MATH 20100 section 3.10

Related Rates Problem Strategy

1) Draw a picture and name the variables and constants. Use t for time. Assume that all variables are differentiable functions of t.

2) Write down the numerical information (in terms of the symbols you have chosen).

3) Write down what you are asked to find (usually a rate, expressed as a derivative).

4) Write an equation that relates the variables. You may have to combine two or more equations to get a single equation that relates the variable whose rate you want to the variables whose rates you know.

5) Differentiate with respect to t. Then express the rate you want in terms of the rates and variables whose values you know.

6) Evaluate. Use known values to find the unknown rate.

2) $S = 4\pi r^2 \quad \frac{dS}{dt} = ?$

$\left[1 \frac{dS}{dt}\right] = 4\pi\left[2r\frac{dr}{dt}\right]$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

4) $2x + 3y = 12, \ \frac{dy}{dt} = -2, \ \frac{dx}{dt} = ?$

$2\left[1\frac{dx}{dt}\right] + 3\left[1\frac{dy}{dt}\right] = [0]$

$2\frac{dx}{dt} + 3\frac{dy}{dt} = 0$

$2\frac{dx}{dt} = -3\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-3}{2}\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-3}{2}(-2)$

$= 3$

6) $x = y^3 - y, \ \frac{dy}{dt} = 5, \ y = 2, \ \frac{dx}{dt} = ?$

$\left[1\frac{dx}{dt}\right] = \left[3y^2\frac{dy}{dt}\right] - \left[1\frac{dy}{dt}\right]$

$\frac{dx}{dt} = 3y^2\frac{dy}{dt} - \frac{dy}{dt}$

$\frac{dx}{dt} = 3(2)^2(5) - (5) = 60 - 5$

$= 55$

8) $x^2y^3 = \frac{4}{27}, \ \frac{dy}{dt} = \frac{1}{2}, \ x = 2, \ \frac{dx}{dt} = ?$

$(x^2)\left[3y^2\frac{dy}{dt}\right] + (y^3)\left[2x\frac{dx}{dt}\right] = [0]$

$3x^2y^2\frac{dy}{dt} + 2xy^3\frac{dx}{dt} = 0$

$2xy^3\frac{dx}{dt} = -3x^2y^2\frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-3x^2y^2}{2xy^3}\frac{dy}{dt} = \frac{-3x}{2y}\frac{dy}{dt}$

$(2)^2y^3 = \frac{4}{27}$

$4y^3 = \frac{4}{27}$

$y^3 = \frac{1}{27}$

$y = \sqrt[3]{\frac{1}{27}}$

$y = \frac{1}{3}$

$\frac{dx}{dt} = \frac{-3(2)}{2\left(\frac{1}{3}\right)}\left(\frac{1}{2}\right)$

$= \frac{-9}{2}$

10) $r + s^2 + v^3 = 12, \ \frac{dr}{dt} = 4, \ \frac{ds}{dt} = -3, \ r = 3 \ s = 1, \ \frac{dv}{dt} = ?$

$\left[1\frac{dr}{dt}\right] + \left[2s\frac{ds}{dt}\right] + \left[3v^2\frac{dv}{dt}\right] = [0]$

$\frac{dr}{dt} + 2s\frac{ds}{dt} + 3v^2\frac{dv}{dt} = 0$

$3v^2\frac{dv}{dt} = -\frac{dr}{dt} - 2s\frac{ds}{dt}$

$\frac{dv}{dt} = \frac{-1}{3v^2}\frac{dr}{dt} - \frac{2s}{3v^2}\frac{ds}{dt}$

10) continued

$$(3)+(1)^2+v^3=12$$
$$3+1+v^3=12$$
$$4+v^3=12$$
$$v^3=8$$
$$v=2$$

$$\frac{dv}{dt}=\frac{-1}{3(2)^2}(4)-\frac{2(1)}{3(2)^2}(-3)$$
$$=\frac{-1}{3}+\frac{1}{2}=\frac{-2}{6}+\frac{3}{6}=\underline{\underline{\frac{1}{6}}}$$

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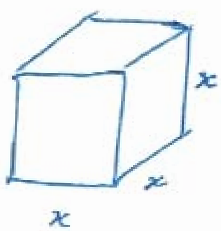
12)



$$S=6x^2 \qquad \frac{dS}{dt}=72\ \text{in}^2/\text{sec}$$
$$\left[1\frac{dS}{dt}\right]=6\left[2x\frac{dx}{dt}\right] \qquad x=3\ \text{in}$$
$$\frac{dS}{dt}=12x\frac{dx}{dt} \qquad \frac{dx}{dt}=?\ \text{then}\ \frac{dV}{dt}=?$$
$$\frac{dx}{dt}=\frac{1}{12x}\frac{dS}{dt} \qquad \frac{dx}{dt}=\frac{1}{12(3)}(72)=\frac{6}{3}=\underline{\underline{2\ \text{in/sec}}}$$

$$V=x^3$$
$$\left[1\frac{dV}{dt}\right]=\left[3x^2\frac{dx}{dt}\right] \qquad \frac{dV}{dt}=3(3)^2(2)=\underline{\underline{54\ \text{in}^3/\text{sec}}}$$
$$\frac{dV}{dt}=3x^2\frac{dx}{dt}$$

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14) $V=\frac{1}{3}\pi r^2 h$

a) $$\left[1\frac{dV}{dt}\right]=\frac{1}{3}\pi r^2\left[1\frac{dh}{dt}\right]$$
$$\frac{dV}{dt}=\frac{1}{3}\pi r^2\frac{dh}{dt}$$

b) $$\left[1\frac{dV}{dt}\right]=\frac{1}{3}\pi h\left[2r\frac{dr}{dt}\right]$$
$$\frac{dV}{dt}=\frac{2}{3}\pi r h\frac{dr}{dt}$$

c) $$\left[1\frac{dV}{dt}\right]=\frac{1}{3}\pi\left\{(r^2)\left[1\frac{dh}{dt}\right]+(h)\left[2r\frac{dr}{dt}\right]\right\}$$
$$\frac{dV}{dt}=\frac{1}{3}\pi\left\{r^2\frac{dh}{dt}+2rh\frac{dr}{dt}\right\}$$
$$=\frac{1}{3}\pi r^2\frac{dh}{dt}+\frac{2}{3}\pi r h\frac{dr}{dt}$$

16) $P = RI^2$

a) $\left[1\frac{dP}{dt}\right] = (R)\left[2I\frac{dI}{dt}\right] + (I^2)\left[1\frac{dR}{dt}\right]$

$$\frac{dP}{dt} = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$$

$$P = RI^2$$

$$\frac{P}{I^2} = R$$

b) $[0] = (R)\left[2I\frac{dI}{dt}\right] + (I^2)\left[1\frac{dR}{dt}\right]$

$$0 = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$$

$$I^2\frac{dR}{dt} = -2RI\frac{dI}{dt}$$

$$\frac{dR}{dt} = \frac{-2RI}{I^2}\frac{dI}{dt} = \underline{\underline{\frac{-2R}{I}\frac{dI}{dt}}} = \frac{-2\left(\frac{P}{I^2}\right)}{I}\frac{dI}{dt} = \underline{\underline{\frac{-2P}{I^3}\frac{dI}{dt}}}$$

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18) $s = \sqrt{x^2 + y^2 + z^2}$

$$s^2 = x^2 + y^2 + z^2$$

a) $\left[2s\frac{ds}{dt}\right] = \left[2x\frac{dx}{dt}\right] + \left[2y\frac{dy}{dt}\right] + \left[2z\frac{dz}{dt}\right]$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$$

$$\frac{ds}{dt} = \frac{2x}{2s}\frac{dx}{dt} + \frac{2y}{2s}\frac{dy}{dt} + \frac{2z}{2s}\frac{dz}{dt}$$

$$\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt} + \frac{y}{s}\frac{dy}{dt} + \frac{z}{s}\frac{dz}{dt}$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}\frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2+z^2}}\frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{dz}{dt}$$

18) continued

b) $\left[2s\frac{ds}{dt}\right]=[0]+\left[2y\frac{dy}{dt}\right]+\left[2z\frac{dz}{dt}\right]$

$$2s\frac{ds}{dt}=2y\frac{dy}{dt}+2z\frac{dz}{dt}$$

$$\frac{ds}{dt}=\frac{2y}{2s}\frac{dy}{dt}+\frac{2z}{2s}\frac{dz}{dt}=\frac{y}{s}\frac{dy}{dt}+\frac{z}{s}\frac{dz}{dt}$$

$$=\frac{y}{\sqrt{x^2+y^2+z^2}}\frac{dy}{dt}+\frac{z}{\sqrt{x^2+y^2+z^2}}\frac{dz}{dt}$$

c) $[0]=\left[2x\frac{dx}{dt}\right]+\left[2y\frac{dy}{dt}\right]+\left[2z\frac{dz}{dt}\right]$

$$0=2x\frac{dx}{dt}+2y\frac{dy}{dt}+2z\frac{dz}{dt}$$

$$2x\frac{dx}{dt}=-2y\frac{dy}{dt}-2z\frac{dz}{dt}$$

$$\frac{dx}{dt}=\frac{-2y}{2x}\frac{dy}{dt}-\frac{2z}{2x}\frac{dz}{dt}=\underline{\underline{\frac{-y}{x}\frac{dy}{dt}-\frac{z}{x}\frac{dz}{dt}}}$$

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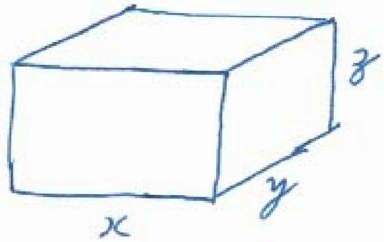
20) $A=\pi r^2$ $\qquad \frac{dr}{dt}=0.01\text{ cm/min}=\frac{1}{100}\text{ cm/min},\ r=50\text{ cm}$

$\left[1\frac{dA}{dt}\right]=\pi\left[2r\frac{dr}{dt}\right]$ $\qquad \frac{dA}{dt}=?$

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt}=2\pi(50)\left(\frac{1}{100}\right)=\underline{\underline{\pi\text{ cm}^2/\text{min}}}$$

22)



$x = 4\,m, \; y = 3\,m, \; z = 2\,m$

$\frac{dx}{dt} = 1\,m/sec, \; \frac{dy}{dt} = -2\,m/sec, \; \frac{dz}{dt} = 1\,m/sec$

a) $V = xyz$

$\ln V = \ln(xyz)$

$\ln V = \ln x + \ln y + \ln z$

$$\left[\frac{1}{V}\frac{dV}{dt}\right] = \left[\frac{1}{x}\frac{dx}{dt}\right] + \left[\frac{1}{y}\frac{dy}{dt}\right] + \left[\frac{1}{z}\frac{dz}{dt}\right]$$

$$\frac{dV}{dt} = \frac{V}{x}\frac{dx}{dt} + \frac{V}{y}\frac{dy}{dt} + \frac{V}{z}\frac{dz}{dt} = \frac{xyz}{x}\frac{dx}{dt} + \frac{xyz}{y}\frac{dy}{dt} + \frac{xyz}{z}\frac{dz}{dt}$$

$$\frac{dV}{dt} = yz\frac{dx}{dt} + xz\frac{dy}{dt} + xy\frac{dz}{dt}$$

$$\frac{dV}{dt} = (3)(2)(1) + (4)(2)(-2) + (4)(3)(1) = 6 - 16 + 12 = \underline{\underline{2\,m^3/sec}}$$

b) $S = 2xy + 2xz + 2yz$

$$\left[1\frac{dS}{dt}\right] = \left\{(2x)\left[1\frac{dy}{dt}\right] + (y)\left[2\frac{dx}{dt}\right]\right\} + \left\{(2x)\left[1\frac{dz}{dt}\right] + (z)\left[2\frac{dx}{dt}\right]\right\}$$
$$+ \left\{(2y)\left[1\frac{dz}{dt}\right] + (z)\left[2\frac{dy}{dt}\right]\right\}$$

$$\frac{dS}{dt} = 2x\frac{dy}{dt} + 2y\frac{dx}{dt} + 2x\frac{dz}{dt} + 2z\frac{dx}{dt} + 2y\frac{dz}{dt} + 2z\frac{dy}{dt}$$

$$\frac{dS}{dt} = 2y\frac{dx}{dt} + 2z\frac{dx}{dt} + 2x\frac{dy}{dt} + 2z\frac{dy}{dt} + 2x\frac{dz}{dt} + 2y\frac{dz}{dt}$$

22) continued

b) $$\frac{dS}{dt} = (2y+2z)\frac{dx}{dt} + (2x+2z)\frac{dy}{dt} + (2x+2y)\frac{dz}{dt}$$

$$\frac{dS}{dt} = (2(3)+2(2))(1) + (2(4)+2(2))(-2) + (2(4)+2(3))(1)$$
$$= (10)(1) + (12)(-2) + (14)(1) = 10 - 24 + 14 = \underline{\underline{0 \text{ m}^2/\text{sec}}}$$

c) $$s = \sqrt{x^2+y^2+z^2}$$

$$s^2 = x^2+y^2+z^2$$

$$\left[2s\frac{ds}{dt}\right] = \left[2x\frac{dx}{dt}\right] + \left[2y\frac{dy}{dt}\right] + \left[2z\frac{dz}{dt}\right]$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$$

$$\frac{ds}{dt} = \frac{2x}{2s}\frac{dx}{dt} + \frac{2y}{2s}\frac{dy}{dt} + \frac{2z}{2s}\frac{dz}{dt}$$

$$\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt} + \frac{y}{s}\frac{dy}{dt} + \frac{z}{s}\frac{dz}{dt}$$

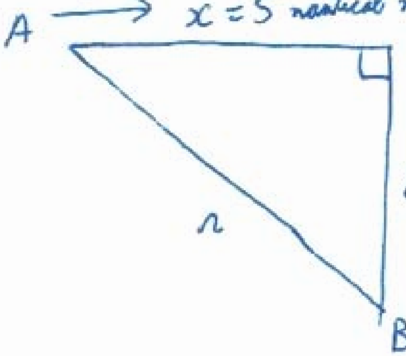
$$s = \sqrt{(4)^2+(3)^2+(2)^2} = \sqrt{16+9+4} = \sqrt{29}$$

$$\frac{ds}{dt} = \frac{(4)}{(\sqrt{29})}(1) + \frac{(3)}{(\sqrt{29})}(-2) + \frac{(2)}{(\sqrt{29})}(1) = \frac{4}{\sqrt{29}} - \frac{6}{\sqrt{29}} + \frac{2}{\sqrt{29}}$$

$$= \frac{6}{\sqrt{29}} - \frac{6}{\sqrt{29}} = \underline{\underline{0 \text{ m/sec}}}$$

24) $\frac{dx}{dt} = -442$ knots

$x = 5$ nautical miles



$y = 12$ nautical miles

$\frac{dy}{dt} = -481$ knots

$r^2 = x^2 + y^2$

$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + \left[2y\frac{dy}{dt}\right]$

$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dr}{dt} = \frac{2x}{2r}\frac{dx}{dt} + \frac{2y}{2r}\frac{dy}{dt}$

$\frac{dr}{dt} = \frac{x}{r}\frac{dx}{dt} + \frac{y}{r}\frac{dy}{dt}$

$r^2 = (5)^2 + (12)^2$

$r^2 = 25 + 144$

$r^2 = 169$

$r = \pm\sqrt{169} = \pm 13$

$r = 13$ nautical miles

$\frac{dr}{dt} = \frac{(5)}{(13)}(-442) + \frac{(12)}{(13)}(-481) = (5)(-34) + (12)(-37)$

$= -170 - 444 = \underline{\underline{-614 \text{ knots}}}$

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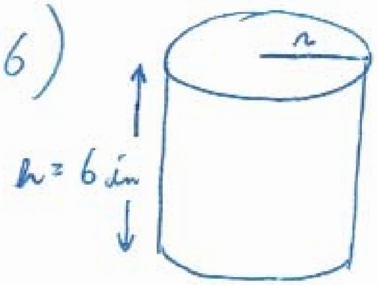
26)



diameter: 3.8 in $\Rightarrow$ radius: $r = 1.9$ in

$\frac{dr}{dt} = \frac{\frac{1}{1000}\text{ in}}{3\text{ min}} = \frac{1}{3000}$ in/min

$\frac{dV}{dt} = ?$

$V = \pi r^2 h$

$V = \pi r^2 (6)$

$V = 6\pi r^2$

$\left[1\frac{dV}{dt}\right] = 6\pi\left[2r\frac{dr}{dt}\right]$

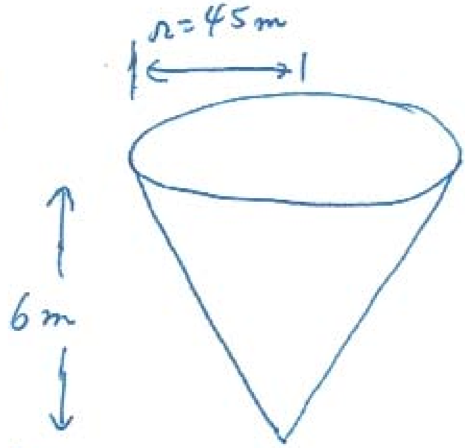
$\frac{dV}{dt} = 12\pi r\frac{dr}{dt}$

$\frac{dV}{dt} = 12\pi(1.9)\left(\frac{1}{3000}\right)$

$= \frac{12\pi(1.9)}{3000} = \frac{12\pi(19)}{30000}$

$= \frac{228\pi}{30000}\text{ in}^3/\text{min} = \underline{\underline{\frac{57\pi}{7500}\text{ in}^3/\text{min}}}$

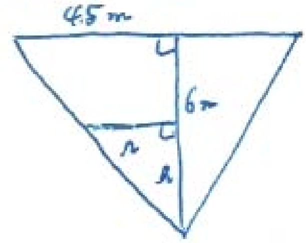
28)



$\frac{dV}{dt} = -50\ m^3/min$

$V = \frac{1}{3}\pi r^2 h$

side view



$\frac{r}{45} = \frac{h}{6}$

$r = \frac{45}{6}h = \frac{15}{2}h$

a) $\frac{dh}{dt} = ?$ when $h = 5\ m$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h = \frac{(15)^2}{3(2)^2}\pi h^3 = \frac{(15)^2}{3(4)}\pi h^3$

$\left[1\frac{dV}{dt}\right] = \frac{(15)^2}{3(4)}\pi\left[3h^2\frac{dh}{dt}\right]$

$\frac{dV}{dt} = \frac{(15)^2}{4}\pi h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{(15)^2\pi h^2}\frac{dV}{dt}$

$\frac{dh}{dt} = \frac{4}{(15)^2\pi(5)^2}(-50) = \frac{-8}{\pi(15)^2}\ m/min$

$= \frac{-8}{\pi(15)^2}(100) = \underline{\underline{\frac{-800}{\pi(15)^2}\ cm/min}}$

b) when $h = 5\ m$ $\frac{dr}{dt} = ?$

$r = \frac{15}{2}h$

$\left[1\frac{dr}{dt}\right] = \frac{15}{2}\left[1\frac{dh}{dt}\right]$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{15}{2}\left(\frac{-8}{\pi(15)^2}\right) = \frac{-4}{15\pi}\ m/min$

$= \frac{-4}{15\pi}(100) = \underline{\underline{\frac{-400}{15\pi}\ cm/min}}$

30)

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

$\frac{dV}{dt} = kS = 4\pi k r^2$

$\left[1\frac{dV}{dt}\right] = \frac{4}{3}\pi\left[3r^2\frac{dr}{dt}\right]$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2}\left(kS\right) = \frac{1}{4\pi r^2}\left(4\pi k r^2\right)$

$\frac{dr}{dt} = k \leftarrow$ constant

so, the radius is increasing at a constant rate.

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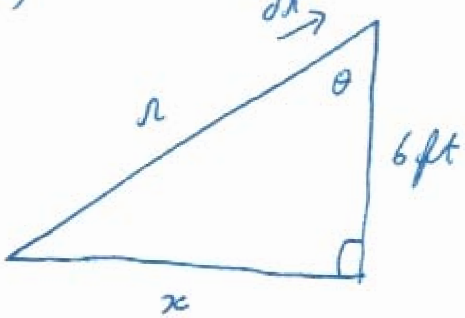
32)

$\frac{dr}{dt} = -2$ ft/sec



$r^2 = x^2 + (6)^2$

a) $r = 10$ ft, $\frac{dx}{dt} = ?$

$(10)^2 = x^2 + (6)^2$

$100 = x^2 + 36$

$64 = x^2$

$\cancel{x} = \pm\sqrt{64} = \pm 8$

$x = 8$ ft

$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + [0]$

$2r\frac{dr}{dt} = 2x\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{2r}{2x}\frac{dr}{dt} = \frac{r}{x}\frac{dr}{dt}$

$\frac{dx}{dt} = \frac{(10)}{(8)}(-2) = \frac{-5}{2}$ ft/sec

b) $\sec\theta = \frac{r}{6}$

$\left[\sec\theta\tan\theta\frac{d\theta}{dt}\right] = \frac{1}{6}\left[1\frac{dr}{d\theta}\right]$

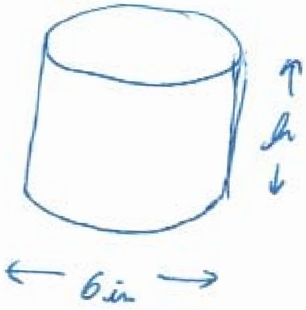
$\sec\theta\tan\theta\frac{d\theta}{dt} = \frac{1}{6}\frac{dr}{d\theta}$

$\frac{d\theta}{dt} = \frac{1}{6\sec\theta\tan\theta}\frac{dr}{d\theta}$

$\frac{d\theta}{dt} = \frac{1}{6\left(\frac{10}{6}\right)\left(\frac{8}{6}\right)}(-2) = \frac{(-2)(6)}{(10)(8)}$

$= \frac{-3}{20}$ rad/sec

34) a)



$r = 3 \text{ in} \qquad \dfrac{dV}{dt} = +10 \text{ in}^3/\text{min}$

$V = \pi r^2 h$

$V = \pi (3)^2 h$

$V = 9\pi h$

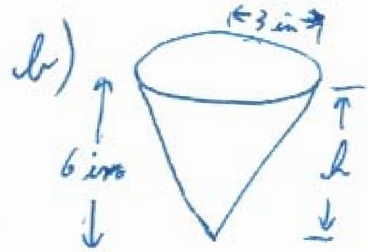
$\left[1 \dfrac{dV}{dt}\right] = 9\pi \left[1 \dfrac{dh}{dt}\right]$

$\dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{1}{9\pi} \dfrac{dV}{dt}$

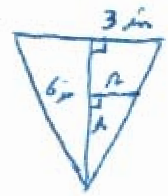
$\dfrac{dh}{dt} = \dfrac{1}{9\pi}(10) = \underline{\underline{\dfrac{10}{9\pi} \text{ in/min}}}$

b)



$h = 5 \text{ in} \qquad \dfrac{dV}{dt} = -10 \text{ in}^3/\text{min}$

$V = \dfrac{1}{3}\pi r^2 h$



$\dfrac{h}{6} = \dfrac{r}{3}$

$r = \dfrac{3}{6} h = \dfrac{1}{2} h$

$V = \dfrac{1}{3}\pi \left(\dfrac{1}{2}h\right)^2 h$

$V = \dfrac{1}{12}\pi h^3$

$\left[1 \dfrac{dV}{dt}\right] = \dfrac{1}{12}\pi \left[3h^2 \dfrac{dh}{dt}\right]$

$\dfrac{dV}{dt} = \dfrac{3}{12}\pi h^2 \dfrac{dh}{dt}$

$\dfrac{dV}{dt} = \dfrac{1}{4}\pi h^2 \dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{4}{\pi h^2} \dfrac{dV}{dt}$

$\dfrac{dh}{dt} = \dfrac{4}{\pi (5)^2}(-10) = \underline{\underline{\dfrac{-8}{5\pi} \text{ in/min}}}$

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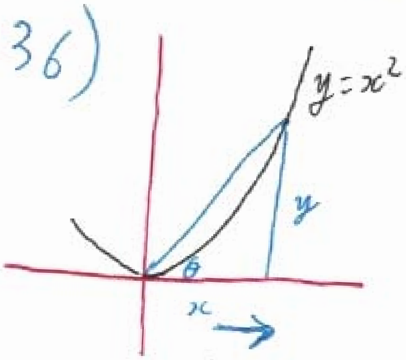
36)



$\dfrac{dx}{dt} = 10 \text{ m/sec}$

at $x = 3$ m

$\dfrac{d\theta}{dt} = ?$

at $x = 3$ m, $y = (3)^2 = 9$ m $\qquad \tan\theta = \dfrac{y}{x} = \dfrac{9 \text{ m}}{3 \text{ m}} = 3$

$\tan\theta = \dfrac{y}{x} = \dfrac{x^2}{x} = x$

$\left[\sec^2\theta \dfrac{d\theta}{dt}\right] = \left[1 \dfrac{dx}{dt}\right]$

$\sec^2\theta \dfrac{d\theta}{dt} = \dfrac{dx}{dt}$

$\dfrac{d\theta}{dt} = \dfrac{1}{\sec^2\theta} \dfrac{dx}{dt}$

$\dfrac{d\theta}{dt} = \dfrac{1}{1+\tan^2\theta} \dfrac{dx}{dt}$

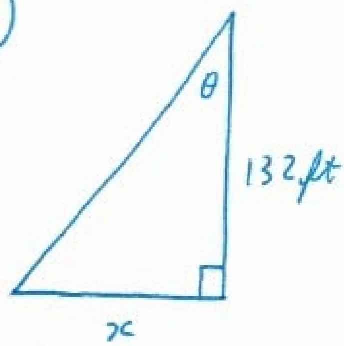
$\dfrac{d\theta}{dt} = \dfrac{1}{1+(3)^2}(10)$

$= \dfrac{1}{1+9}(10)$

$= \dfrac{1}{10}(10)$

$\underline{\underline{\dfrac{d\theta}{dt} = 1 \text{ rad/sec}}}$

38)



$$\tan\theta = \frac{x}{132} \qquad \text{at } x=0,\ \frac{dx}{dt} = ?$$

$$\left[\sec^2\theta \frac{d\theta}{dt}\right] = \frac{1}{132}\left[1\frac{dx}{dt}\right]$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{132}\frac{dx}{dt}$$

$$\frac{dx}{dt} = -264 \text{ ft/sec}$$

$$\frac{d\theta}{dt} = \frac{1}{132\sec^2\theta}\frac{dx}{dt} = \frac{1}{132(1+\tan^2\theta)}\frac{dx}{dt}$$

at $x=0$, $\theta=0$ and $\sec\theta = 1$

$$\frac{d\theta}{dt} = \frac{1}{132(1)}(-264) = \underline{\underline{-2 \text{ rad/sec}}}$$

at 1 second later: $x = 264$ ft and $\frac{dx}{dt} = +264$ ft/sec

at $\frac{1}{2}$ second later: $x = \frac{264 \text{ ft}}{2} = 132$ ft

and here $\tan\theta = \frac{x}{132} = \frac{(132)}{132} = 1$

$$\frac{d\theta}{dt} = \frac{1}{132(1+\tan^2\theta)}\frac{dx}{dt} = \frac{1}{132(1+(1)^2)}(264) = \frac{1}{132(2)}(264) = \frac{1}{264}(264)$$

$$= \underline{\underline{1 \text{ rad/sec}}}$$

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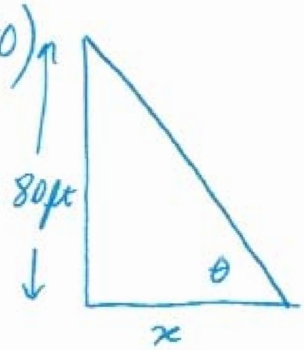
40)



$$\frac{d\theta}{dt} = 0.27°/\text{min} = 0.27°\left(\frac{\pi}{180°}\right)\text{rad}$$

$$= \left(\frac{27}{100}\right)\left(\frac{\pi}{180}\right)\text{rad} = \left(\frac{3}{100}\right)\left(\frac{\pi}{20}\right)\text{rad}$$

$$\frac{d\theta}{dt} = \frac{3\pi}{2000}\text{rad}, \quad x = 60 \text{ ft}, \quad \frac{dx}{dt} = ?$$

40) continued

$$\tan\theta = \frac{80}{x} = 80x^{-1}$$

$$\left[\sec^2\theta \frac{d\theta}{dt}\right] = 80\left[-1x^{-2}\frac{dx}{dt}\right]$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-80}{x^2}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-x^2\sec^2\theta}{80}\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \frac{-x^2(1+\tan^2\theta)}{80}\frac{d\theta}{dt}$$

when $x=60$

$$\tan\theta = \frac{80}{x} = \frac{80}{60} = \frac{8}{6} = \frac{4}{3}$$

$$\frac{dx}{dt} = \frac{-(60)^2\left(1+\left(\frac{4}{3}\right)^2\right)}{80}\left(\frac{3\pi}{2000}\right)$$

$$= \frac{-(60)^2\left(1+\frac{16}{9}\right)}{80}\left(\frac{3\pi}{2000}\right)$$

$$= \frac{-(60)^2\left(\frac{25}{9}\right)}{80}\left(\frac{3\pi}{2000}\right) = \frac{-(20)^2(25)}{80}\left(\frac{3\pi}{2000}\right)$$

$$= \frac{-(2)(25)(3\pi)}{80(10)} = \frac{-(25)(3\pi)}{80(5)} = \underline{\underline{\frac{-3\pi}{16}\text{ ft/min}}}$$

$$\frac{dx}{dt} = \frac{-3\pi}{16}(12)\text{ in/min} = \underline{\underline{\frac{-9\pi}{4}\text{ in/min}}}$$

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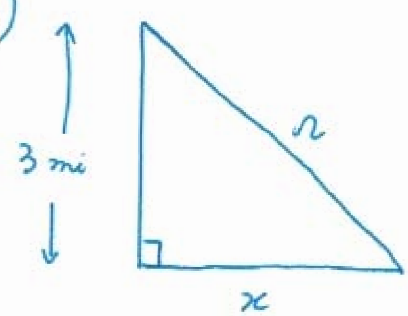
42)



$$\frac{dr}{dt} = -160\text{ mph} \qquad r = 5\text{ mi} \qquad \frac{dx}{dt} = ?$$

$$x^2 + (3)^2 = r^2$$

$$\left[2x\frac{dx}{dt}\right] + [0] = \left[2r\frac{dr}{dt}\right]$$

$$2x\frac{dx}{dt} = 2r\frac{dr}{dt}$$

$$\frac{dx}{dt} = \frac{2r}{2x}\frac{dr}{dt} = \frac{r}{x}\frac{dr}{dt}$$

$$x^2 + (3)^2 = (5)^2$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = \pm\sqrt{16} = \pm 4$$

$$x = 4\text{ miles}$$

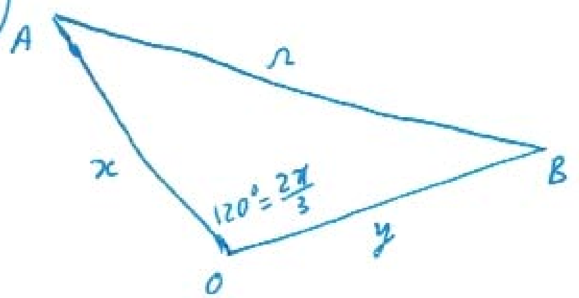
$$\frac{dx}{dt} = \frac{(5)}{(4)}(-160) = (5)(-40) = -200\text{ mph}$$

speed of plane + speed of car $= \left|\frac{dx}{dt}\right| = |-200|$ mph

120 mph + speed of car = 200 mph

speed of car = <u>80 mph</u>

44)



$\frac{dx}{dt} = 14$ knots , $x = 5$ nautical miles

$\frac{dy}{dt} = 21$ knots, $y = 3$ nautical miles

$\cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

$$r^2 = x^2 + y^2 - 2xy\cos\left(\frac{2\pi}{3}\right)$$
$$r^2 = x^2 + y^2 - 2xy\left(\frac{-1}{2}\right)$$
$$r^2 = x^2 + y^2 + xy$$

$$r^2 = (5)^2 + (3)^2 + (5)(3)$$
$$r^2 = 25 + 9 + 15 = 49$$
$$r = \pm\sqrt{49} = \pm 7 \Rightarrow r = 7 \text{ nautical miles}$$

$$\left[2r\frac{dr}{dt}\right] = \left[2x\frac{dx}{dt}\right] + \left[2y\frac{dy}{dt}\right] + \left\{(x)\left[1\frac{dy}{dt}\right] + (y)\left[\cancel{1}\frac{dx}{dt}\right]\right\}$$

$$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dr}{dt} = \frac{2x}{2r}\frac{dx}{dt} + \frac{2y}{2r}\frac{dy}{dt} + \frac{x}{2r}\frac{dy}{dt} + \frac{y}{2r}\frac{dx}{dt}$$

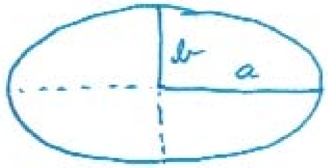
$$\frac{dr}{dt} = \frac{x}{r}\frac{dx}{dt} + \frac{y}{r}\frac{dy}{dt} + \frac{x}{2r}\frac{dy}{dt} + \frac{y}{2r}\frac{dx}{dt}$$

$$\frac{dr}{dt} = \frac{(5)}{(7)}(14) + \frac{(3)}{(7)}(21) + \frac{(5)}{2(7)}(21) + \frac{(3)}{2(7)}(14)$$

$$= (5)(2) + (3)(3) + \frac{5(3)}{2} + (3)(1)$$

$$= 10 + 9 + \frac{15}{2} + 3 = 22 + \frac{15}{2} = \frac{44}{2} + \frac{15}{2} = \underline{\underline{\frac{59}{2} \text{ knots}}}$$

46)



ellipse Major Axis : $2a$

minor axis : $2b$

Area $= \pi a b$

slick is constant 9 in thick. $9_{in} = 9\left(\frac{1}{12}\right) ft = \frac{9}{12} = \frac{3}{4} ft$

Major Axis of the slick $2a = 2\,mi = 2(5280) ft \Rightarrow a = 5280\, ft$

length is increasing at the rate of 30 ft/hr

$$\frac{d}{dt}(2a) = 30\, ft/hr \Longrightarrow \frac{da}{dt} = 15\, ft/hr$$

minor axis of the slick $2b = \frac{3}{4}\, mi = \frac{3}{4}(5280) ft \Rightarrow b = \frac{3}{8}(5280) ft$

width is increasing at the rate of 10 ft/hr

$$\frac{d}{dt}(2b) = 10\, ft/hr \Longrightarrow \frac{db}{dt} = 5\, ft/hr$$

$$V = (\text{thickness})\,\text{Area} = \left(\frac{3}{4}\right)(\pi a b) = \frac{3\pi}{4} a b$$

$$\left[1 \frac{dV}{dt}\right] = \frac{3\pi}{4}\left\{(a)\left[1 \frac{db}{dt}\right] + (b)\left[1 \frac{da}{dt}\right]\right\}$$

$$\frac{dV}{dt} = \frac{3\pi}{4}\left\{a \frac{db}{dt} + b \frac{da}{dt}\right\}$$

$$\frac{dV}{dt} = \frac{3\pi}{4}\left\{(5280)(5) + \left(\frac{3}{8}(5280)\right)(15)\right\} = \frac{3\pi}{4}(5280)\left\{5 + \frac{45}{8}\right\}$$

$$= \frac{3\pi}{4}(5280)\left\{\frac{40}{8} + \frac{45}{8}\right\} = \frac{3\pi}{4}(5280)\left\{\frac{85}{8}\right\} = \frac{3\pi}{4}(660)\{85\}$$

$$= 3\pi(165)\{85\}\ ft^3/hr$$