Definition

$y = \sin^{-1} x$ is the number in $[\frac{-\pi}{2}, \frac{\pi}{2}]$ for which $\sin y = x$.

$y = \cos^{-1} x$ is the number in $[0, \pi]$ for which $\cos y = x$.

$y = \tan^{-1} x$ is the number in $(\frac{-\pi}{2}, \frac{\pi}{2})$ for which $\tan y = x$.

$y = \cot^{-1} x$ is the number in $(0, \pi)$ for which $\cot y = x$.

$y = \sec^{-1} x$ is the number in $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ for which $\sec y = x$.

$y = \csc^{-1} x$ is the number in $[\frac{-\pi}{2}, 0) \cup (0 \frac{\pi}{2}]$ for which $\csc y = x$.

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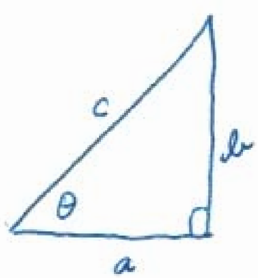
useful information from precalculus class:



$c = \sqrt{a^2+b^2}$ $\quad \sin\theta = \frac{b}{c}$ $\quad \csc\theta = \frac{c}{b}$

$b = \sqrt{c^2-a^2}$ $\quad \cos\theta = \frac{a}{c}$ $\quad \sec\theta = \frac{c}{a}$

$a = \sqrt{c^2-b^2}$ $\quad \tan\theta = \frac{b}{a}$ $\quad \cot\theta = \frac{a}{b}$

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Instead of memorizing 6 formulas for derivatives of Inverse Trigonometric Functions, we can use the procedure given below (also a modified procedure is used for 6 additional formulas of Inverse Hyperbolic Trigonometric Functions).

Step 1: Convert into regular trigonometric form.

Step 2: Use the implicit differentiation

Step 3: Convert back into original form (draw the appropriate triangle to simplify your answer)

demostration of the procedure:

$$\frac{d}{dv}(\cos^{-1} v)$$

$$\theta = \cos^{-1} v$$

$$\Downarrow$$

Step 1

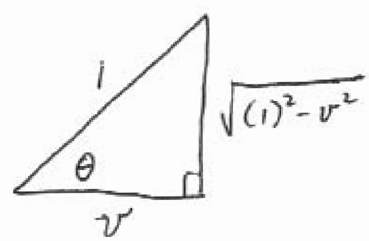
$$\cos\theta = v = \frac{v}{1}$$

Step 2

$$\left[-\sin\theta \frac{d\theta}{dv}\right] = [1]$$

$$\frac{d\theta}{dv} = \frac{-1}{\sin\theta}$$

Step 3:



$$\frac{d\theta}{dv} = \frac{-1}{\left(\frac{\sqrt{(1)^2 - v^2}}{1}\right)}$$

$$\frac{d\theta}{dv} = \frac{-1}{\sqrt{1 - v^2}}$$

{restriction}

$$\frac{d}{dv}(\cos^{-1} v) = \frac{-1}{\sqrt{1 - v^2}} \quad |v| < 1$$

$\frac{d}{dv}(\csc^{-1} v)$

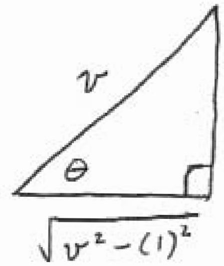
Step 1

$\theta = \csc^{-1} v$

$\Downarrow$

$\csc\theta = v = \frac{v}{1}$

Step 3



Step 2

$\left[-\csc\theta \cot\theta \frac{d\theta}{dv}\right] = [1]$

$\frac{d\theta}{dv} = \frac{-1}{\csc\theta \cot\theta}$

$\frac{d\theta}{dv} = \frac{-1}{\left(\frac{v}{1}\right)\left(\frac{\sqrt{v^2-(1)^2}}{1}\right)}$

$\frac{d\theta}{dv} = \frac{-1}{v\sqrt{v^2-1}}$

{restriction}

$\Downarrow$

$|v| > 1$

$\frac{d}{dv}(\csc^{-1} v) = \frac{-1}{|v|\sqrt{v^2-1}}$

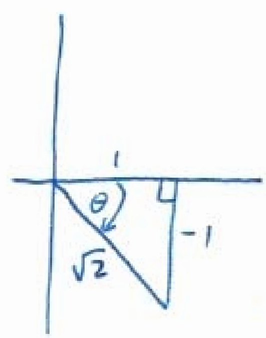
abs. value on v is necessary because θ may be in $[-\frac{\pi}{2}, 0)$ and this in to ensure that we get the correct sign (pos. or neg.) on our answer.

2-a) $\arctan(-1) = \tan^{-1}(-1)$

$\theta = \tan^{-1}(-1)$

$\Downarrow$

$\tan\theta = -1 = \frac{-1}{1}$



$\theta = \frac{-\pi}{4}$

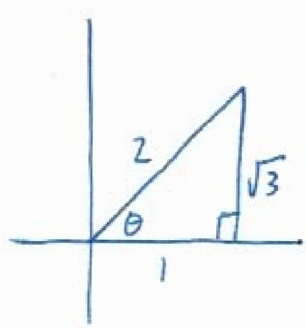
$\underline{\underline{\tan^{-1}(-1) = \frac{-\pi}{4}}}$

2-b) $\tan^{-1}\sqrt{3}$

$\theta = \tan^{-1}\sqrt{3}$

$\Downarrow$

$\tan\theta = \sqrt{3} = \frac{\sqrt{3}}{1}$



$\theta = \frac{\pi}{3}$

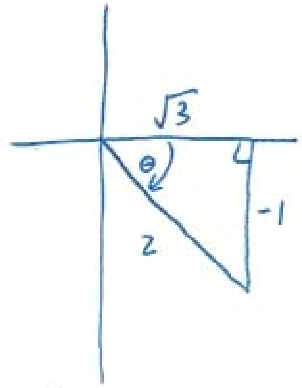
$\underline{\underline{\tan^{-1}\sqrt{3} = \frac{\pi}{3}}}$

2-c) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\Downarrow$

$\tan\theta = \frac{-1}{\sqrt{3}}$



$\theta = \frac{-\pi}{6}$

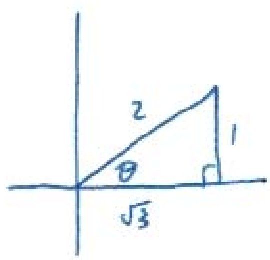
$\underline{\underline{\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}}}$

4-a) $\sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{6}$

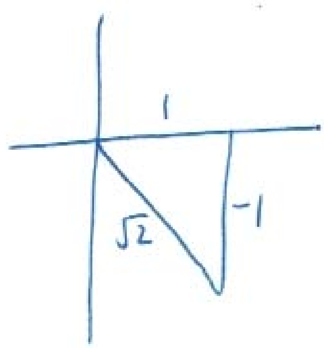
$\underline{\underline{\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}}}$

4-b) $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$\theta = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$\Downarrow$

$\sin\theta = \frac{-1}{\sqrt{2}}$



$\theta = \frac{-\pi}{4}$

$\underline{\underline{\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{-\pi}{4}}}$

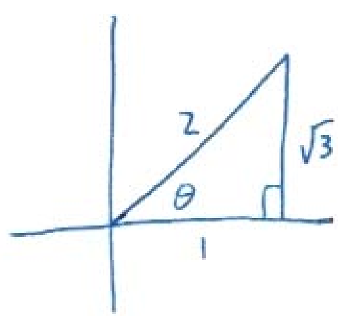
4-c) $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Downarrow$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$



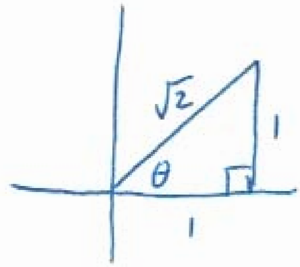
$\underline{\underline{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}}}$

6-a) $\csc^{-1}\sqrt{2}$

$\theta = \csc^{-1}\sqrt{2}$

$\Downarrow$

$\csc\theta = \sqrt{2} = \frac{\sqrt{2}}{1}$



$\theta = \frac{\pi}{4}$

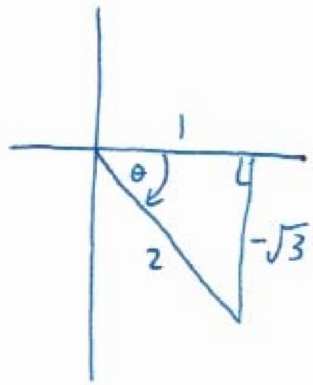
$\underline{\underline{\csc^{-1}\sqrt{2} = \frac{\pi}{4}}}$

6-b) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\theta = \csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\Downarrow$

$\csc\theta = \frac{-2}{\sqrt{3}}$



$\theta = \frac{-\pi}{3}$

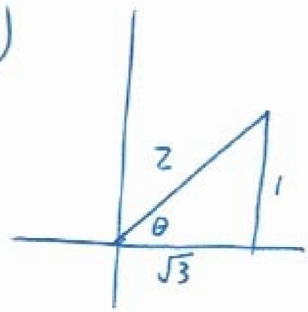
$\underline{\underline{\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \frac{-\pi}{3}}}$

6-c) $\operatorname{arccsc} 2 = \csc^{-1}(2)$

$\theta = \csc^{-1}(2)$

$\Downarrow$

$\csc\theta = 2 = \frac{2}{1}$



$\theta = \frac{\pi}{6}$

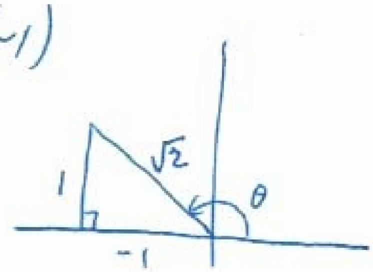
$\underline{\underline{\csc^{-1}(2) = \frac{\pi}{6}}}$

8-a) $\operatorname{arccot}(-1) = \cot^{-1}(-1)$

$\theta = \cot^{-1}(-1)$

$\Downarrow$

$\cot\theta = -1 = \frac{-1}{1}$



$\theta = \frac{3\pi}{4}$

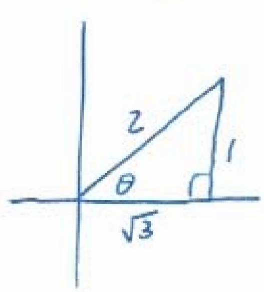
$\underline{\underline{\cot^{-1}(-1) = \frac{3\pi}{4}}}$

8-b) $\cot^{-1}(\sqrt{3})$

$\theta = \cot^{-1}(\sqrt{3})$

$\Downarrow$

$\cot\theta = \sqrt{3} = \frac{\sqrt{3}}{1}$



$\theta = \frac{\pi}{6}$

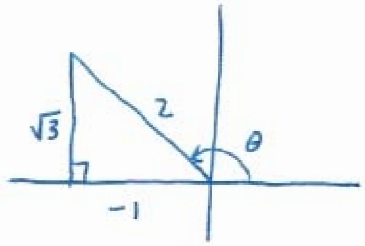
$\underline{\underline{\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}}}$

8-c) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\theta = \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\Downarrow$

$\cot\theta = \frac{-1}{\sqrt{3}}$



$\theta = \frac{2\pi}{3}$

$\underline{\underline{\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}}}$

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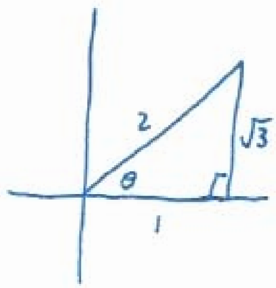
10) $\sec\big(\underbrace{\cos^{-1}\tfrac{1}{2}}_{\theta}\big)$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\cos\theta = \frac{1}{2}$



$\underline{\underline{\sec\left(\cos^{-1}\frac{1}{2}\right) = \frac{2}{1} = 2}}$

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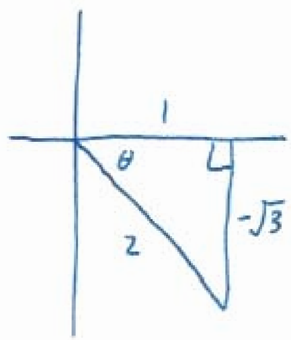
12) $\cot\Big(\underbrace{\sin^{-1}\left(-\tfrac{\sqrt{3}}{2}\right)}_{\theta}\Big)$

$\theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$\Downarrow$

$\sin\theta = \frac{-\sqrt{3}}{2}$



$\cot\left(\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right) = \frac{1}{-\sqrt{3}}$

$= \underline{\underline{\frac{-1}{\sqrt{3}}}}$

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14) $\lim\limits_{x \to -1^+} \cos^{-1}x = \cos^{-1}(-1^+) = \underline{\underline{\pi}}$

16) $\lim\limits_{x \to -\infty} \tan^{-1}x = \underline{\underline{\frac{-\pi}{2}}}$

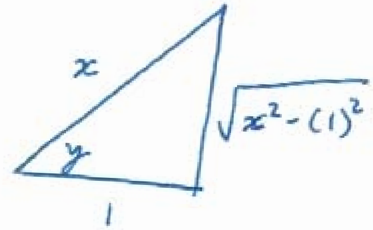
18) $\lim\limits_{x \to -\infty} \sec^{-1}x = \lim\limits_{x \to -\infty} \cos^{-1}\left(\frac{1}{x}\right) = \cos^{-1}(0^-) = \underline{\underline{\frac{\pi}{2}}}$

20) $\lim\limits_{x \to -\infty} \csc^{-1}x = \lim\limits_{x \to -\infty} \sin^{-1}\left(\frac{1}{x}\right) = \sin^{-1}(0^-) = \underline{\underline{0}}$

22) $y = \cos^{-1}\left(\frac{1}{x}\right)$

$\Downarrow$

$\cos y = \frac{1}{x} = x^{-1}$



$\left[-\sin y \frac{dy}{dx}\right] = \left[-1x^{-2}\right]$

$-\sin y \frac{dy}{dx} = \frac{-1}{x^2}$

$\frac{dy}{dx} = \frac{-1}{x^2(-\sin y)} = \frac{1}{x^2 \sin y}$

$= \frac{1}{x^2\left(\frac{\sqrt{x^2-(1)^2}}{x}\right)} = \underline{\underline{\frac{1}{|x|\sqrt{x^2-1}}}}$

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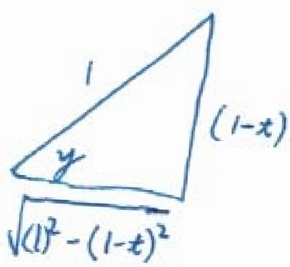
24) $y = \sin^{-1}(1-t)$

$\Downarrow$

$\sin y = 1-t = \frac{1-t}{1}$



$\left[\cos y \frac{dy}{dt}\right] = \left[-1\right]$

$\frac{dy}{dt} = \frac{-1}{\cos y} = \frac{-1}{\left(\frac{\sqrt{(1)^2-(1-t)^2}}{1}\right)} = \underline{\underline{\frac{-1}{\sqrt{1-(1-t)^2}}}}$

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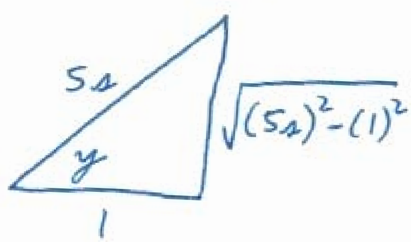
26) $y = \sec^{-1} 5s$

$\Downarrow$

$\sec y = 5s = \frac{5s}{1}$



$\left[\sec y \tan y \frac{dy}{ds}\right] = \left[5\right]$

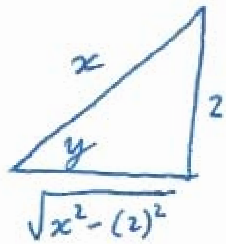
$\frac{dy}{ds} = \frac{5}{\sec y \tan y} = \frac{5}{\left(\frac{5s}{1}\right)\left(\frac{\sqrt{(5s)^2-(1)^2}}{1}\right)} = \underline{\underline{\frac{1}{|s|\sqrt{25s^2-1}}}}$

28) $y = \csc^{-1}\frac{x}{2}$

$\Downarrow$

$\csc y = \frac{x}{2}$

$\left[-\csc y \cot y \frac{dy}{dx}\right] = \left[\frac{1}{2}\right]$



$\frac{dy}{dx} = \frac{-1}{2\csc y \cot y}$

$= \frac{-1}{2\left(\frac{x}{2}\right)\left(\frac{\sqrt{x^2-(2)^2}}{2}\right)}$

$= \frac{-1}{\frac{x\sqrt{x^2-4}}{2}} = \underline{\underline{\frac{-2}{|x|\sqrt{x^2-4}}}}$

---

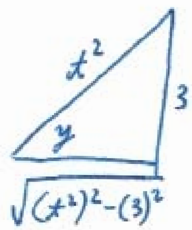
30) $y = \sin^{-1}\frac{3}{t^2}$

$\Downarrow$

$\sin y = \frac{3}{t^2} = 3t^{-2}$

$\left[\cos y \frac{dy}{dt}\right] = \left[-6t^{-3}\right]$

$\cos y \frac{dy}{dt} = \frac{-6}{t^3}$



$\frac{dy}{dt} = \frac{-6}{t^3 \cos y}$

$= \frac{-6}{t^3\left(\frac{\sqrt{(t^2)^2-(3)^2}}{t^2}\right)}$

$= \underline{\underline{\frac{-6}{t\sqrt{t^4-9}}}}$

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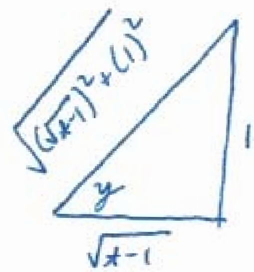
32) $y = \cot^{-1}\sqrt{t-1}$

$\Downarrow$

$\cot y = \sqrt{t-1} = (t-1)^{\frac{1}{2}}$

$\left[-\csc^2 y \frac{dy}{dt}\right] = \left[\frac{1}{2}(t-1)^{-\frac{1}{2}}(1)\right]$

$-\csc^2 y \frac{dy}{dt} = \frac{1}{2\sqrt{t-1}}$



$\frac{dy}{dt} = \frac{-1}{2\sqrt{t-1}\csc^2 y}$

$= \frac{-1}{2\sqrt{t-1}\left(\frac{\sqrt{(\sqrt{t-1})^2+(1)^2}}{1}\right)^2}$

$= \frac{-1}{2\sqrt{t-1}\,((t-1)+1)}$

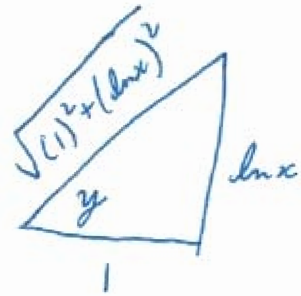
$= \underline{\underline{\frac{-1}{2t\sqrt{t-1}}}}$

34) $y = \tan^{-1}(\ln x)$

$$\Downarrow$$

$\tan y = \ln x$

$$\left[\sec^2 y \frac{dy}{dx}\right] = \left[\frac{1}{x}(1)\right]$$



$$\frac{dy}{dx} = \frac{1}{x \sec^2 y}$$

$$= \frac{1}{x\left(\frac{\sqrt{(1)^2+(\ln x)^2}}{1}\right)^2}$$

$$= \frac{1}{x(1+(\ln x)^2)}$$

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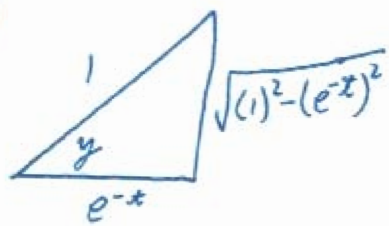
36) $y = \cos^{-1}(e^{-t})$

$$\Downarrow$$

$\cos y = e^{-t}$

$$\left[-\sin y \frac{dy}{dt}\right] = \left[e^{-t}(-1)\right]$$



$$\frac{dy}{dt} = \frac{e^{-t}}{\sin y}$$

$$= \frac{e^{-t}}{\left(\frac{\sqrt{(1)^2-(e^{-t})^2}}{1}\right)}$$

$$= \frac{e^{-t}}{\sqrt{1-e^{-2t}}}$$

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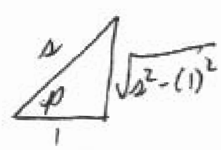
38) $p = \sec^{-1} s$

$$\Downarrow$$

$\sec p = s$



$$\left[\sec p \tan p \frac{dp}{ds}\right] = [1]$$

$$\frac{dp}{ds} = \frac{1}{\sec p \tan p}$$

$$= \frac{1}{\left(\frac{s}{1}\right)\left(\frac{\sqrt{s^2-(1)^2}}{1}\right)} = \frac{1}{|s|\sqrt{s^2-1}}$$

$$y = \sqrt{s^2-1} - \sec^{-1} s$$

$$= (s^2-1)^{\frac{1}{2}} - \sec^{-1} s$$

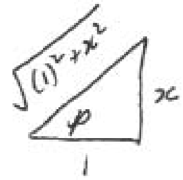
$$\frac{dy}{ds} = \left[\frac{1}{2}(s^2-1)^{-\frac{1}{2}}(2s)\right] - \left[\frac{1}{|s|\sqrt{s^2-1}}\right]$$

$$= \frac{s}{\sqrt{s^2-1}} - \frac{1}{|s|\sqrt{s^2-1}}$$

40) $p = \cot^{-1}\frac{1}{x}$

$\Downarrow$

$\cot p = \frac{1}{x} = x^{-1}$



$\left[-\csc^2 p \frac{dp}{dx}\right] = \left[-1x^{-2}\right]$

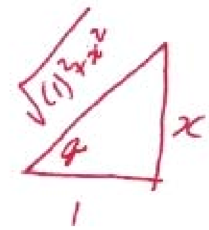
$$\frac{dp}{dx} = \frac{1}{x^2\csc^2 p}$$

$$= \frac{1}{x^2\left(\frac{\sqrt{(1)^2+x^2}}{x}\right)^2} = \frac{1}{1+x^2}$$

$q = \tan^{-1} x$

$\Downarrow$

$\tan q = x = \frac{x}{1}$



$\left[\sec^2 q \frac{dq}{dx}\right] = [1]$

$$\frac{dq}{dx} = \frac{1}{\sec^2 q} = \frac{1}{\left(\frac{\sqrt{(1)^2+x^2}}{1}\right)^2} = \frac{1}{1+x^2}$$

$y = \cot^{-1}\frac{1}{x} - \tan^{-1} x$

$$\frac{dy}{dx} = \left[\frac{1}{1+x^2}\right] - \left[\frac{1}{1+x^2}\right] = \underline{\underline{0}}$$

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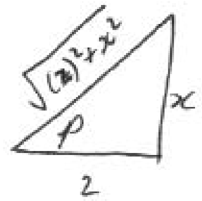
42) $p = \tan^{-1}\left(\frac{x}{2}\right)$

$\Downarrow$

$\tan p = \frac{x}{2}$



$\left[\sec^2 p \frac{dp}{dx}\right] = \left[\frac{1}{2}\right]$

$$\frac{dp}{dx} = \frac{1}{2\sec^2 p} = \frac{1}{2\left(\frac{\sqrt{(2)^2+x^2}}{2}\right)^2}$$

$$= \frac{1}{\frac{4+x^2}{2}} = \frac{2}{4+x^2}$$

$y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$

$$\frac{dy}{dx} = \left[\frac{1}{x^2+4}(2x)\right] - \left\{(x)\left[\frac{2}{4+x^2}\right] + \left(\tan^{-1}\left(\frac{x}{2}\right)\right)[1]\right\}$$

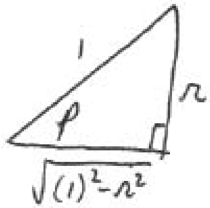
$$= \frac{2x}{x^2+4} - \left\{\frac{2x}{4+x^2} + \tan^{-1}\left(\frac{x}{2}\right)\right\} = \frac{2x}{x^2+4} - \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right)$$

$$= \underline{\underline{-\tan^{-1}\left(\frac{x}{2}\right)}}$$

44) $p = \sin^{-1} r$

$\Downarrow$

$\sin p = r$



$$\left[\cos p \frac{dp}{dr}\right] = [1]$$

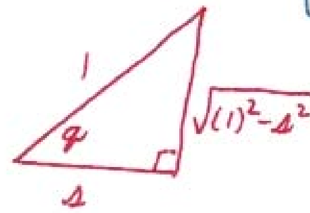
$$\frac{dp}{dr} = \frac{1}{\cos p} = \frac{1}{\left(\frac{\sqrt{(1)^2 - r^2}}{1}\right)} = \frac{1}{\sqrt{1-r^2}}$$

let $r = x + y$

$q = \cos^{-1} s$

$\Downarrow$

$\cos q = s$



$$\left[-\sin q \frac{dq}{ds}\right] = [1]$$

$$\frac{dq}{ds} = \frac{-1}{\sin q} = \frac{-1}{\left(\frac{\sqrt{(1)^2 - s^2}}{1}\right)} = \frac{-1}{\sqrt{1-s^2}}$$

let $s = x - y$

$$\sin^{-1}(x+y) + \cos^{-1}(x-y) = \frac{5\pi}{6}$$

$$\left[\frac{1}{\sqrt{1-(x+y)^2}}\left(1 + 1\frac{dy}{dx}\right)\right] + \left[\frac{-1}{\sqrt{1-(x-y)^2}}\left(1 - 1\frac{dy}{dx}\right)\right] = [0]$$

$$\frac{1}{\sqrt{1-(x+y)^2}} + \left(\frac{1}{\sqrt{1-(x+y)^2}}\right)\frac{dy}{dx} - \frac{1}{\sqrt{1-(x-y)^2}} + \left(\frac{1}{\sqrt{1-(x-y)^2}}\right)\frac{dy}{dx} = 0$$

$$\left(\frac{1}{\sqrt{1-(x+y)^2}}\right)\frac{dy}{dx} + \left(\frac{1}{\sqrt{1-(x-y)^2}}\right)\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-y)^2}} - \frac{1}{\sqrt{1-(x+y)^2}}$$

$$\frac{dy}{dx}\left(\frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{\sqrt{1-(x-y)^2}}\right) = \frac{1}{\sqrt{1-(x-y)^2}} - \frac{1}{\sqrt{1-(x+y)^2}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{\sqrt{1-(x-y)^2}} - \frac{1}{\sqrt{1-(x+y)^2}}}{\frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{\sqrt{1-(x-y)^2}}} = \left(\frac{\frac{1}{\sqrt{1-(x-y)^2}} - \frac{1}{\sqrt{1-(x+y)^2}}}{\frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{\sqrt{1-(x-y)^2}}}\right)\left(\frac{\frac{\sqrt{1-(x-y)^2}\,\sqrt{1-(x+y)^2}}{1}}{\frac{\sqrt{1-(x-y)^2}\,\sqrt{1-(x+y)^2}}{1}}\right)$$

44) continued

$$\frac{dy}{dx} = \frac{\sqrt{1-(x+y)^2} - \sqrt{1-(x-y)^2}}{\sqrt{1-(x-y)^2} + \sqrt{1-(x+y)^2}}$$

at $P(0, \frac{1}{2})$

$$\left.\frac{dy}{dx}\right|_{x=0,\, y=\frac{1}{2}} = \frac{\sqrt{1-((0)+(\frac{1}{2}))^2} - \sqrt{1-((0)-(\frac{1}{2}))^2}}{\sqrt{1-((0)-(\frac{1}{2}))^2} + \sqrt{1-((0)+(\frac{1}{2}))^2}}$$

$$= \frac{\sqrt{1-(\frac{1}{2})^2} - \sqrt{1-(\frac{-1}{2})^2}}{\sqrt{1-(\frac{-1}{2})^2} + \sqrt{1-(\frac{1}{2})^2}} = \frac{\sqrt{1-\frac{1}{4}} - \sqrt{1-\frac{1}{4}}}{\sqrt{1-\frac{1}{4}} + \sqrt{1-\frac{1}{4}}}$$

$$= \frac{\sqrt{\frac{3}{4}} - \sqrt{\frac{3}{4}}}{\sqrt{\frac{3}{4}} + \sqrt{\frac{3}{4}}} = 0$$

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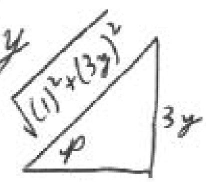
46) $p = \tan^{-1} 3y$



$\Downarrow$

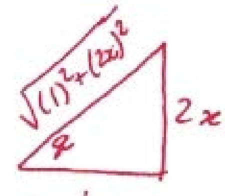
$\tan p = 3y$

$$\left[\sec^2 p \frac{dp}{dy}\right] = [3]$$

$$\frac{dp}{dy} = \frac{3}{\sec^2 p} = \frac{3}{\left(\frac{\sqrt{(1)^2+(3y)^2}}{1}\right)^2}$$

$$= \frac{3}{1+(3y)^2}$$

$q = \tan^{-1} 2x$



$\Downarrow$

$\tan q = 2x$

$$\left[\sec^2 q \frac{dq}{dx}\right] = [2]$$

$$\frac{dq}{dx} = \frac{2}{\sec^2 q} = \frac{2}{\left(\frac{\sqrt{(1)^2+(2x)^2}}{1}\right)^2}$$

$$= \frac{2}{1+(2x)^2}$$

46) continued

$$16(\tan^{-1}3y)^2 + 9(\tan^{-1}2x)^2 = 2\pi^2$$

$$16\left[2(\tan^{-1}3y)^1\left(\frac{3}{1+(3y)^2}\right)\frac{dy}{dx}\right] + 9\left[2(\tan^{-1}2x)\left(\frac{2}{1+(2x)^2}\right)\right] = [0]$$

$$\frac{16(2)(3)\tan^{-1}3y}{1+(3y)^2}\frac{dy}{dx} = \frac{-9(2)(2)\tan^{-1}2x}{1+(2x)^2}$$

$$\frac{dy}{dx} = \frac{-9(2)(2)(\tan^{-1}2x)(1+(3y)^2)}{16(2)(3)(\tan^{-1}3y)(1+(2x)^2)} = \frac{-3(\tan^{-1}2x)(1+(3y)^2)}{8(\tan^{-1}3y)(1+(2x)^2)}$$

at $P\left(\frac{\sqrt{3}}{2}, \frac{1}{3}\right)$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\sqrt{3}}{2},\, y=\frac{1}{3}} = \frac{-3\left(\tan^{-1}2\left(\frac{\sqrt{3}}{2}\right)\right)\left(1+\left(3\left(\frac{1}{3}\right)\right)^2\right)}{8\left(\tan^{-1}3\left(\frac{1}{3}\right)\right)\left(1+\left(2\left(\frac{\sqrt{3}}{2}\right)\right)^2\right)} = \frac{-3(\tan^{-1}\sqrt{3})(1+(1)^2)}{8(\tan^{-1}1)(1+(\sqrt{3})^2)}$$

$$= \frac{-3\left(\frac{\pi}{3}\right)(2)}{8\left(\frac{\pi}{4}\right)(4)} = \frac{-2\pi}{8\pi} = \underline{\underline{\frac{-1}{4}}}$$

52-a) $\csc^{-1}\left(\frac{1}{2}\right)$ not defined because $|\csc\theta| > 1$

$\theta = \csc^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\csc\theta = \frac{1}{2}$

52-b) $\csc^{-1} 2$

$\theta = \csc^{-1} 2$ defined because $|\csc\theta| > 1$

$\Downarrow$

$\csc\theta = 2$

54-a) $\cot^{-1}\left(\frac{-1}{2}\right)$

$\theta = \cot^{-1}\left(\frac{-1}{2}\right)$ defined because $-\infty < \cot\theta < \infty$

$\Downarrow$

$\cot\theta = \frac{-1}{2}$

54-b) $\cos^{-1}(-5)$

$\theta = \cos^{-1}(-5)$ not defined because $|\cos\theta| < 1$

$\Downarrow$

$\cos\theta = -5$