(x)\,dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

Note: On any integral, dx inform us about what is the variable of the function (the variable that we need to use our rules of integration) and all other letters are constants.

For example:

1) $\int xyt\,dx$ integrate with respect to x, x is the variable where we apply the rules below, y and t are constants, so $\int xyt\,dx = yt\left[\frac{x^2}{2}\right] + C = \frac{1}{2}x^2yt + C$

2) $\int xyt\,dy$ integrate with respect to y, y is the variable where we apply the rules below, x and t are constants, so $\int xyt\,dy = xt\left[\frac{y^2}{2}\right] + C = \frac{1}{2}xy^2t + C$

3) $\int xyt\,dt$ integrate with respect to t, t is the variable where we apply the rules below ,x and y are constants, so $\int xyt\,dt = xy\left[\frac{t^2}{2}\right] + C = \frac{1}{2}xyt^2 + C$

Instead of listing the Antiderivative formulas on pg. 283 (but still copy them into your notes), I listed a part of the Integration Table from section 8.1 because these formats of the formulas are the ones you will be using on your next course.

$\int k\,dt = kt + C$ (any number k)	$\int a^t\,dt = \frac{a^t}{\ln a} + C \quad (a>0, a\neq 1)$	$\int \csc^2 t\,dt = -\cot t + C$
$\int t^n\,dt = \frac{t^{n+1}}{n+1} + C \quad (n \neq 1)$	$\int \sin t\,dt = -\cos t + C$	$\int \sec t \tan t\,dt = \sec t + C$
$\int \frac{1}{t}\,dt = \ln\lvert t\rvert + C$	$\int \cos t\,dt = \sin t + C$	$\int \csc t \cot t\,dt = -\csc t + C$
$\int e^t\,dt = e^t + C$	$\int \sec^2 t\,dt = \tan t + C$	$\int \frac{1}{a^2+t^2}\,dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right) + C$

2-a) $\int 6x\,dx = 6\left[\frac{x^2}{2}\right] + C = 3x^2 + C$

2-b) $\int x^7\,dx = \left[\frac{x^8}{8}\right] + C = \frac{1}{8}x^8 + C$

2-c) $\int x^7 - 6x + 8\,dx = \left[\frac{x^8}{8}\right] - 6\left[\frac{x^2}{2}\right] + 8[x] + C = \frac{1}{8}x^8 - 3x^2 + 8x + C$

4-a) $\int 2x^{-3}\,dx = 2\left[\frac{x^{-2}}{-2}\right] + C = -x^{-2} + C = \frac{-1}{x^2} + C$

4-b) $\int \frac{x^{-3}}{2} + x^2\,dx = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right] + \left[\frac{x^3}{3}\right] + C = \frac{-1}{4x^2} + \frac{1}{3}x^3 + C$

4-c) $\int (-x^{-3} + x - 1)\,dx = -\left[\frac{x^{-2}}{-2}\right] + \left[\frac{x^2}{2}\right] - [x] + C = \frac{1}{2x^2} + \frac{1}{2}x^2 - x + C$

6-a) $\int -\frac{2}{x^3}\,dx = \int -2x^{-3}\,dx = -2\left[\frac{x^{-2}}{-2}\right] + C = x^{-2} + C = \frac{1}{x^2} + C$

6-b) $\int \frac{1}{2x^3}\,dx = \int \frac{1}{2}x^{-3}\,dx = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right] + C = \frac{-1}{4}x^{-2} + C = \frac{-1}{4x^2} + C$

6-c) $\int x^3 - \frac{1}{x^3}\,dx = \int (x^3 - x^{-3})\,dx = \left[\frac{x^4}{4}\right] - \left[\frac{x^{-2}}{-2}\right] + C = \frac{1}{4}x^4 + \frac{1}{2x^2} + C$

8-a) $\int \frac{4}{3}\sqrt[3]{x}\,dx = \int \frac{4}{3}x^{\frac{1}{3}}\,dx = \frac{4}{3}\left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right] + C = x^{\frac{4}{3}} + C = (\sqrt[3]{x})^4 + C$

8-b) $\int \frac{1}{3\sqrt[3]{x}}\,dx = \int \frac{1}{3}x^{-\frac{1}{3}}\,dx = \frac{1}{3}\left[\frac{x^{2/3}}{\frac{2}{3}}\right] + C = \frac{1}{2}x^{2/3} + C = \frac{1}{2}(\sqrt[3]{x})^2 + C$

8-c) $\int \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\,dx = \int (x^{\frac{1}{3}} + x^{-\frac{1}{3}})\,dx = \left[\frac{x^{4/3}}{\frac{4}{3}}\right] + \left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right] + C = \frac{3}{4}(\sqrt[3]{x})^4 + \frac{3}{2}(\sqrt[3]{x})^2 + C$

10-a) $\int \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{1}{2}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = x^{\frac{1}{2}} + C = \sqrt{x} + C$

10-b) $\int -\frac{1}{2}x^{-\frac{3}{2}}\,dx = \frac{-1}{2}\left[\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right] + C = x^{-\frac{1}{2}} + C = \frac{1}{\sqrt{x}} + C$

10-c) $\int -\frac{3}{2}x^{-\frac{5}{2}}\,dx = \frac{-3}{2}\left[\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}\right] + C = x^{-\frac{3}{2}} + C = \frac{1}{(\sqrt{x})^3} + C$

12-a) $\int \frac{1}{3x}\,dx = \int \left(\frac{1}{3}\right)\frac{1}{x}\,dx = \frac{1}{3}\left[\ln|x|\right] + C = \frac{1}{3}\ln|x| + C$

12-b) $\int \frac{2}{5x}\,dx = \int \left(\frac{2}{5}\right)\frac{1}{x}\,dx = \frac{2}{5}\left[\ln|x|\right] + C = \frac{2}{5}\ln|x| + C$

12-c) $\int \left(1 + \frac{4}{3x} - \frac{1}{x^2}\right)dx = \int \left(1 + \left(\frac{4}{3}\right)\left(\frac{1}{x}\right) - x^{-2}\right)dx$

$$= [x] + \left(\frac{4}{3}\right)\left[\ln|x|\right] - \left[\frac{x^{-1}}{-1}\right] + C$$

$$= x + \frac{4}{3}\ln|x| + \frac{1}{x} + C$$

14-a) $\int \pi \cos \pi x\,dx = \int \cos(\pi x)(\pi\,dx)$

$$= \int \cos(p)(dp) = \sin p + C$$

let $p = \pi x$

$$= \sin(\pi x) + C$$

$\frac{dp}{dx} = \pi \Rightarrow dp = \pi\,dx$

14-b) $\int \frac{\pi}{2} \cos \frac{\pi}{2} x\,dx = \int \cos\left(\frac{\pi}{2}x\right)\left(\frac{\pi}{2}dx\right)$

let $p = \frac{\pi}{2}x$ $\quad = \int \cos(p)(dp) = \sin p + C$

$\frac{dp}{dx} = \frac{\pi}{2} \Rightarrow dp = \frac{\pi}{2}dx$ $\quad = \sin\left(\frac{\pi}{2}x\right) + C$

14-c) $\int \left(\cos\frac{\pi x}{2} + \pi \cos x\right)dx = \int \cos\left(\frac{\pi}{2}x\right)dx + \int \pi \cos x\,dx$

$\int \cos\left(\frac{\pi}{2}x\right)dx = \int \cos p\left(\frac{2}{\pi}dp\right)$

$p = \frac{\pi}{2}x$ $\quad = \frac{2}{\pi}[\sin p] + C_1$

$dp = \frac{\pi}{2}dx$ $\quad = \frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right) + C_1$

$\frac{2}{\pi}dp = dx$

$$= \left[\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)\right] + \pi[\sin x] + C$$

$$= \frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right) + \pi \sin x + C$$

16-a) $\int \csc^2 x\,dx = [-\cot x] + C = -\cot x + C$

16-b) $\int -\frac{3}{2}\csc^2\frac{3x}{2}\,dx = \int -\csc^2\left(\frac{3}{2}x\right)\left(\frac{3}{2}dx\right)$

$p = \frac{3}{2}x$

$dp = \frac{3}{2}dx$

$= \int -\csc^2(p)\,(dp) = -[-\cot p] + C$

$= \cot\left(\frac{3}{2}x\right) + C$

16-c) $\int(1 - 8\csc^2 2x)\,dx = \int 1\,dx - \int 8\csc^2(2x)\,dx$

$\int 8\csc^2(2x)\,dx = \int 4\csc^2(2x)(2\,dx)$

$p = 2x$

$dp = 2\,dx$

$= \int 4\csc^2(p)(dp)$

$= 4[-\cot(p)] + c_1$

$= -4\cot(2x) + c_1$

$= [x] - [-4\cot(2x)] + C$

$= x + 4\cot(2x) + C$

18-a) $\int \sec x \tan x\,dx = \sec x + C$

18-b) $\int 4\sec 3x \tan 3x\,dx = \int 4\sec(p)\tan(p)\left(\frac{1}{3}dp\right)$

$p = 3x$

$dp = 3\,dx$

$\frac{1}{3}dp = dx$

$= \int \frac{4}{3}\sec(p)\tan(p)\,dp$

$= \frac{4}{3}[\sec p] + C$

$= \frac{4}{3}\sec(3x) + C$

18-c) $\int \sec\frac{\pi x}{2}\tan\frac{\pi x}{2}\,dx = \int \sec(p)\tan(p)\left(\frac{2}{\pi}dp\right)$

$p = \frac{\pi}{2}x$

$dp = \frac{\pi}{2}dx$

$\frac{2}{\pi}dp = dx$

$= \int \frac{2}{\pi}\sec(p)\tan(p)\,dp$

$= \frac{2}{\pi}[\sec p] + C$

$= \frac{2}{\pi}\sec\left(\frac{\pi}{2}x\right) + C$

20-a) $\int e^{-2x}\,dx = \int e^{p}\left(\frac{-1}{2}\,dp\right) = \int \frac{-1}{2} e^{p}\,dp$

$p = -2x$

$dp = -2\,dx$

$\frac{-1}{2}\,dp = dx$

$= \frac{-1}{2}\left[e^{p}\right] + C$

$= \frac{-1}{2} e^{-2x} + C = \frac{-1}{2e^{2x}} + C$

20-b) $\int e^{\frac{4x}{3}}\,dx = \int e^{p}\left(\frac{3}{4}\,dp\right) = \int \frac{3}{4} e^{p}\,dp$

$p = \frac{4}{3}x$

$dp = \frac{4}{3}\,dx$

$\frac{3}{4}\,dp = dx$

$= \frac{3}{4}\left[e^{p}\right] + C$

$= \frac{3}{4} e^{\frac{4x}{3}} + C$

20-c) $\int e^{\frac{-x}{5}}\,dx = \int e^{p}(-5\,dp) = \int -5 e^{p}\,dp$

$p = \frac{-x}{5}$

$dp = \frac{-1}{5}\,dx$

$-5\,dp = dx$

$= -5\left[e^{p}\right] + C$

$= -5 e^{\frac{-x}{5}} + C = \frac{-5}{e^{\frac{x}{5}}} + C$

22-a) $\int x^{\sqrt{3}}\,dx = \left[\frac{x^{(\sqrt{3}+1)}}{(\sqrt{3}+1)}\right] + C = \frac{1}{1+\sqrt{3}}\, x^{(1+\sqrt{3})} + C$

22-b) $\int x^{\pi}\,dx = \left[\frac{x^{(\pi+1)}}{(\pi+1)}\right] + C = \frac{1}{\pi+1}\, x^{(\pi+1)} + C$

22-c) $\int x^{(\sqrt{2}-1)}\,dx = \left[\frac{x^{\sqrt{2}}}{\sqrt{2}}\right] + C = \frac{1}{\sqrt{2}}\, x^{\sqrt{2}} + C$

24-a) $\int \left(x-\left(\frac{1}{2}\right)^x\right)dx = \int x\,dx - \int \left(\frac{1}{2}\right)^x dx$

$$= \left[\frac{x^2}{2}\right] - \left[\frac{\left(\frac{1}{2}\right)^x}{\ln\left(\frac{1}{2}\right)}\right] + C$$

$$= \frac{1}{2}x^2 - \frac{1}{\ln\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)^x + C$$

24-b) $\int (x^2+2^x)\,dx = \int x^2\,dx + \int 2^x\,dx$

$$= \left[\frac{x^3}{3}\right] + \left[\frac{2^x}{\ln 2}\right] + C$$

$$= \frac{1}{3}x^3 + \frac{1}{\ln 2}2^x + C$$

24-c) $\int (\pi^x - x^{-1})\,dx = \int \pi^x\,dx - \int \frac{1}{x}\,dx$

$$= \left[\frac{\pi^x}{\ln \pi}\right] - \left[\ln|x|\right] + C$$

$$= \frac{1}{\ln \pi}\pi^x - \ln|x| + C$$

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26) $\int (5-6x)\,dx = 5[x] - 6\left[\frac{x^2}{2}\right] + C = 5x - 3x^2 + C$

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28) $\int \left(\frac{t^2}{2} + 4t^3\right)dt = \frac{1}{2}\left[\frac{t^3}{3}\right] + 4\left[\frac{t^4}{4}\right] + C = \frac{1}{6}t^3 + t^4 + C$

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30) $\int (1-x^2-3x^5)\,dx = [x] - \left[\frac{x^3}{3}\right] - 3\left[\frac{x^6}{6}\right] + C$

$$= x - \frac{1}{3}x^3 - \frac{1}{2}x^6 + C$$

32) $\int\left(\frac{1}{5}-\frac{2}{x^3}+2x\right)dx=\int\left(\frac{1}{5}-2x^{-3}+2x\right)dx$

$=\frac{1}{5}[x]-2\left[\frac{x^{-2}}{-2}\right]+2\left[\frac{x^2}{2}\right]+C=\frac{1}{5}x+\frac{1}{x^2}+x^2+C$

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34) $\int x^{-\frac{5}{4}}dx=\left[\frac{x^{-\frac{1}{4}}}{\frac{-1}{4}}\right]+C=-4x^{-\frac{1}{4}}+C=\frac{-4}{\sqrt[4]{x}}+C$

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36) $\int\left(\frac{\sqrt{x}}{2}+\frac{2}{\sqrt{x}}\right)dx=\int\left(\frac{1}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\right)dx$

$=\frac{1}{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]+2\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]+C=\frac{1}{3}x^{\frac{3}{2}}+4x^{\frac{1}{2}}+C=\frac{1}{3}(\sqrt{x})^3+4\sqrt{x}+C$

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38) $\int\left(\frac{1}{7}-\frac{1}{y^{5/4}}\right)dy=\int\left(\frac{1}{7}-y^{-5/4}\right)dy=\frac{1}{7}[y]-\left[\frac{y^{-\frac{1}{4}}}{\frac{-1}{4}}\right]+C$

$=\frac{1}{7}y+4y^{-\frac{1}{4}}+C=\frac{1}{7}y+\frac{4}{\sqrt[4]{y}}+C$

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40) $\int x^{-3}(x+1)\,dx=\int\left(x^{-2}+x^{-3}\right)dx=\left[\frac{x^{-1}}{-1}\right]+\left[\frac{x^{-2}}{-2}\right]+C$

$=-x^{-1}-\frac{1}{2}x^{-2}+C=\frac{-1}{x}-\frac{1}{2x^2}+C$

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42) $\int\frac{4+\sqrt{t}}{t^3}dt=\int\left(\frac{4}{t^3}+\frac{\sqrt{t}}{t^3}\right)dt=\int\left(4t^{-3}+t^{-5/2}\right)dt$

$=4\left[\frac{t^{-2}}{-2}\right]+\left[\frac{t^{-\frac{3}{2}}}{\frac{-3}{2}}\right]+C=-2t^{-2}-\frac{2}{3}t^{-\frac{3}{2}}+C$

$=\frac{-2}{t^2}-\frac{2}{3t^{3/2}}+C=\frac{-2}{t^2}-\frac{2}{3(\sqrt{t})^3}+C$

44) $\int(-5\sin t)\,dt = -5[-\cos t] + C = 5\cos t + C$

46) $\int 3\cos 5\theta\, d\theta = \int 3\cos(p)\left(\frac{1}{5}dp\right) = \int \frac{3}{5}\cos(p)(dp)$

$p = 5\theta$ $\quad = \frac{3}{5}[\sin(p)] + C$

$dp = 5\,d\theta$ $\quad = \frac{3}{5}\sin(5\theta) + C$

$\frac{1}{5}dp = d\theta$

48) $\int\left(-\frac{\sec^2 x}{3}\right)dx = \frac{-1}{3}[\tan x] + C = \frac{-1}{3}\tan x + C$

50) $\int \frac{2}{5}\sec\theta\tan\theta\, d\theta = \frac{2}{5}[\sec\theta] + C = \frac{2}{5}\sec\theta + C$

52) $\int(2e^x - 3e^{-2x})dx = \int 2e^x dx - \int 3e^{-2x}dx$

$\int 3e^{-2x}dx = \int 3e^p\left(\frac{-1}{2}dp\right)$ $\quad = 2[e^x] - \left[\frac{-3}{2}e^{-2x}\right] + C$

$p = -2x$ $\quad = \int \frac{-3}{2}e^p dp$ $\quad = 2e^x + \frac{3}{2}e^{-2x} + C$

$dp = -2\,dx$ $\quad = \frac{-3}{2}[e^p] + C_1$

$\frac{-1}{2}dp = dx$ $\quad = \frac{-3}{2}e^{-2x} + C_1$ $\quad = 2e^x + \frac{3}{2e^{2x}} + C$

54) $\int(1.3)^x dx = \left[\frac{(1.3)^x}{\ln(1.3)}\right] + C = \frac{1}{\ln(1.3)}(1.3)^x + C$

56) $\int\frac{1}{2}(\csc^2 x - \csc x\cot x)dx = \frac{1}{2}\{[-\cot x] - [-\csc x]\} + C$

$= \frac{1}{2}\{-\cot x + \csc x\} + C \equiv \frac{1}{2}\csc x - \frac{1}{2}\cot x + C$

58) $\int (2\cos 2x - 3\sin 3x)\,dx = \int 2\cos 2x\,dx - \int 3\sin 3x\,dx$

$= [\sin(2x)] - [-\cos(3x)] + C$

$= \sin(2x) + \cos(3x) + C$

$\int 2\cos 2x\,dx = \int \cos(2x)(2\,dx)$

$p = 2x \qquad = \int \cos(p)(dp)$

$dp = 2\,dx \qquad = [\sin(p)] + C_1$

$= \sin(2x) + C_1$

$\int 3\sin 3x\,dx = \int \sin(3x)(3\,dx)$

$q = 3x \qquad = \int \sin(q)(dq)$

$dq = 3\,dx \qquad = [-\cos(q)] + C_2$

$= -\cos(3x) + C_2$

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60) $\int \frac{1-\cos 6t}{2}\,dt = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6t\right)dt = \int \frac{1}{2}\,dt - \int \frac{1}{2}\cos 6t\,dt$

$\int \frac{1}{2}\cos 6t\,dt = \int \frac{1}{2}\cos(p)\left(\frac{1}{6}dp\right)$

$p = 6t \qquad = \int \frac{1}{12}\cos(p)(dp)$

$dp = 6\,dt \qquad = \frac{1}{12}[\sin(p)] + C_1$

$\frac{1}{6}dp = dt \qquad = \frac{1}{12}\sin(6t) + C_1$

$= \frac{1}{2}[t] - \left[\frac{1}{12}\sin(6t)\right] + C$

$= \frac{1}{2}t - \frac{1}{12}\sin(6t) + C$

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62) $\int \left(\frac{2}{\sqrt{1-y^2}} - \frac{1}{y^{1/4}}\right)dy = \int \frac{2}{\sqrt{1-y^2}}\,dy - \int y^{-1/4}\,dy$

$\int \frac{2}{\sqrt{1-y^2}}\,dy = \int 2\left(\frac{1}{\sqrt{(1)^2-y^2}}\right)dy$

$= 2\left[\sin^{-1}\left(\frac{y}{1}\right)\right] + C_1$

$= 2\sin^{-1}y + C_1$

$= \left[2\sin^{-1}y\right] - \left[\frac{y^{\frac{3}{4}}}{\frac{3}{4}}\right] + C$

$= 2\sin^{-1}y - \frac{4}{3}y^{\frac{3}{4}} + C$

$= 2\sin^{-1}y - \frac{4}{3}\left(\sqrt[4]{y}\right)^3 + C$

64) $\int x^{\sqrt{2}-1}\,dx = \left[\frac{x^{\sqrt{2}}}{\sqrt{2}}\right] + C = \frac{1}{\sqrt{2}}x^{\sqrt{2}} + C$

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66) $\int (2+\tan^2\theta)\,d\theta = \int (1+1+\tan^2\theta)\,d\theta = \int (1+(1+\tan^2\theta))\,d\theta$

$= \int (1+\sec^2\theta)\,d\theta = [\theta] + [\tan\theta] + C = \theta + \tan\theta + C$

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68) $\int (1-\cot^2 x)\,dx = \int (1-(\csc^2 x - 1))\,dx = \int (1-\csc^2 x + 1)\,dx$

$= \int (2-\csc^2 x)\,dx = 2[x] - [-\cot x] + C = 2x + \cot x + C$

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70) $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta = \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,d\theta = \int \left(\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \frac{\sin\theta}{1}}\right)\left(\frac{\frac{\sin\theta}{1}}{\frac{\sin\theta}{1}}\right)d\theta$

$= \int \frac{1}{1-\sin^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta}\,d\theta = \int \sec^2\theta\,d\theta = [\tan\theta] + C = \tan\theta + C$

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71) $\int (7x-2)^3\,dx = \frac{(7x-2)^4}{28} + C$

$\frac{d}{dx}\left(\frac{(7x-2)^4}{28} + C\right) = \frac{1}{28}\left[4(7x-2)^3(7)\right] + [0] = \frac{1}{28}\left[28(7x-2)^3\right] = (7x-2)^3$

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72) $\int (3x+5)^{-2}\,dx = -\frac{(3x+5)^{-1}}{3} + C$

$\frac{d}{dx}\left(-\frac{(3x+5)^{-1}}{3} + C\right) = \frac{-1}{3}\left[-1(3x+5)^{-2}(3)\right] + [0] = (3x+5)^{-2}$

74) $\int \csc^2\left(\frac{x-1}{3}\right)dx = -3\cot\left(\frac{x-1}{3}\right)+C$

$$\frac{d}{dx}\left(-3\cot\left(\frac{x-1}{3}\right)+C\right) = -3\left[\csc^2\left(\frac{x-1}{3}\right)\left(\frac{1}{3}\right)\right]+[0] = \csc^2\left(\frac{x-1}{3}\right)$$

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76) $\int \frac{1}{(x+1)^2}\,dx = \frac{x}{x+1}+C$

$$\frac{d}{dx}\left(\frac{x}{x+1}+C\right) = \frac{(x+1)[1]-(x)[1]}{(x+1)^2}+[0] = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

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78) $\int x e^x\,dx = xe^x - e^x + C$

$$\frac{d}{dx}\left(xe^x - e^x + C\right) = \{(x)[e^x(1)]+(e^x)[1]\} - [e^x(1)] + [0]$$

$$= \{xe^x + e^x\} - e^x = xe^x$$

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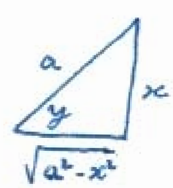
80) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C$

$$\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)+C\right) = \left[\frac{1}{\sqrt{a^2-x^2}}\right]+[0] = \frac{1}{\sqrt{a^2-x^2}}$$

$y = \sin^{-1}\left(\frac{x}{a}\right)$

$\Downarrow$

$\sin y = \frac{x}{a}$



$\left[\cos y \frac{dy}{dx}\right] = \frac{1}{a}[1]$

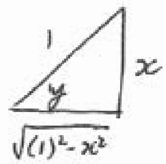
$$\frac{dy}{dx} = \frac{1}{a\cos y} = \frac{1}{a\left(\frac{\sqrt{a^2-x^2}}{a}\right)} = \frac{1}{\sqrt{a^2-x^2}}$$

82) $\int (\sin^{-1}x)^2\,dx = x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\,\sin^{-1}x + C$

$y = \sin^{-1}x$

$\Downarrow$

$\sin y = x = \frac{x}{1}$



$\left[\cos y \frac{dy}{dx}\right] = [1]$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\left(\frac{\sqrt{(1)^2-x^2}}{1}\right)} = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d}{dx}\left(x(\sin^{-1}x)^2 - 2x + 2\sqrt{1-x^2}\,\sin^{-1}x + C\right)$$

$$= \left\{(x)\left[2(\sin^{-1}x)^1\left(\frac{1}{\sqrt{1-x^2}}\right)\right] + \left((\sin^{-1}x)^2\right)[1]\right\} - 2[1] + \left\{\left(2\sqrt{1-x^2}\right)\left[\frac{1}{\sqrt{1-x^2}}\right] + (\sin^{-1}x)\left[2\left(\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\right)\right]\right\} + [0]$$

$$= \frac{2x(\sin^{-1}x)}{\sqrt{1-x^2}} + (\sin^{-1}x)^2 - 2 + 2 - \frac{2x(\sin^{-1}x)}{\sqrt{1-x^2}} = (\sin^{-1}x)^2$$

84-a) $\int \tan\theta \sec^2\theta\,d\theta = \frac{\sec^3\theta}{3} + C$

$\frac{d}{d\theta}\left(\frac{\sec^3\theta}{3} + C\right) = \frac{1}{3}\left[3\sec^2\theta(\sec\theta\tan\theta(1))\right] + [0]$

$= \sec^3\theta\tan\theta$ $\boxed{\text{wrong}}$

84-b) $\int \tan\theta \sec^2\theta\,d\theta = \frac{1}{2}\tan^2\theta + C$

$\boxed{\text{right}}$

$\frac{d}{d\theta}\left(\frac{1}{2}\tan^2\theta + C\right) = \frac{1}{2}\left[2\tan\theta(\sec^2\theta(1))\right] + [0] = \tan\theta\sec^2\theta$

84-c) $\int \tan\theta \sec^2\theta\,d\theta = \frac{1}{2}\sec^2\theta + C$

$\frac{d}{d\theta}\left(\frac{1}{2}\sec^2\theta + C\right) = \frac{1}{2}\left[2\sec\theta(\sec\theta\tan\theta(1))\right] + [0] = \tan\theta\sec^2\theta$

$\boxed{\text{right}}$

86-a) $\int \sqrt{2x+1}\,dx = \sqrt{x^2+x+c}$

$$\frac{d}{dx}\left(\sqrt{x^2+x+c}\right) = \frac{d}{dx}\left((x^2+x+c)^{\frac{1}{2}}\right) = \left[\frac{1}{2}(x^2+x+c)^{-\frac{1}{2}}(2x+1)\right]$$

$$= \frac{2x+1}{2\sqrt{x^2+x+c}} \quad \boxed{\text{wrong}}$$

86-b) $\int \sqrt{2x+1}\,dx = \sqrt{x^2+x} + C$

$$\frac{d}{dx}\left(\sqrt{x^2+x} + C\right) = \frac{d}{dx}\left((x^2+x)^{\frac{1}{2}} + C\right) = \left[\frac{1}{2}(x^2+x)^{-\frac{1}{2}}(2x+1)\right] + [0]$$

$$= \frac{2x+1}{2\sqrt{x^2+x}} \quad \boxed{\text{wrong}}$$

86-c) $\int \sqrt{2x+1}\,dx = \frac{1}{3}\left(\sqrt{2x+1}\right)^3 + C$

$$\frac{d}{dx}\left(\frac{1}{3}\left(\sqrt{2x+1}\right)^3 + C\right) = \frac{d}{dx}\left(\frac{1}{3}(2x+1)^{\frac{3}{2}} + C\right) = \frac{1}{3}\left[\frac{3}{2}(2x+1)^{\frac{1}{2}}(2)\right] + [0]$$

$$= (2x+1)^{\frac{1}{2}} = \sqrt{2x+1} \quad \boxed{\text{right}}$$

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88) $\int \frac{x\cos(x^2) - \sin(x^2)}{x^2}\,dx = \frac{\sin(x^2)}{x} + C$

$$\frac{d}{dx}\left(\frac{\sin(x^2)}{x} + C\right) = \frac{(x)\left[\cos(x^2)(2x)\right] - (\sin(x^2))[1]}{(x)^2} + [0]$$

$$= \frac{2x^2\cos(x^2) - \sin(x^2)}{x^2}$$

$$\boxed{\text{wrong}}$$

90) $\frac{dy}{dx} = -x$, $y = 1$ when $x = -1$

$y(x) = \int -x\,dx = -\left[\frac{x^2}{2}\right] + C = \frac{-1}{2}x^2 + C$

$(1) = \frac{-1}{2}(-1)^2 + C$

$1 = \frac{-1}{2} + C$

$\frac{3}{2} = C$

$y(x) = \frac{-1}{2}x^2 + \frac{3}{2}$

picture (d)

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92) $\frac{dy}{dx} = 10 - x$, $y(0) = -1$

$y(x) = \int (10-x)\,dx = 10[x] - \left[\frac{x^2}{2}\right] + C = 10x - \frac{1}{2}x^2 + C$

$(-1) = 10(0) - \frac{1}{2}(0)^2 + C$

$-1 = C$

$\underline{y(x) = 10x - \frac{1}{2}x^2 - 1}$

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94) $\frac{dy}{dx} = 9x^2 - 4x + 5$ $y(-1) = 0$

$y(x) = \int (9x^2 - 4x + 5)\,dx = 9\left[\frac{x^3}{3}\right] - 4\left[\frac{x^2}{2}\right] + 5[x] + C = 3x^3 - 2x^2 + 5x + C$

$(0) = 3(-1)^3 - 2(-1)^2 + 5(-1) + C$

$0 = -3 - 2 - 5 + C$

$0 = -10 + C$

$10 = C$

$y(x) = 3x^3 - 2x^2 + 5x + 10$

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96) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$ $y(4) = 0$

$y(x) = \int \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{1}{2}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = \sqrt{x} + C$

$(0) = \sqrt{(4)} + C$

$0 = 2 + C$

$-2 = C$

$y(x) = \sqrt{x} - 2$

98) $\frac{ds}{dt} = \cos t + \sin t \qquad s(\pi) = 1$

$s(t) = \int (\cos t + \sin t)\,dt = [\sin t] + [-\cos t] + C = \sin t - \cos t + C$

$(1) = \sin(\pi) - \cos(\pi) + C$

$1 = (0) - (-1) + C$

$1 = 1 + C$

$0 = C$

$s(t) = \sin t - \cos t + 0$

$= \sin t - \cos t$

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100) $\frac{dr}{d\theta} = \cos \pi\theta \qquad r(0) = 1$

$r(\theta) = \int \cos(\pi\theta)\,d\theta = \int \cos(p)\left(\frac{1}{\pi}dp\right) = \int \frac{1}{\pi}\cos(p)(dp)$

$p = \pi\theta$

$dp = \pi\, d\theta$

$\frac{1}{\pi}dp = d\theta$

$= \frac{1}{\pi}[\sin(p)] + C = \frac{1}{\pi}\sin(\pi\theta) + C$

$(1) = \frac{1}{\pi}\sin(\pi(0)) + C$

$1 = \frac{1}{\pi}\sin(0) + C$

$1 = \frac{1}{\pi}(0) + C$

$1 = C$

$r(\theta) = \frac{1}{\pi}\sin(\pi\theta) + 1$

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102) $\frac{dv}{dt} = 8t + \csc^2 t \qquad v\left(\frac{\pi}{2}\right) = -7$

$v(t) = \int (8t + \csc^2 t)\,dt = 8\left[\frac{t^2}{2}\right] + [-\cot t] + C = 4t^2 - \cot t + C$

$(-7) = 4\left(\frac{\pi}{2}\right)^2 - \cot\left(\frac{\pi}{2}\right) + C$

$-7 = 4\left(\frac{\pi^2}{4}\right) - (0) + C$

$-7 = \pi^2 + C$

$-7 - \pi^2 = C$

$v(t) = 4t^2 - \cot t - 7 - \pi^2$

104) $\dfrac{dv}{dt} = \dfrac{8}{1+t^2} + \sec^2 t \qquad v(0) = 1$

$$v(t) = \int\left(\frac{8}{1+t^2} + \sec^2 t\right)dt = \int \frac{8}{(1)^2+t^2}\,dt + \int \sec^2 t\,dt$$

$$= 8\left[\frac{1}{1}\tan^{-1}\left(\frac{t}{1}\right)\right] + [\tan t] + C = 8\tan^{-1} t + \tan t + C$$

$(1) = 8\tan^{-1}(0) + \tan(0) + C$

$1 = 8(0) + (0) + C \qquad\qquad v(t) = 8\tan^{-1} t + \tan t + 1$

$1 = C$

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106) $\dfrac{d^2y}{dx^2} = 0 \qquad y'(0) = \left.\dfrac{dy}{dx}\right|_{x=0} = 2 \qquad y(0) = 0$

$\dfrac{dy}{dx} = \int 0\,dx = C_1 \qquad (2) = C_1 \qquad \dfrac{dy}{dx} = 2$

$y(x) = \int 2\,dx = 2[x] + C_2 = 2x + C_2$

$(0) = 2(0) + C_2 \qquad\qquad y(x) = 2x$

$0 = C_2$

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108) $\dfrac{d^2s}{dt^2} = \dfrac{3t}{8} = \dfrac{3}{8}t \qquad \left.\dfrac{ds}{dt}\right|_{t=4} = 3 \qquad s(4) = 4$

$\dfrac{ds}{dt} = \int \dfrac{3}{8}t\,dt = \dfrac{3}{8}\left[\dfrac{t^2}{2}\right] + C_1 = \dfrac{3}{16}t^2 + C_1$

$(3) = \frac{3}{16}(4)^2 + C_1 \qquad 3 = 3 + C_1$

$3 = \frac{3}{16}(16) + C_1 \Rightarrow 0 = C_1$

$\dfrac{ds}{dt} = \dfrac{3}{16}t^2 + 0 = \dfrac{3}{16}t^2$

$s(t) = \int \dfrac{3}{16}t^2\,dt = \dfrac{3}{16}\left[\dfrac{t^3}{3}\right] + C_2 = \dfrac{1}{16}t^3 + C_2$

$(4) = \frac{1}{16}(4)^3 + C_2 \qquad\qquad s(t) = \frac{1}{16}t^3 + 0 = \frac{1}{16}t^3$

$4 = 4 + C_2 \Rightarrow 0 = C_2$

1(0) $\frac{d^3\theta}{dt^3} = 0$ $\theta''(0) = \frac{d^2\theta}{dt^2}\Big|_{x=0} = -2$ $\theta'(0) = \frac{d\theta}{dt}\Big|_{x=0} = \frac{-1}{2}$ $\theta(0) = \sqrt{2}$

$\frac{d^2\theta}{dt^2} = \int 0\, dt = C_1$ $(-2) = C_1$ $\frac{d^2\theta}{dt^2} = -2$

$\frac{d\theta}{dt} = \int -2\, dt = -2[t] + C_2 = -2t + C_2$ $(\frac{-1}{2}) = -2(0) + C_2$

$\frac{-1}{2} = C_2$

$\frac{d\theta}{dt} = -2t - \frac{1}{2}$

$\theta(t) = \int (-2t - \frac{1}{2})\, dt = -2\left[\frac{t^2}{2}\right] - \frac{1}{2}[t] + C_3 = -t^2 - \frac{1}{2}t + C_3$

$(\sqrt{2}) = -(0)^2 - \frac{1}{2}(0) + C_3$ $\theta(t) = -t^2 - \frac{1}{2}t + \sqrt{3}$

$\sqrt{2} = C_3$

1(2) $y^{(4)} = \frac{d^4y}{dx^4} = -\cos x + 8\sin 2x$ $y'''(0) = \frac{d^3y}{dx^3}\Big|_{x=0} = 0$ $y''(0) = \frac{d^2y}{dx^2}\Big|_{x=0} = 1$

$y'(0) = \frac{dy}{dx}\Big|_{x=0} = 1$ $y(0) = 3$

$\frac{d^3y}{dx^3} = \int(-\cos x + 8\sin 2x)\, dx$

$= -[\sin x] + 8\left[\frac{-\cos 2x}{2}\right] + C_1 = -\sin x - 4\cos 2x + C_1$

$(0) = -\sin(0) - 4\cos 2(0) + C_1$

$0 = -(0) - 4(1) + C_1$

$4 = C_1$

$\frac{d^3y}{dx^3} = -\sin x - 4\cos 2x + 4$

$\frac{d^2y}{dx^2} = \int(-\sin x - 4\cos 2x + 4)\, dx = -[-\cos x] - 4\left[\frac{\sin 2x}{2}\right] + 4[x] + C_2$

$= \cos x - 2\sin 2x + 4x + C_2$

$(1) = \cos(0) - 2\sin 2(0) + 4(0) + C_2$

$1 = (1) - 2(0) + (0) + C_2$

$1 = 1 + C_2$

$0 = C_2$

$\frac{d^2y}{dx^2} = \cos x - 2\sin 2x + 4x + 0$

$= \cos x - 2\sin 2x + 4x$

112) continued

$$\frac{dy}{dx} = \int (\cos x - 2\sin 2x + 4x)\,dx = [\sin x] - 2\left[\frac{-\cos 2x}{2}\right] + 4\left[\frac{x^2}{2}\right] + C_3$$

$$= \sin x + \cos 2x + 2x^2 + C_3$$

$(1) = \sin(0) + \cos 2(0) + 2(0)^2 + C_3$

$1 = (0) + (1) + (0) + C_3$

$0 = C_3$

$$\frac{dy}{dx} = \sin x + \cos 2x + 2x^2 + 0$$
$$= \sin x + \cos 2x + 2x^2$$

$$y(x) = \int (\sin x + \cos 2x + 2x^2)\,dx = [-\cos x] + \left[\frac{\sin 2x}{2}\right] + 2\left[\frac{x^3}{3}\right] + C_4$$

$$= -\cos x + \frac{1}{2}\sin 2x + \frac{2}{3}x^3 + C_4$$

$(3) = -\cos(0) + \frac{1}{2}\sin 2(0) + \frac{2}{3}(0)^3 + C_4$

$3 = -(1) + \frac{1}{2}(0) + \frac{2}{3}(0) + C_4$

$4 = C_4$

$$y(x) = -\cos x + \frac{1}{2}\sin 2x + \frac{2}{3}x^3 + 4$$

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114) a) i) $\frac{d^2y}{dx^2} = 6x$ ii) passes $(0,1) \Rightarrow y(0) = 1$

has horizontal tangent line $\left.\frac{dy}{dx}\right|_{x=0} = 0$

$$\frac{dy}{dx} = \int 6x\,dx = 6\left[\frac{x^2}{2}\right] + C_1 = 3x^2 + C_1$$

$(0) = 3(0)^2 + C_1$

$0 = C_1$

$$\frac{dy}{dx} = 3x^2 + 0 = 3x^2$$

$$y(x) = \int 3x^2\,dx = 3\left[\frac{x^3}{3}\right] + C_2 = x^3 + C_2$$

$(1) = (0)^3 + C_2$

$1 = C_2$

$$y(x) = x^3 + 1$$

b) Only one, because any other possible function would be different from $y(x) = x^3 + 1$. by a constant must be none due to the initial conditions.