Corollary 3 Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).

If $f'(x) = \frac{df}{dx} > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.

If $f'(x) = \frac{df}{dx} < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

First Derivative Test for Local Extrema

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right,

1) if f' changes from negative to positive at c, then f has a local minimum at c;

2) if f' changes from positive to negative at c, then f has a local maximum at c;

3) if f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local extremum at c.

2) $f'(x) = (x+1)(x+2) = \frac{df}{dx}$

a) critical points:

$0 = \frac{df}{dx} = (x+2)(x+1)$

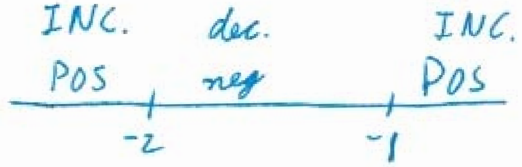
$x+2=0$ | $x+1=0$

$x=-2$ | $x=-1$

c) Local Max: at $x=-2$

local min at $x=-1$

b) $\frac{df}{dx}$



$\frac{df}{dx}\Big|_{x=-3} = ((-3)+1)((-3)+2) = (-2)(-1) = 2 > 0$

$\frac{df}{dx}\Big|_{x=\frac{-3}{2}} = \left(\left(\frac{-3}{2}\right)+1\right)\left(\left(\frac{-3}{2}\right)+2\right) = \left(\frac{-1}{2}\right)\left(\frac{1}{2}\right) < 0$

$\frac{df}{dx}\Big|_{x=0} = ((0)+1)((0)+2) = (1)(2) = 2 > 0$

Increasing: $(-\infty,-2) \cup (-1,\infty)$

decreasing: $(-2,-1)$

---

4) $f'(x) = (x-1)^2(x+2)^2 = \frac{df}{dx}$

a) critical points:

$0 = \frac{df}{dx} = (x+2)^2(x-1)^2$

$(x+2)^2=0$ | $(x-1)^2=0$

$x+2=0$ | $x-1=0$

$x=2$ | $x=1$

b) $\frac{df}{dx}$

Increasing: $(-\infty,-2) \cup (-2,1) \cup (1,\infty)$

decreasing: none

c) Local Max: none
local min: none } No local extrema

6) $f'(x) = (x-7)(x+1)(x+5) = \frac{df}{dx}$

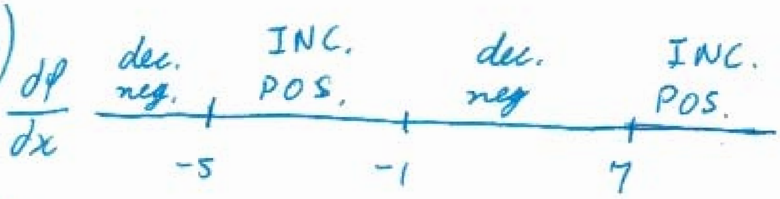
a) critical points:

$0 = \frac{df}{dx} = (x+5)(x+1)(x-7)$

$x+5=0 \quad | \quad x+1=0 \quad | \quad x-7=0$

$x=-5 \quad | \quad x=-1 \quad | \quad x=7$

b)



Increasing: $(-5,-1) \cup (7,\infty)$

decreasing: $(-\infty,-5) \cup (-1,7)$

c) Local Max at $x=-1$

local min at $x=-5$ and $x=7$

---

8) $f'(x) = \frac{(x-2)(x+4)}{(x+1)(x-3)} = \frac{df}{dx}$, $x \neq -1, x \neq 3$

a) critical points:

$0 = \frac{df}{dx} = \frac{(x-2)(x+4)}{(x+1)(x-3)}$

$0 = (x+4)(x-2)$

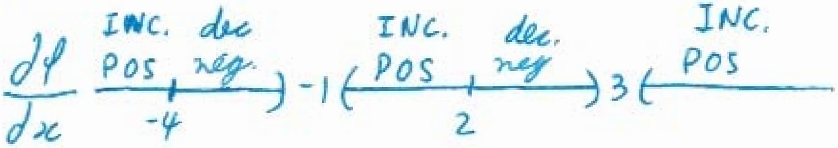
$x+4=0 \quad | \quad x-2=0$

$x=-4 \quad | \quad x=2$

also

$x=-1, \; x=3$

b)



Increasing: $(-\infty,-4) \cup (-1,2) \cup (3,\infty)$

decreasing: $(-4,-1) \cup (2,3)$

c) Local Max at $x=-4$ and $x=2$

local min: none

10) $f'(x) = 3 - \frac{6}{\sqrt{x}} = \frac{df}{dx}$, $x \neq 0$

a) critical points:

$0 = \frac{df}{dx} = 3 - \frac{6}{\sqrt{x}}$

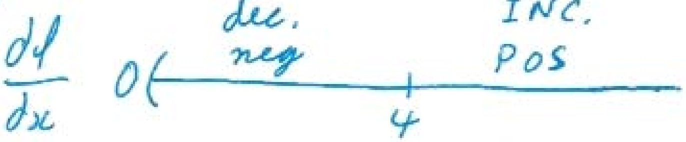
$\frac{6}{\sqrt{x}} = 3$

$\frac{6}{3} = \sqrt{x}$ also

$2 = \sqrt{x}$ $x = 0$

$4 = x$

b)



Increasing : $(4, \infty)$

decreasing : $(0, 4)$

c) Local Max : none

local min at $x = 4$

---

12) $f'(x) = x^{-\frac{1}{2}}(x-3) = \frac{df}{dx} = \frac{x-3}{\sqrt{x}}$

a) critical points

$0 = \frac{df}{dx} = \frac{x-3}{\sqrt{x}}$

$0 = x - 3$ | also $\sqrt{x} = 0$

$x = 3$ | also $x = 0$

b)

Increasing : $(3, \infty)$

decreasing : $(0, 3)$

c) Local Max : none

local min at $x = 3$

14) $f'(x) = (\sin x + \cos x)(\sin x - \cos x) = \frac{df}{dx}$, $0 \le x \le 2\pi$

a) critical points:

$0 = \frac{df}{dx} = (\sin x + \cos x)(\sin x - \cos x)$

$\sin x + \cos x = 0$

$\sin x = -\cos x$

$\frac{\sin x}{-\cos x} = 1$

$-\tan x = 1$

$\tan x = -1$

Q II | Q IV

$x = \frac{3\pi}{4}$ | $x = \frac{7\pi}{4}$

$\sin x - \cos x = 0$

$\sin x = \cos x$

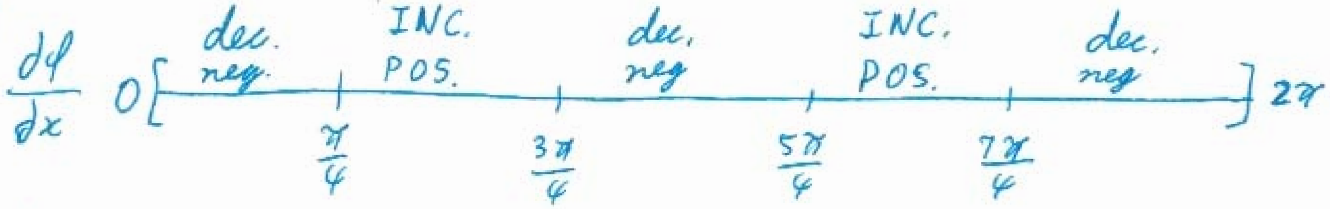
$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

Q I | Q III

$x = \frac{\pi}{4}$ | $x = \frac{5\pi}{4}$

b)



Increasing: $(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{7\pi}{4})$

decreasing: $[0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

c) Local Max at $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$ also $x = 0$ "endpoint"

local min at $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ also $x = 2\pi$ "endpoint"

16) domain: [-4, 4]

a) $\frac{df}{dx}$

Increasing: $(-4, \approx -3.25) \cup (\approx -1.5, 1) \cup (2, 4)$ decreasing: $(\approx -3.25, \approx -1.5) \cup (1, 2)$

b) Absolute Maximum: $(4, 2)$ absolute minimum: $(\approx -1.5, -1)$

Local Max: $(\approx -3.25, 1)$ local min: $(-4, 0)$ and $(2, 0)$

---

18) domain: $(-4, 4]$ but composed of 3 pieces $(-4, -1) \cup [-1, 1] \cup (1, 4]$

a) $\frac{df}{dx}$

Increasing: $(-4, \approx -2.5) \cup (-1, 1) \cup (3, 4)$ decreasing: $(\approx -2.5, -1) \cup (1, 3)$

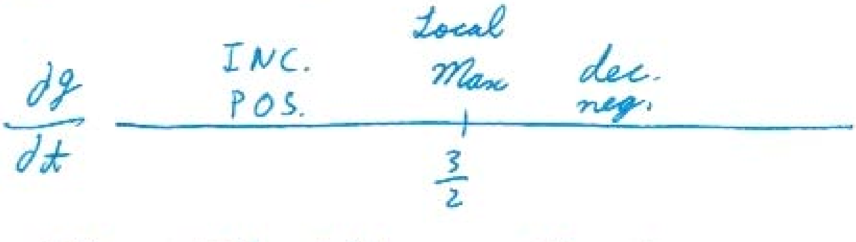
b) Abs. Max: none abs. min: none

Local. Max: $(\approx -2.5, 1)$, $(1, 2)$, $(4, 2)$ local min: $(-1, 0)$, $(3, 1)$

---

20) $g(t) = -3t^2 + 9t + 5$ domain: $(-\infty, \infty)$

$\frac{dg}{dt} = -6t + 9$



critical point:

$0 = \frac{dg}{dt} = -6t + 9$

$g\left(\frac{3}{2}\right) = -3\left(\frac{3}{2}\right)^2 + 9\left(\frac{3}{2}\right) + 5 = \frac{-27}{4} + \frac{27}{2} + 5$

$0 = -6t + 9$

$= \frac{-27}{4} + \frac{54}{4} + 5 = \frac{27}{4} + 5 = \frac{27}{4} + \frac{20}{4} = \frac{47}{4}$

$6t = 9$

$t = \frac{9}{6} = \frac{3}{2}$

a) Increasing: $\left(-\infty, \frac{3}{2}\right)$ decreasing: $\left(\frac{3}{2}, \infty\right)$

b) Local Max: $\left(\frac{3}{2}, \frac{47}{4}\right)$ Abs. Max: $\left(\frac{3}{2}, \frac{47}{4}\right)$

local min: none abs min: none

22) $h(x) = 2x^3 - 18x$ domain: $(-\infty, \infty)$

$$\frac{dh}{dx} = 6x^2 - 18$$

critical points:

$$0 = \frac{dh}{dx} = 6x^2 - 18$$

$$0 = 6x^2 - 18$$

$$0 = 6(x^2 - 3)$$

$$0 = 6(x+\sqrt{3})(x-\sqrt{3})$$

$x + \sqrt{3} = 0$ | $x - \sqrt{3} = 0$

$x = -\sqrt{3}$ | $x = \sqrt{3}$

$$h(-\sqrt{3}) = 2(-\sqrt{3})^3 - 18(-\sqrt{3}) = -6\sqrt{3} + 18\sqrt{3} = 12\sqrt{3}$$

$$h(\sqrt{3}) = 2(\sqrt{3})^3 - 18(\sqrt{3}) = 6\sqrt{3} - 18\sqrt{3} = -12\sqrt{3}$$

a) Increasing: $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

decreasing: $(-\sqrt{3}, \sqrt{3})$

b) Local Max: $(-\sqrt{3}, 12\sqrt{3})$ Abs Max: none

local min: $(\sqrt{3}, -12\sqrt{3})$ abs min: none

---

24) $f(\theta) = 6\theta - \theta^3$ domain $(-\infty, \infty)$

$$\frac{df}{d\theta} = 6 - 3\theta^2$$

critical points:

$$0 = \frac{df}{d\theta} = 6 - 3\theta^2$$

$$0 = 6 - 3\theta^2$$

$$0 = 3(2 - \theta^2)$$

$$0 = 3(\sqrt{2} + \theta)(\sqrt{2} - \theta)$$

$\sqrt{2} + \theta = 0$ | $\sqrt{2} - \theta = 0$

$\theta = -\sqrt{2}$ | $\theta = \sqrt{2}$

$$f(-\sqrt{2}) = 6(-\sqrt{2}) - (-\sqrt{2})^3 = -6\sqrt{2} + 2\sqrt{2} = -4\sqrt{2}$$

$$f(\sqrt{2}) = 6(\sqrt{2}) - (\sqrt{2})^3 = 6\sqrt{2} - 2\sqrt{2} = 4\sqrt{2}$$

a) Increasing: $(-\sqrt{2}, \sqrt{2})$

decreasing: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

b) Local Max: $(\sqrt{2}, 4\sqrt{2})$ Abs Max: none

local min: $(-\sqrt{2}, -4\sqrt{2})$ abs min: none

26) $h(r) = (r+7)^3$ domain: $(-\infty, \infty)$

$\frac{dh}{dr} = 3(r+7)^2(1) = 3(r+7)^2$

critical point:

$0 = \frac{dh}{dr} = 3(r+7)^2$

$0 = 3(r+7)^2$

$0 = (r+7)^2$

$0 = r+7$

$r = -7$

$h(-7) = ((-7)+7)^3 = (0)^3 = 0$

a) Increasing: $(-\infty, -7) \cup (-7, \infty)$

decreasing: none

b) Local Max: none Abs Max: none

local min: none abs min: none

At $r = -7$ is an example of an inflection point which will be shown in section 4.4.

28) $g(x) = x^4 - 4x^3 + 4x^2$ domain: $(-\infty, \infty)$

$\frac{dg}{dx} = 4x^3 - 12x^2 + 8x$

critical points:

$0 = \frac{dg}{dx} = 4x^3 - 12x^2 + 8x$

$0 = 4x^3 - 12x^2 + 8x$

$0 = 4x(x^2 - 3x + 2)$

$0 = 4x(x-1)(x-2)$

$4x = 0 \mid x-1 = 0 \mid x-2 = 0$

$x = 0 \mid x = 1 \mid x = 2$

$g(0) = (0)^4 - 4(0)^3 + 4(0)^2 = 0$

$g(1) = (1)^4 - 4(1)^3 + 4(1)^2 = 1 - 4 + 4 = 1$

$g(2) = (2)^4 - 4(2)^3 + 4(2)^2 = (2)^2\{(2)^2 - 4(2) + 4\} = (2)^2\{0\} = 0$

a) Increasing: $(0,1) \cup (2, \infty)$

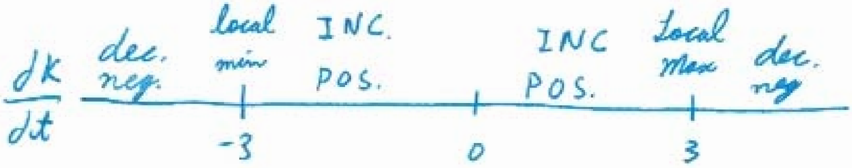
decreasing: $(-\infty, 1) \cup (1, 2)$

b) Local Max: $(1,1)$ Abs Max: none

local min: none abs min: $(0,0), (2,0)$

30) $K(t) = 15t^3 - t^5$ domain: $(-\infty, \infty)$

$$\frac{dK}{dt} = 45t^2 - 5t^4$$



critical points:

$$0 = \frac{dK}{dt} = 45t^2 - 5t^4$$

$$0 = 45t^2 - 5t^4$$

$$0 = 5t^2(9 - t^2)$$

$$0 = (3+t)(5t^2)(3-t)$$

$3+t=0$, $t=-3$ | $5t^2=0$, $t^2=0$, $t=0$ | $3-t=0$, $t=3$

$$K(-3) = 15(-3)^3 - (-3)^5 = (3)^3\{-15 + (3)^2\} = (27)\{-15+9\}$$
$$= (27)\{-6\} = -162$$

$$K(3) = 15(3)^3 - (3)^5 = (3)^3\{15 - (3)^2\} = (27)\{15-9\}$$
$$= (27)\{6\} = 162$$

$$K(0) = 15(0)^3 - (0)^5 = 0$$

a) Increasing: $(-3,0) \cup (0,3)$

decreasing: $(-\infty,-3)$, $(3,\infty)$

b) Local Max: $(3,162)$ Abs Max: none

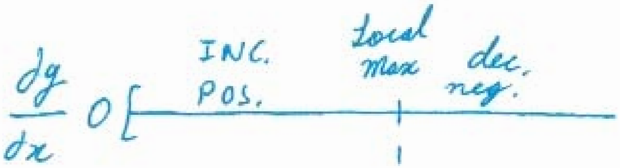
local min: $(-3,-162)$ abs min: none

at $t=0$, another example of inflection point

- - - - - - - - - - - - - - - - - -

32) $g(x) = 4\sqrt{x} - x^2 + 3 = 4x^{\frac{1}{2}} - x^2 + 3$ domain: $[0,\infty)$

$$\frac{dg}{dx} = 4\left[\tfrac{1}{2}x^{-\frac{1}{2}}\right] - [2x] + [0] = \frac{2}{\sqrt{x}} - 2x$$



critical points:

$$0 = \frac{dg}{dx} = \frac{2}{\sqrt{x}} - 2x \;;\quad 0 = 2 - 2(\sqrt{x})^3$$

$$0 = 2(1 - (\sqrt{x})^3)$$

$$0 = \frac{2}{\sqrt{x}} - 2x \qquad 0 = 1 - (\sqrt{x})^3$$

$$0 = \frac{2}{\sqrt{x}} - 2(\sqrt{x})^2 \qquad (\sqrt{x})^3 = 1$$

$$0 = \frac{2}{\sqrt{x}} - \frac{2(\sqrt{x})^3}{\sqrt{x}} \qquad \sqrt{x} = 1$$

$$x = 1$$

$$0 = \frac{2 - 2(\sqrt{x})^3}{\sqrt{x}}$$

also $x=0$

$$g(0) = 4\sqrt{(0)} - (0)^2 + 3 = 3$$

$$g(1) = 4\sqrt{(1)} - (1)^2 + 3 = 4 - 1 + 3 = 6$$

a) Increasing: $(0,1)$

decreasing: $(1,\infty)$

b) Local Max: $(1,6)$ Abs Max: $(1,6)$

local min: $(0,1)$ abs min: none

34)

$g(x) = x^2\sqrt{5-x} = x^2(5-x)^{\frac{1}{2}}$ domain: $(-\infty, 5]$

$$\frac{dg}{dx} = (x^2)\left[\tfrac{1}{2}(5-x)^{-\frac{1}{2}}(-1)\right] + \left((5-x)^{\frac{1}{2}}\right)[2x] = \frac{-x^2}{2\sqrt{5-x}} + \frac{2x\sqrt{5-x}}{1}\left(\frac{2\sqrt{5-x}}{2\sqrt{5-x}}\right)$$

$$= \frac{-x^2}{2\sqrt{5-x}} + \frac{4x(5-x)}{2\sqrt{5-x}} = \frac{-x^2+4x(5-x)}{2\sqrt{5-x}} = \frac{-x^2+20x-4x^2}{2\sqrt{5-x}} = \frac{20x-5x^2}{2\sqrt{5-x}}$$

$$= \frac{5x(4-x)}{2\sqrt{5-x}}$$

critical points:

$$0 = \frac{dg}{dx} = \frac{5x(4-x)}{2\sqrt{5-x}}$$

$$0 = \frac{5x(4-x)}{2\sqrt{5-x}}$$

$0 = 5x(4-x)$

$5x = 0$ | $4-x=0$

$x = 0$ | $x = 4$

also $5-x=0$

$x=5$

$g(0) = (0)^2\sqrt{5-(0)} = (0)\sqrt{5} = 0$

$g(4) = (4)^2\sqrt{5-(4)} = (16)\sqrt{1} = 16$

$g(5) = (5)^2\sqrt{5-(5)} = (25)\sqrt{0} = 0$

a) Increasing: $(0,4)$

decreasing: $(-\infty, 0), (4,5)$

b) Local Max: $(4,16)$ Abs Max: none

local min: $(0,0)$, $(5,0)$ which is also abs min

---

36) $f(x) = \frac{x^3}{3x^2+1}$

V.A.: $3x^2+1=0$

no solution

V.A.: none

domain: $(-\infty, \infty)$

$$\frac{df}{dx} = \frac{(3x^2+1)[3x^2]-(x^3)[6x]}{(3x^2+1)^2}$$

$$= \frac{9x^4+3x^2-6x^4}{(3x^2+1)^2} = \frac{3x^4+3x^2}{(3x^2+1)^2} = \frac{3x^2(x^2+1)}{(3x^2+1)^2}$$

36) continued

critical point:

$0 = \frac{df}{dx} = \frac{3x^2(x^2+1)}{(3x^2+1)^2}$

$0 = \frac{3x^2(x^2+1)}{(3x^2+1)^2}$

$0 = 3x^2(x^2+1)$

$3x^2 = 0$ | $x^2+1=0$

$x^2 = 0$ | no solution

$x = 0$ |

$\frac{df}{dx}$ ——— INC POS ——— 0 ——— INC POS ———

$f(0) = \frac{(0)^3}{3(0)^2+1} = \frac{0}{1} = 0$

a) Increasing: $(-\infty,0)\cup(0,\infty)$

decreasing: none

b) Local Max: none   Abs Max: none

local min: none   abs min: none

at $x=0$, another example of inflection point

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38) $g(x) = x^{\frac{2}{3}}(x+5) = (\sqrt[3]{x})^2(x+5)$ domain: $(-\infty,\infty)$

$$\frac{dg}{dx} = \left(x^{\frac{2}{3}}\right)\left[1\right] + (x+5)\left[\frac{2}{3}x^{-\frac{1}{3}}\right] = (\sqrt[3]{x})^2 + \frac{2(x+5)}{3(\sqrt[3]{x})} = \frac{(\sqrt[3]{x})^2}{1}\left(\frac{3(\sqrt[3]{x})}{3(\sqrt[3]{x})}\right) + \frac{2(x+5)}{3(\sqrt[3]{x})}$$

$$= \frac{3(\sqrt[3]{x})^3 + 2(x+5)}{3(\sqrt[3]{x})} = \frac{3x+2x+10}{3(\sqrt[3]{x})} = \frac{5x+10}{3(\sqrt[3]{x})} = \frac{5(x+2)}{3(\sqrt[3]{x})}$$

critical points:

$0 = \frac{dg}{dx} = \frac{5(x+2)}{3(\sqrt[3]{x})}$

$0 = \frac{5(x+2)}{3(\sqrt[3]{x})}$

$0 = 5(x+2)$

$x+2=0$

$x=-2$

also

$x=0$

$\frac{dg}{dx}$ ——— INC. POS. ——— Local Max -2 ——— dec. neg. ———)0(——— INC POS. ———

$g(-2) = (\sqrt[3]{(-2)})^2((-2)+5) = (\sqrt[3]{4})(3) = 3(\sqrt[3]{4})$

$g(0) = (\sqrt[3]{(0)})^2((0)+5) = (0)(5) = 0$

a) Increasing: $(-\infty,-2)$, $(0,\infty)$

decreasing: $(-2,0)$

b) Local Max: $(-2, 3(\sqrt[3]{4}))$ Abs Max: none

local min: $(0,0)$ abs min: none

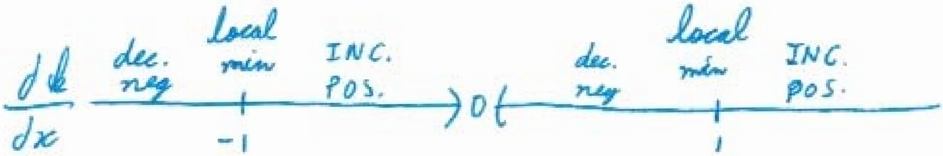
40) $k(x) = x^{2/3}(x^2-4) = (\sqrt[3]{x})^2(x^2-4)$ domain: $(-\infty, \infty)$

$$\frac{dk}{dx} = (x^{2/3})[2x] + (x^2-4)\left[\frac{2}{3}x^{-\frac{1}{3}}\right] = 2x(\sqrt[3]{x})^2 + \frac{2(x^2-4)}{3(\sqrt[3]{x})}$$

$$= \frac{2x(\sqrt[3]{x})^2}{1}\left(\frac{3(\sqrt[3]{x})}{3(\sqrt[3]{x})}\right) + \frac{2(x^2-4)}{3(\sqrt[3]{x})} = \frac{6x(\sqrt[3]{x})^3+2(x^2-4)}{3(\sqrt[3]{x})} = \frac{6x(x)+2x^2-8}{3(\sqrt[3]{x})}$$

$$= \frac{6x^2+2x^2-8}{3(\sqrt[3]{x})} = \frac{8x^2-8}{3(\sqrt[3]{x})} = \frac{8(x^2-1)}{3(\sqrt[3]{x})}$$

critical points:



$0 = \frac{dk}{dx} = \frac{8(x^2-1)}{3(\sqrt[3]{x})}$

$0 = \frac{8(x^2-1)}{3(\sqrt[3]{x})}$

$0 = 8(x^2-1)$

$0 = 8(x+1)(x-1)$

$x+1=0$ | $x-1=0$

$x=-1$ | $x=1$

also $x=0$

$k(-1) = (\sqrt[3]{(-1)})^2((-1)^2-4) = (1)(-3) = -3$

$k(0) = (\sqrt[3]{(0)})^2((0)^2-4) = (0)(-4) = 0$

$k(1) = (\sqrt[3]{(1)})^2((1)^2-4) = (1)(-3) = -3$

a) Increasing: $(-1, 0)$, $(1, \infty)$

decreasing: $(-\infty, -1)$, $(0, 1)$

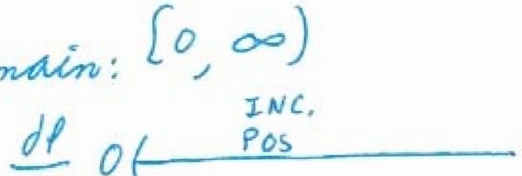
b) Local Max: $(0,0)$ Abs Max: none

local min: $(-1,-3)$, $(1,-3)$ which is also abs. min.

---

42) $f(x) = e^{\sqrt{x}} = e^{(x^{\frac{1}{2}})}$ domain: $[0, \infty)$

$$\frac{df}{dx} = e^{(x^{\frac{1}{2}})}\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$



$f(0) = e^{\sqrt{(0)}} = e^0 = 1$

critical point:

$0 = \frac{df}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

$0 = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

| $0 = e^{\sqrt{x}}$

| no solution

| but $x = 0$

a) Increasing: $(0, \infty)$ decreasing: none

b) Local Max: none Abs. Max: none

local min: $(0,1)$ which is also abs. min.

44) $f(x) = x^2 \ln x$ domain: $(0, \infty)$

$$\frac{df}{dx} = (x^2)\left[\frac{1}{x}(1)\right] + (\ln x)[2x] = x + 2x \ln x = x(1 + 2\ln x)$$

critical point:

$0 = \frac{df}{dx} = x(1 + 2\ln x)$

$0 = x(1 + 2\ln x)$

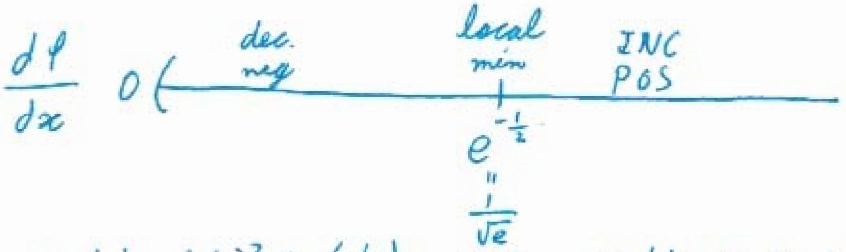
$x = 0$ discard $x = 0$ not in domain | $1 + 2\ln x = 0$

$2 \ln x = -1$

$\ln x = \frac{-1}{2}$

$\Downarrow$

$x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$



$$f\left(\frac{1}{\sqrt{e}}\right) = \left(\frac{1}{\sqrt{e}}\right)^2 \ln\left(\frac{1}{\sqrt{e}}\right) = \left(\frac{1}{e}\right) \ln\left(e^{-\frac{1}{2}}\right) = \left(\frac{1}{e}\right)\left(\frac{-1}{2}\right) = \frac{-1}{2e}$$

a) Increasing: $\left(\frac{1}{\sqrt{e}}, \infty\right)$

decreasing: $\left(0, \frac{1}{\sqrt{e}}\right)$

b) Local Max: none Abs Max: none

local min: $\left(\frac{1}{\sqrt{e}}, \frac{-1}{2e}\right)$ which is also abs. min.

---

46) $g(x) = x^2 - 2x - 4\ln x$ domain: $(0, \infty)$

$$\frac{dg}{dx} = [2x] - 2[1] - 4\left[\frac{1}{x}(1)\right] = 2x - 2 - \frac{4}{x} = \frac{2x}{1}\left(\frac{x}{x}\right) - \frac{2}{1}\left(\frac{x}{x}\right) - \frac{4}{x} = \frac{2x^2 - 2x - 4}{x}$$

$$= \frac{2(x^2 - x - 2)}{x} = \frac{2(x+1)(x-2)}{x}$$

critical points:

$0 = \frac{dg}{dx} = \frac{2(x+1)(x-2)}{x}$

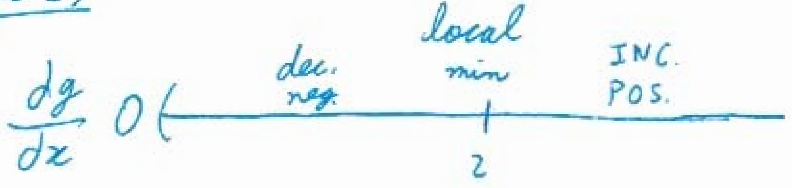
$0 = \frac{2(x+1)(x-2)}{x}$

$0 = 2(x+1)(x-2)$

$x + 1 = 0$ | $x - 2 = 0$

$x = -1$ | $x = 2$

discard ↑ not in domain → also discard $x = 0$



$g(2) = (2)^2 - 2(2) - 4\ln(2) = 4 - 4 - 4\ln 2 = -4\ln 2$

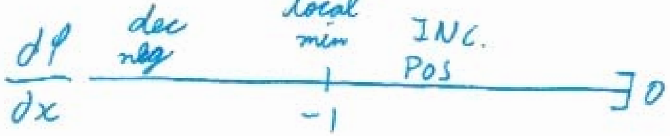
a) Increasing: $(2, \infty)$ decreasing: $(0, 2)$

b) Local Max: none Abs. Max: none

local min: $(2, -4\ln 2)$ which is also abs. min.

48) $f(x) = (x+1)^2$ $\quad -\infty < x \le 0$

a) $\frac{df}{dx} = 2(x+1)'(1) = 2(x+1)$



critical point:

$0 = \frac{df}{dx} = 2(x+1)$

$0 = 2(x+1)$

$0 = x+1$

$x = -1$

$f(-1) = ((-1)+1)^2 = (0)^2 = 0$

$f(0) = ((0)+1)^2 = (1)^2 = 1$

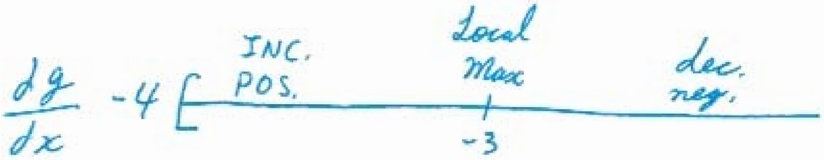
Local Max: $(0, 1)$

local min: $(-1, 0)$

also endpoint: $x = 0$ $\quad$ b) Abs. Max: none $\quad$ abs. min: $(-1, 0)$

50) $g(x) = -x^2 - 6x - 9$ $\quad -4 \le x < \infty$

a) $\frac{dg}{dx} = -2x - 6$



critical point:

$0 = \frac{dg}{dx} = -2x - 6$

$0 = -2x - 6$

$2x = -6$

$x = -3$

also endpoint: $x = -4$

$g(-4) = -(-4)^2 - 6(-4) - 9 = -16 + 24 - 9 = -1$

$g(-3) = -(-3)^2 - 6(-3) - 9 = -9 + 18 - 9 = 0$

Local Max: $(-3, 0)$

local min: $(-4, -1)$

b) Abs. Max: $(-3, 0)$ $\quad$ abs. min: none

52) $f(t) = t^3 - 3t^2$ $\quad -\infty < t \le 3$

a) $\frac{df}{dt} = 3t^2 - 6t$

critical point

$0 = \frac{df}{dt} = 3t^2 - 6t$

$0 = 3t^2 - 6t$

$0 = 3t(t-2)$

$3t = 0$ | $t - 2 = 0$

$t = 0$ | $t = 2$

also endpoint: $t = 3$

52) continued

$f(0) = (0)^3 - 3(0)^2 = 0$

$f(2) = (2)^3 - 3(2)^2 = 8 - 12 = -4$

$f(3) = (3)^3 - 3(3)^2 = 27 - 27 = 0$

Local Max: $(0,0)$, $(3,0)$

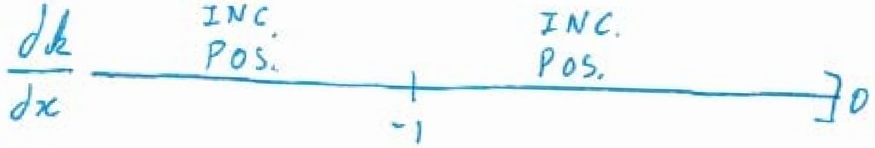
local min: $(2,-4)$

b) Abs Max: $(0,0)$, $(3,0)$ abs min: none

---

54) $k(x) = x^3 + 3x^2 + 3x + 1$ $\quad -\infty < x \le 0$

a) $\frac{dk}{dx} = 3x^2 + 6x + 3$



critical points:

$0 = \frac{dk}{dx} = 3x^2 + 6x + 3$

$0 = 3x^2 + 6x + 3$

$0 = 3(x^2 + 2x + 1)$

$0 = 3(x+1)(x+1)$

$0 = 3(x+1)^2$

$(x+1)^2 = 0$

$x + 1 = 0$

$x = -1$

also endpoint: $x = 0$

$k(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

$k(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 1$

Local Max: $(0,1)$

local min: none

at $x = -1$, another example of inflection point

b) Abs Max: $(0,1)$

abs min: none

---

56) $f(x) = \sqrt{x^2 - 2x - 3} = (x^2 - 2x - 3)^{\frac{1}{2}}$ $\quad 3 \le x < \infty$

a) $\frac{df}{dx} = \frac{1}{2}(x^2 - 2x - 3)^{-\frac{1}{2}}(2x - 2) = \frac{2x - 2}{2\sqrt{x^2 - 2x - 3}} = \frac{2(x-1)}{2\sqrt{x^2 - 2x - 3}} = \frac{x - 1}{\sqrt{x^2 - 2x - 3}}$

56) continued

critical points:

$0 = \frac{df}{dx} = \frac{x-1}{\sqrt{x^2-2x-3}}$

$0 = \frac{x-1}{\sqrt{x^2-2x-3}}$

$0 = x-1$

$x = 1$ ← discard → $x = -1$

not in $[3, \infty)$

also check denominator = 0

$0 = \sqrt{x^2-2x-3}$

$0 = x^2-2x-3$

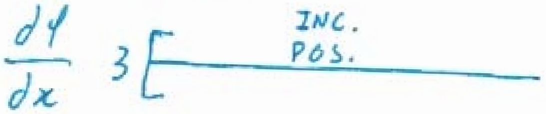
$0 = (x+1)(x-3)$

$x+1=0$ | $x-3=0$

$x=-1$ | $x=3$ ← this is also an endpoint

$f(3) = \sqrt{(3)^2-2(3)-3}$

$= \sqrt{9-6-3} = \sqrt{0} = 0$



Local Max: none

local min: $(3,0)$

b) Abs Max: none   abs. min: $(3,0)$

- - - - - - - - - - - - - -

58) $g(x) = \frac{x^2}{4-x^2}$ $-2 < x \leq 1$

V.A.: $4-x^2=0$

$(2+x)(2-x)=0$

$2+x=0$ | $2-x=0$

$x=-2$ | $x=2$

a) $\frac{dg}{dx} = \frac{(4-x^2)[2x]-(x^2)[-2x]}{(4-x^2)^2} = \frac{2x\{(4-x^2)[1]-(x^2)[-1]\}}{(4-x^2)^2} = \frac{2x\{4-x^2+x^2\}}{(4-x^2)^2}$

$= \frac{2x\{4\}}{(4-x^2)^2} = \frac{8x}{(4-x^2)^2}$

critical point

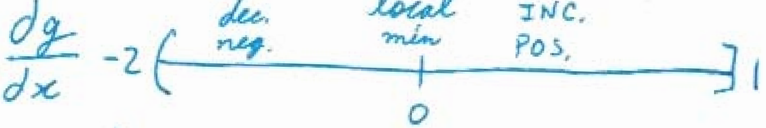
$0 = \frac{dg}{dx} = \frac{8x}{(4-x^2)^2}$

$0 = \frac{8x}{(4-x^2)^2}$

$0 = 8x$

$x = 0$

also endpoint: $x = 1$



$g(0) = \frac{(0)^2}{4-(0)^2} = \frac{0}{4} = 0$

$g(1) = \frac{(1)^2}{4-(1)^2} = \frac{1}{3}$

Local Max: $(1, \frac{1}{3})$

local min: $(0,0)$

b) Abs Max: none

abs min: $(0,0)$

60) $f(x) = \sin x - \cos x$ $\qquad 0 \le x \le 2\pi$

$\frac{df}{dx} = [\cos x(1)] - [-\sin x(1)] = \cos x + \sin x$

critical points:

$0 = \frac{df}{dx} = \cos x + \sin x$

$0 = \cos x + \sin x$

$-\cos x = \sin x$

$-1 = \frac{\sin x}{\cos x}$

$-1 = \tan x$

Q II	Q III
$x = \frac{3\pi}{4}$	$x = \frac{7\pi}{4}$

also endpoints: $x = 0$, $x = 2\pi$

$f(0) = \sin(0) - \cos(0) = (0) - (1) = -1$

$f(2\pi) = \sin(2\pi) - \cos(2\pi) = (0) - (1) = -1$

$f(\frac{3\pi}{4}) = \sin(\frac{3\pi}{4}) - \cos(\frac{3\pi}{4}) = (\frac{1}{\sqrt{2}}) - (\frac{-1}{\sqrt{2}}) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$f(\frac{7\pi}{4}) = \sin(\frac{7\pi}{4}) - \cos(\frac{7\pi}{4}) = (\frac{-1}{\sqrt{2}}) - (\frac{1}{\sqrt{2}}) = \frac{-2}{\sqrt{2}} = -\sqrt{2}$

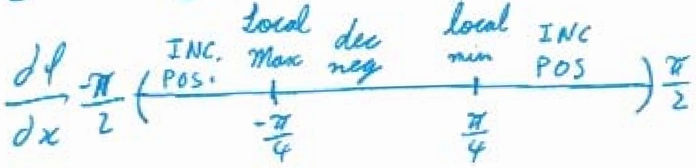
Local Max: $(\frac{3\pi}{4}, \sqrt{2})$, $(2\pi, -1)$

local min: $(0, -1)$, $(\frac{7\pi}{4}, -\sqrt{2})$

---

62) $f(x) = -2x + \tan x$ $\qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\frac{df}{dx} = -2[1] + [\sec^2 x(1)] = -2 + \sec^2 x$



critical point

$0 = \frac{df}{dx} = -2 + \sec^2 x$

$0 = -2 + \sec^2 x$

$2 = \sec^2 x$

$2 = \frac{1}{\cos^2 x} \Rightarrow \frac{2}{1} = \frac{1}{\cos^2 x}$

$\frac{1}{2} = \cos^2 x$

$\pm\sqrt{\frac{1}{2}} = \cos x$

$\cos x = \frac{-1}{\sqrt{2}}$

discard
not in
$\frac{-\pi}{2} < x < \frac{\pi}{2}$

$\cos x = \frac{1}{\sqrt{2}}$

Q IV	Q I
$x = \frac{-\pi}{4}$	$x = \frac{\pi}{4}$

$f(\frac{-\pi}{4}) = -2(\frac{-\pi}{4}) + \tan(\frac{-\pi}{4}) = \frac{\pi}{2} + (-1) = \frac{\pi}{2} - 1$

$f(\frac{\pi}{4}) = -2(\frac{\pi}{4}) + \tan(\frac{\pi}{4}) = \frac{-\pi}{2} + (1) = 1 - \frac{\pi}{2}$

Local Max: $(\frac{-\pi}{4}, \frac{\pi}{2} - 1)$

local min: $(\frac{\pi}{4}, 1 - \frac{\pi}{2})$

64) $f(x) = -2\cos x - \cos^2 x \qquad -\pi \le x \le \pi$

$$\frac{df}{dx} = -2[-\sin x(1)] - [2\cos x(\sin x(1))] = 2\sin x - 2\sin x\cos x$$

critical point

$$0 = \frac{df}{dx} = 2\sin x - 2\sin x\cos x$$

$$0 = 2\sin x - 2\sin x\cos x$$

$$0 = 2\sin x(1 - \cos x)$$

$2\sin x = 0$

$\sin x = 0$

$x = 0$ | $x = -\pi$, $x = \pi$ also endpoints

| $1 - \cos x = 0$

$1 = \cos x$

$x = 0$

$f(-\pi) = -2\cos(-\pi) - \cos^2(-\pi) = -2(-1) - (-1)^2$
$= 2 - 1 = 1$

$f(0) = -2\cos(0) - \cos^2(0) = -2(1) - (1)^2 = -3$

$f(\pi) = -2\cos(\pi) - \cos^2(\pi) = -2(-1) - (-1)^2$
$= 2 - 1 = 1$

Local Max: $(-\pi, 1)$, $(\pi, 1)$

local min: $(0, -3)$

---

66) $f(x) = \sec^2 x - 2\tan x \qquad \frac{-\pi}{2} < x < \frac{\pi}{2}$

$$\frac{df}{dx} = [2\sec x(\sec x\tan x(1))] - 2[\sec^2 x(1)] = 2\sec^2 x\tan x - 2\sec^2 x$$

critical points

$$0 = \frac{df}{dx} = 2\sec^2 x\tan x - 2\sec^2 x$$

$$0 = 2\sec^2 x\tan x - 2\sec^2 x$$

$$0 = 2\sec^2 x(\tan x - 1)$$

$2\sec^2 x = 0$

$\sec^2 x = 0$

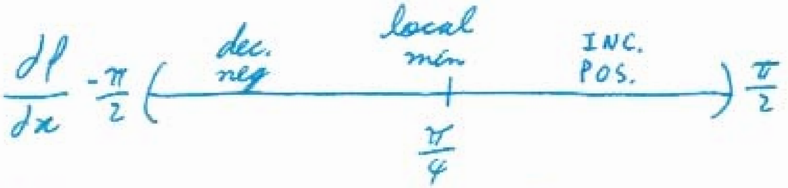
discard

$\sec x \ge 1$

| $\tan x - 1 = 0$

$\tan x = 1$

$x = \frac{\pi}{4}$

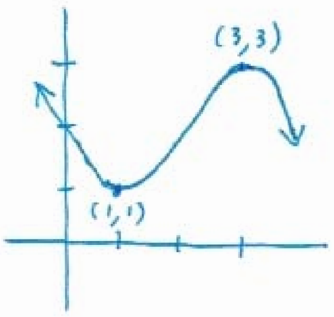


$f(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4}) - 2\tan(\frac{\pi}{4}) = (\sqrt{2})^2 - 2(1)$
$= 2 - 2 = 0$

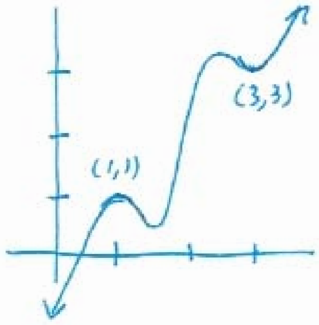
Local Max: none

local min: $(\frac{\pi}{4}, 0)$

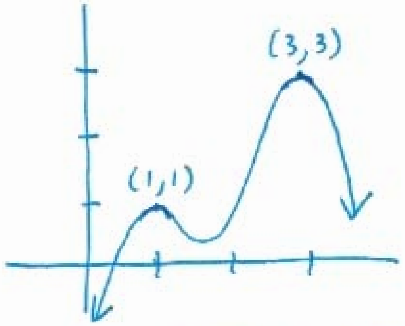
72) a) local min (1,1)
Local Max (3,3)



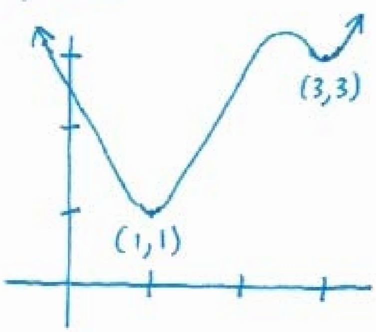
b) Local Max (1,1)
local min (3,3)



c) Local Max: (1,1), (3,3)



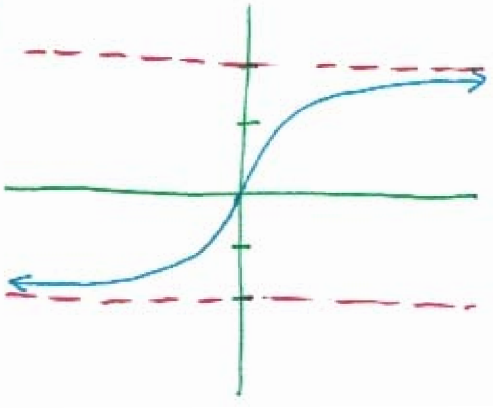
d) local min: (1,1), (3,3)



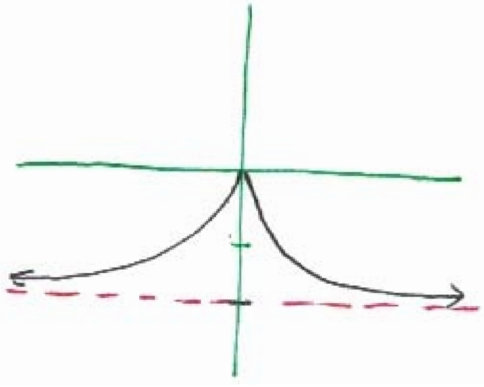
74) a) $h(0)=0$, $-2 \le h(x) \le 2$ for all x, $\frac{dh}{dx} \to \infty$ as $x \to 0^-$ and $\frac{dh}{dx} \to \infty$ as $x \to 0^+$.

b) $h(0)=0$, $-2 \le h(x) \le 0$ for all x, $\frac{dh}{dx} \to \infty$ as $x \to 0^-$ and $\frac{dh}{dx} \to -\infty$ as $x \to 0^+$.

a)



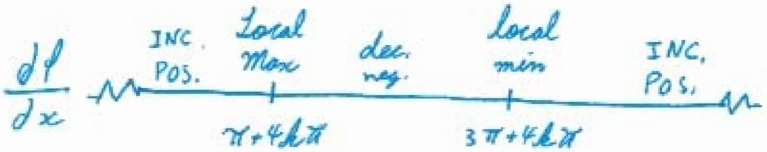
b)



82) $f(x) = 2e^{\sin(\frac{x}{2})}$

$$\frac{df}{dx} = 2\left[e^{\sin(\frac{x}{2})}\left(\cos(\tfrac{x}{2})(\tfrac{1}{2})\right)\right] = \cos(\tfrac{x}{2})\, e^{\sin(\frac{x}{2})}$$

critical points



$$0 = \frac{df}{dx} = \cos(\tfrac{x}{2})\, e^{\sin(\frac{x}{2})}$$

$$0 = \cos(\tfrac{x}{2})\, e^{\sin(\frac{x}{2})}$$

$f(\pi) = 2e^{\sin(\frac{(\pi)}{2})} = 2e^{(1)} = 2e$

$f(3\pi) = 2e^{\sin(\frac{(3\pi)}{2})} = 2e^{(-1)} = \frac{2}{e}$

$e^{\sin(\frac{x}{2})} = 0$

discard

$e^{\sin(\frac{x}{2})} > 0$

for all x

$\cos(\frac{x}{2}) = 0$

$\frac{x}{2} = \frac{\pi}{2}$ | $\frac{x}{2} = \frac{3\pi}{2}$

$\frac{x}{2} = \frac{\pi}{2} + 2k\pi$ | $\frac{x}{2} = \frac{3\pi}{2} + 2k\pi$

$x = \pi + 4k\pi$ | $x = 3\pi + 4k\pi$

Local Max of $2e$ when

$x = \pi + 4k\pi$

local min of $\frac{2}{e}$ when

$x = 3\pi + 4k\pi$

where k is any integer

84-a) $e^x \geq 1 + x$ if $x \geq 0$

$e^x = 1 + x$

$e^x - x - 1 = 0$

let $f(x) = e^x - x - 1$

$$\frac{df}{dx} = [e^x(1)] - [1] - [0] = e^x - 1$$

critical point

$0 = \frac{df}{dx} = e^x - 1$

$0 = e^x - 1$

$1 = e^x$

$e^0 = e^x$

$x = 0$

$f(0) = e^{(0)} - (0) - 1 = 1 - 0 - 1 = 0$

the function $f(x)$ is increasing on $(0, \infty)$

which means that $f(x) \geq 0$ for $x \geq 0$

$e^x - x - 1 \geq 0$

and $e^x \geq 1 + x$ for $x \geq 0$

84-b) $e^x \geq 1 + x + \frac{1}{2}x^2$

$$e^x = 1 + x + \frac{1}{2}x^2$$

$$e^x - \frac{1}{2}x^2 - x - 1 = 0$$

let $g(x) = e^x - \frac{1}{2}x^2 - x - 1$

$$\frac{dg}{dx} = [e^x(1)] - \frac{1}{2}[2x] - [1] - [0] = e^x - x - 1 = f(x) \text{ "from part a"}$$

using the result of part a, $f(x) \geq 0$

$$\frac{dg}{dx} \geq 0 \quad \text{for } x \geq 0$$

$$e^x - x - 1 \geq 0$$

this means that $g(x)$ is increasing on $(0, \infty)$.

$$g(0) = e^{(0)} - \frac{1}{2}(0)^2 - (0) - 1 = (1) - 0 - 0 - 1 = 0$$

which means that $g(x) \geq 0$ for $x \geq 0$

$$e^x - \frac{1}{2}x^2 - x - 1 \geq 0$$

and $e^x \geq 1 + x + \frac{1}{2}x^2$ for $x \geq 0$.