MATH 20100 section 3.7

Implicit Differentiation

1) Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.
2) Collect the terms with $\frac{dy}{dx}$ on one side of the equation and solve for $\frac{dy}{dx}$.

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2) $x^3 + y^3 = 18xy$

$$\left[3x^2\right] + \left[3y^2\frac{dy}{dx}\right] = 18\left\{(x)\left[1\frac{dy}{dx}\right] + (y)\left[1\right]\right\}$$

$$3x^2 + 3y^2\frac{dy}{dx} = 18x\frac{dy}{dx} + 18y$$

$$3y^2\frac{dy}{dx} - 18x\frac{dy}{dx} = 18y - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 - 18x\right) = 18y - 3x^2$$

$$\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x} = \frac{3(6y - x^2)}{3(y^2 - 6x)} = \frac{6y - x^2}{y^2 - 6x}$$

4) $x^3 - xy + y^3 = 1$

$$[3x^2] - \{(x)[1\frac{dy}{dx}] + (y)[1]\} + [3y^2\frac{dy}{dx}] = 0$$

$$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$$

$$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

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6) $(3xy + 7)^2 = 6y$

$$[2(3xy+7)'((3x)[1\frac{dy}{dx}] + (y)[3])] = 6[1\frac{dy}{dx}]$$

$$6x(3xy+7)\frac{dy}{dx} + 6y(3xy+7) = 6\frac{dy}{dx}$$

$$6y(3xy+7) = 6\frac{dy}{dx} - 6x(3xy+7)\frac{dy}{dx}$$

$$6y(3xy+7) = \frac{dy}{dx}(6 - 6x(3xy+7))$$

$$\frac{6y(3xy+7)}{6 - 6x(3xy+7)} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6y(3xy+7)}{6(1 - x(3xy+7))} = \frac{y(3xy+7)}{1 - x(3xy+7)} = \frac{3xy^2 + 7y}{1 - 3x^2y - 7x}$$

8) $x^3 = \dfrac{2x - y}{x + 3y} \Rightarrow x^3(x+3y) = 2x - y$

$x^4 + 3x^3 y = 2x - y$

$[4x^3] + \{(3x^3)[1\frac{dy}{dx}] + (y)[9x^2]\} = 2[1] - [1\frac{dy}{dx}]$

$4x^3 + 3x^3 \dfrac{dy}{dx} + 9x^2 y = 2 - \dfrac{dy}{dx}$

$3x^3 \dfrac{dy}{dx} + \dfrac{dy}{dx} = 2 - 4x^3 - 9x^2 y$

$\dfrac{dy}{dx}(3x^3 + 1) = 2 - 4x^3 - 9x^2 y$

$\dfrac{dy}{dx} = \dfrac{2 - 4x^3 - 9x^2 y}{3x^3 - 1}$

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10) $xy = \cot(xy)$

$\{(x)[1\frac{dy}{dx}] + (y)[1]\} = -\csc^2(xy)(\{(x)[1\frac{dy}{dx}] + (y)[1]\})$

$x\dfrac{dy}{dx} + y = -x\csc^2(xy)\dfrac{dy}{dx} - y\csc^2(xy)$

$x\dfrac{dy}{dx} + x\csc^2(xy)\dfrac{dy}{dx} = -y - y\csc^2(xy)$

$\dfrac{dy}{dx}(x + x\csc^2(xy)) = -y - y\csc^2(xy)$

$\dfrac{dy}{dx} = \dfrac{-y - y\csc^2(xy)}{x + x\csc^2(xy)} = \dfrac{-y(1 + \csc^2(xy))}{x(1 + \csc^2(xy))} = \dfrac{-y}{x}$

12) $x^4 + \sin y = x^3 y^2$

$$[4x^3] + \left[\cos y\left(\frac{dy}{dx}\right)\right] = \left\{(x^3)\left[2y\frac{dy}{dx}\right] + (y^2)[3x^2]\right\}$$

$$4x^3 + \cos y\frac{dy}{dx} = 2x^3y\frac{dy}{dx} + 3x^2y^2$$

$$\cos y\frac{dy}{dx} - 2x^3y\frac{dy}{dx} = 3x^2y^2 - 4x^3$$

$$\frac{dy}{dx}(\cos y - 2x^3y) = 3x^2y^2 - 4x^3$$

$$\frac{dy}{dx} = \frac{3x^2y^2 - 4x^3}{\cos y - 2x^3y}$$

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14) $x\cos(2x+3y) = y\sin x$

$$\left\{(x)\left[-\sin(2x+3y)\left(2+3\frac{dy}{dx}\right)\right] + (\cos(2x+3y))[1]\right\}$$

$$= \left\{(y)[\cos x(1)] + (\sin x)\left[1\frac{dy}{dx}\right]\right\}$$

$$-2x\sin(2x+3y) - 3x\sin(2x+3y)\frac{dy}{dx} + \cos(2x+3y)$$

$$= y\cos x + \sin x\frac{dy}{dx}$$

$$\cos(2x+3y) - 2x\sin(2x+3y) - y\cos x = \sin x\frac{dy}{dx} + 3x\sin(2x+3y)\frac{dy}{dx}$$

$$\cos(2x+3y) - 2x\sin(2x+3y) - y\cos x = \frac{dy}{dx}(\sin x + 3x\sin(2x+3y))$$

$$\frac{\cos(2x+3y) - 2x\sin(2x+3y) - y\cos x}{\sin x + 3x\sin(2x+3y)} = \frac{dy}{dx}$$

16) $e^{x^2y} = 2x + 2y$

$$\left[e^{x^2y}\left(\left\{(x^2)\left[1\frac{dy}{dx}\right] + (y)\left[2x\right]\right\}\right)\right] = 2\left[1\right] + 2\left[1\frac{dy}{dx}\right]$$

$$x^2e^{x^2y}\frac{dy}{dx} + 2xy\,e^{x^2y} = 2 + 2\frac{dy}{dx}$$

$$x^2e^{x^2y}\frac{dy}{dx} - 2\frac{dy}{dx} = 2 - 2xy\,e^{x^2y}$$

$$\frac{dy}{dx}\left(x^2e^{x^2y} - 2\right) = 2 - 2xy\,e^{x^2y}$$

$$\frac{dy}{dx} = \frac{2 - 2xy\,e^{x^2y}}{x^2e^{x^2y} - 2}$$

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18) $r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4}$

$$r - 2\theta^{\frac{1}{2}} = \frac{3}{2}\theta^{\frac{2}{3}} + \frac{4}{3}\theta^{\frac{3}{4}}$$

$$\left[1\frac{dr}{d\theta}\right] - 2\left[\frac{1}{2}\theta^{-\frac{1}{2}}\right] = \frac{3}{2}\left[\frac{2}{3}\theta^{-\frac{1}{3}}\right] + \frac{4}{3}\left[\frac{3}{4}\theta^{-\frac{1}{4}}\right]$$

$$\frac{dr}{d\theta} - \frac{1}{\sqrt{\theta}} = \frac{1}{\sqrt[3]{\theta}} + \frac{1}{\sqrt[4]{\theta}}$$

$$\frac{dr}{d\theta} = \frac{1}{\sqrt{\theta}} + \frac{1}{\sqrt[3]{\theta}} + \frac{1}{\sqrt[4]{\theta}}$$

20) $\cos r + \cot\theta = e^{r\theta}$

$$\left[-\sin r \frac{dr}{d\theta}\right] + \left[-\csc^2\theta\,(1)\right] = \left[e^{r\theta}\left(\left\{(r)[1] + (\theta)\left[1\frac{dr}{d\theta}\right]\right\}\right)\right]$$

$$-\sin r\frac{dr}{d\theta} - \csc^2\theta = re^{r\theta} + \theta e^{r\theta}\frac{dr}{d\theta}$$

$$-\csc^2\theta - re^{r\theta} = \theta e^{r\theta}\frac{dr}{d\theta} + \sin r\frac{dr}{d\theta}$$

$$-\csc^2\theta - re^{r\theta} = \frac{dr}{d\theta}\left(\theta e^{r\theta} + \sin r\right)$$

$$\frac{-\csc^2\theta - re^{r\theta}}{\theta e^{r\theta} + \sin r} = \frac{dr}{d\theta}$$

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22) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

$$\left[\frac{2}{3}x^{-\frac{1}{3}}\right] + \left[\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx}\right] = 0$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0 \Rightarrow$$

$$\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = \frac{-2}{3}x^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{\frac{-2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = \frac{-x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = \frac{-y^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \frac{-\sqrt[3]{y}}{\sqrt[3]{x}}$$

$$\frac{d^2y}{dx^2} = \frac{\left(x^{\frac{1}{3}}\right)\left[-\frac{1}{3}y^{-\frac{2}{3}}\frac{dy}{dx}\right] - \left(-y^{\frac{1}{3}}\right)\left[\frac{1}{3}x^{-\frac{2}{3}}\right]}{\left(x^{\frac{1}{3}}\right)^2}$$

$$= \frac{\left(\sqrt[3]{x}\right)\left[\frac{-1}{3\left(\sqrt[3]{y}\right)^2}\left(\frac{dy}{dx}\right)\right] + \left(\sqrt[3]{y}\right)\left[\frac{1}{3\left(\sqrt[3]{x}\right)^2}\right]}{\left(\sqrt[3]{x}\right)^2} = \frac{\frac{-\sqrt[3]{x}}{3\left(\sqrt[3]{y}\right)^2}\left(\frac{-\sqrt[3]{y}}{\sqrt[3]{x}}\right) + \frac{\sqrt[3]{y}}{3\left(\sqrt[3]{x}\right)^2}}{\left(\sqrt[3]{x}\right)^2}$$

$$= \left(\frac{\frac{\sqrt[3]{x}}{3\left(\sqrt[3]{x}\right)\left(\sqrt[3]{y}\right)} + \frac{\sqrt[3]{y}}{3\left(\sqrt[3]{x}\right)^2}}{\frac{\left(\sqrt[3]{x}\right)^2}{1}}\right)\left(\frac{\frac{3\left(\sqrt[3]{x}\right)^2\left(\sqrt[3]{y}\right)}{1}}{\frac{3\left(\sqrt[3]{x}\right)^2\left(\sqrt[3]{y}\right)}{1}}\right) = \frac{\left(\sqrt[3]{x}\right)^2 + \left(\sqrt[3]{y}\right)^2}{3\left(\sqrt[3]{x}\right)^4\left(\sqrt[3]{y}\right)}$$

24) $y^2 - 2x = 1 - 2y$

$$\left[2y\frac{dy}{dx}\right] - [2] = [0] - \left[2\frac{dy}{dx}\right] \Rightarrow$$

$$2y\frac{dy}{dx} - 2 = -2\frac{dy}{dx}$$

$$2y\frac{dy}{dx} + 2\frac{dy}{dx} = 2$$

$$\frac{dy}{dx}(2y+2) = 2$$

$$\frac{dy}{dx} = \frac{2}{2y+2} = \frac{2}{2(y+1)} = \frac{1}{y+1}$$

$$\frac{dy}{dx} = (y+1)^{-1}$$

$$\frac{d^2y}{dx^2} = \left[-1(y+1)^{-2}\left(1\frac{dy}{dx}\right)\right]$$

$$= \frac{-\frac{dy}{dx}}{(y+1)^2} = \frac{-\left(\frac{1}{y+1}\right)}{(y+1)^2} = \frac{-1}{(y+1)^3}$$

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26) $xy + y^2 = 1$

$$\left\{(x)\left[1\frac{dy}{dx}\right] + (y)[1]\right\} + \left[2y\frac{dy}{dx}\right] = 0$$

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0 \Rightarrow$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$$

$$\frac{dy}{dx}(x+2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2y}$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left[-1\frac{dy}{dx}\right] - (-y)\left[1+2\frac{dy}{dx}\right]}{(x+2y)^2} = \frac{-x\frac{dy}{dx} - 2y\frac{dy}{dx} + y + 2y\frac{dy}{dx}}{(x+2y)^2}$$

$$= \frac{-x\frac{dy}{dx} + y}{(x+2y)^2} = \frac{-x\left(\frac{-y}{x+2y}\right) + y}{(x+2y)^2} = \left(\frac{\frac{xy}{x+2y} + \frac{y}{1}}{\frac{(x+2y)^2}{1}}\right)\left(\frac{\frac{x+2y}{1}}{\frac{x+2y}{1}}\right)$$

$$= \frac{xy + y(x+2y)}{(x+2y)^3} = \frac{xy + xy + 2y^2}{(x+2y)^3} = \frac{2xy + 2y^2}{(x+2y)^3} = \frac{2y(x+y)}{(x+2y)^3}$$

28) $\ln y = xe^y - 2$

$\downarrow$ see section 3.8

$$\left[\frac{1}{y}\frac{dy}{dx}\right] = \left\{(x)\left[e^y\frac{dy}{dx}\right] + (e^y)[1]\right\} - [0]$$

$$\frac{1}{y}\frac{dy}{dx} = xe^y\frac{dy}{dx} + e^y \qquad \Rightarrow \qquad \frac{dy}{dx}\left(\frac{1}{y} - xe^y\right) = e^y$$

$$\frac{1}{y}\frac{dy}{dx} - xe^y\frac{dy}{dx} = e^y \qquad \frac{dy}{dx} = \frac{e^y}{\frac{1}{y} - xe^y} = \frac{e^y}{y^{-1} - xe^y}$$

$$\frac{d^2y}{dx^2} = \frac{(y^{-1} - xe^y)\left[e^y\frac{dy}{dx}\right] - (e^y)\left[-1y^{-2} - \left\{(x)\left[e^y\frac{dy}{dx}\right] + (e^y)[1]\right\}\right]}{(y^{-1} - xe^y)^2}$$

$$= \frac{\left(\frac{1}{y} - xe^y\right)\left[e^y\left(\frac{e^y}{\frac{1}{y} - xe^y}\right)\right] - (e^y)\left[\frac{-1}{y^2} - xe^y\left(\frac{e^y}{\frac{1}{y} - xe^y}\right) - e^y\right]}{\left(\frac{1}{y} - xe^y\right)^2}$$

$$= \frac{e^{2y} + \frac{e^y}{y^2} + \frac{xe^{3y}}{\frac{1}{y} - xe^y} + e^{2y}}{\left(\frac{1}{y} - xe^y\right)^2}$$

$$= \left(\frac{\frac{2e^{2y}}{1} + \frac{e^y}{y^2} + \frac{xe^{3y}}{\frac{1}{y} - xe^y}}{\frac{\left(\frac{1}{y} - xe^y\right)^2}{1}}\right)\left(\frac{\frac{y^2\left(\frac{1}{y} - xe^y\right)}{1}}{\frac{y^2\left(\frac{1}{y} - xe^y\right)}{1}}\right)$$

$$= \frac{2y^2e^{2y}\left(\frac{1}{y} - xe^y\right) + e^y\left(\frac{1}{y} - xe^y\right) + xy^2e^{3y}}{y^2\left(\frac{1}{y} - xe^y\right)^3}$$

28) continued

$$\frac{d^2y}{dx^2} = \frac{2ye^{2y} - 2xy^2e^{3y} + \frac{e^y}{y} - xe^{2y} + xy^2e^{3y}}{y^2\left(\frac{1}{y} - xe^y\right)^3}$$

$$= \left(\frac{\underbrace{2ye^{2y}}_{1} - \underbrace{xy^2e^{3y}}_{1} + \frac{e^y}{y} - \underbrace{xe^{2y}}_{1}}{\underbrace{y^2\left(\frac{1}{y} - xe^y\right)^3}_{1}}\right)\left(\frac{\frac{y}{1}}{\frac{y}{1}}\right)$$

$$= \frac{2y^2e^{2y} - xy^3e^{3y} + e^y - xye^{2y}}{y^3\left(\frac{1}{y} - xe^y\right)^3}$$

$$= \frac{2y^2e^{2y} - xy^3e^{3y} + e^y - xye^{2y}}{\left(y\left(\frac{1}{y} - xe^y\right)\right)^3}$$

$$= \frac{2y^2e^{2y} - xy^3e^{3y} + e^y - xye^{2y}}{(1 - xye^y)^3}$$

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30) $xy + y^2 = 1$ $\qquad \left.\frac{d^2y}{dx^2}\right|_{x=0,y=-1} = ?$

from exercise 26, $\frac{d^2y}{dx^2} = \frac{2y(x+y)}{(x+2y)^3}$

$$\left.\frac{d^2y}{dx^2}\right|_{x=0,y=-1} = \frac{2(-1)((0)+(-1))}{((0)+2(-1))^3} = \frac{2(-1)(-1)}{(-2)^3} = \frac{-1}{4}$$

32) $(x^2+y^2)^2 = (x-y)^2$ slope at $(1,0)$ and $(1,-1)$

$$\left[2(x^2+y^2)'\left(2x+2y\frac{dy}{dx}\right)\right] = \left[2(x-y)'\left(1-1\frac{dy}{dx}\right)\right]$$

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx} = 2(x-y)-2(x-y)\frac{dy}{dx}$$

$$4y(x^2+y^2)\frac{dy}{dx}+2(x-y)\frac{dy}{dx} = 2(x-y)-4x(x^2+y^2)$$

$$\frac{dy}{dx}\left(4y(x^2+y^2)+2(x-y)\right) = 2(x-y)-4x(x^2+y^2)$$

$$\frac{dy}{dx} = \frac{2(x-y)-4x(x^2+y^2)}{4y(x^2+y^2)+2(x-y)} = \frac{2\left((x-y)-2x(x^2+y^2)\right)}{2\left(2y(x^2+y^2)+(x-y)\right)}$$

$$= \frac{(x-y)-2x(x^2+y^2)}{2y(x^2+y^2)+(x-y)}$$

at $(1,0)$: $\left.\frac{dy}{dx}\right|_{x=1,y=0} = \frac{((1)-(0))-2(1)((1)^2+(0)^2)}{2(0)((1)^2+(0)^2)+((1)-(0))} = \frac{(1)-2(1)(1)}{2(0)(1)+(1)} = \frac{-1}{1} = -1$

at $(1,-1)$: $\left.\frac{dy}{dx}\right|_{x=1,y=-1} = \frac{((1)-(-1))-2(1)((1)^2+(-1)^2)}{2(-1)((1)^2+(-1)^2)+((1)-(-1))} = \frac{(2)-2(1)(2)}{2(-1)(2)+(2)}$

$$= \frac{2-4}{-4+2} = \frac{-2}{-2} = 1$$

34) $x^2 + y^2 = 25$, $(3, -4)$

$[2x] + [2y \frac{dy}{dx}] = 0$

$2x + 2y \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

a) $m = \frac{dy}{dx}\Big|_{x=3,\,y=-4} = \frac{-(3)}{(-4)} = \frac{3}{4}$

$y - (-4) = \frac{3}{4}(x - (3))$

$y + 4 = \frac{3}{4}(x - 3)$

$y + 4 = \frac{3}{4}x - \frac{9}{4}$

$y = \frac{3}{4}x - \frac{25}{4}$

b) $m_2 = \frac{-1}{m_1} = \frac{-1}{(\frac{3}{4})} = \frac{-4}{3}$

$y - (-4) = \frac{-4}{3}(x - (3))$

$y + 4 = \frac{-4}{3}(x - 3)$

$y + 4 = \frac{-4}{3}x + 4$

$y = \frac{-4}{3}x$

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36) $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$

$[2y \frac{dy}{dx}] - 2[1] - 4[1 \frac{dy}{dx}] - [0] = 0$

$2y \frac{dy}{dx} - 2 - 4 \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} - 4 \frac{dy}{dx} = 2$

$\frac{dy}{dx}(2y - 4) = 2$

$\frac{dy}{dx} = \frac{2}{2y-4} = \frac{2}{2(y-2)} = \frac{1}{y-2}$

a) $m = \frac{dy}{dx}\Big|_{x=-2,\,y=1} = \frac{1}{(1)-2} = \frac{1}{-1}$

$m = -1$

$y - (1) = -1(x - (-2))$

$y - 1 = -1(x + 2)$

$y - 1 = -x - 2$

$y = -x - 1$

b) $m_2 = \frac{-1}{m_1} = \frac{-1}{(-1)} = 1$

$y - (1) = 1(x - (-2))$

$y - 1 = 1(x + 2)$

$y - 1 = x + 2$

$y = x + 3$

38) $x^2 - \sqrt{3}\,xy + 2y^2 = 5$, $(\sqrt{3}, 2)$

$[2x] - \sqrt{3}\{(x)[1\frac{dy}{dx}] + (y)[1]\} + 2[2y\frac{dy}{dx}] = 0$

$2x - \sqrt{3}x\frac{dy}{dx} - \sqrt{3}y + 4y\frac{dy}{dx} = 0$

$4y\frac{dy}{dx} - \sqrt{3}x\frac{dy}{dx} = \sqrt{3}y - 2x$

$\frac{dy}{dx}(4y - \sqrt{3}x) = \sqrt{3}y - 2x$

$\frac{dy}{dx} = \frac{\sqrt{3}y - 2x}{4y - \sqrt{3}x}$

a) $m = \frac{dy}{dx}\Big|_{x=\sqrt{3},\,y=2} = \frac{\sqrt{3}(2) - 2(\sqrt{3})}{4(2) - \sqrt{3}(\sqrt{3})}$

$= \frac{2\sqrt{3} - 2\sqrt{3}}{8 - 3} = \frac{0}{5} = 0$

$y - (2) = 0(x - (\sqrt{3}))$

$y - 2 = 0(x - \sqrt{3})$

$y - 2 = 0$

$y = 2$ {horizontal line}

b) $x = \sqrt{3}$ {vertical line}

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40) $x\sin(2y) = y\cos(2x)$, $(\frac{\pi}{4}, \frac{\pi}{2})$

$(x)[\cos(2y)(2\frac{dy}{dx})] + (\sin(2y))[1] = (y)[-\sin(2x)(2)] + (\cos(2x))[1\frac{dy}{dx}$

$2x\cos(2y)\frac{dy}{dx} + \sin(2y) = -2y\sin(2x) + \cos(2x)\frac{dy}{dx}$

$\sin(2y) + 2y\sin(2x) = \cos(2x)\frac{dy}{dx} - 2x\cos(2y)\frac{dy}{dx}$

$\sin(2y) + 2y\sin(2x) = \frac{dy}{dx}(\cos(2x) - 2x\cos(2y))$

$\frac{\sin(2y) + 2y\sin(2x)}{\cos(2x) - 2x\cos(2y)} = \frac{dy}{dx}$

a) $m = \frac{dy}{dx}\Big|_{x=\frac{\pi}{4},\,y=\frac{\pi}{2}} = \frac{\sin(2(\frac{\pi}{2})) + 2(\frac{\pi}{2})\sin(2(\frac{\pi}{4}))}{\cos(2(\frac{\pi}{4})) - 2(\frac{\pi}{4})\cos(2(\frac{\pi}{2}))}$

$= \frac{\sin(\pi) + \pi\sin(\frac{\pi}{2})}{\cos(\frac{\pi}{2}) - \frac{\pi}{2}\cos(\pi)} = \frac{(0) + \pi(1)}{(0) - \frac{\pi}{2}(-1)} = 2$

$y - (\frac{\pi}{2}) = 2(x - (\frac{\pi}{4}))$

$y - \frac{\pi}{2} = 2(x - \frac{\pi}{4}) \Rightarrow y = 2x$

$y - \frac{\pi}{2} = 2x - \frac{\pi}{2}$

b) $m_2 = \frac{-1}{m_1} = \frac{-1}{(2)} = \frac{-1}{2}$

$y - (\frac{\pi}{2}) = \frac{-1}{2}(x - (\frac{\pi}{4}))$

$y - \frac{\pi}{2} = \frac{-1}{2}(x - \frac{\pi}{4}) \Rightarrow y = \frac{-1}{2}x + \frac{5\pi}{8}$

$y - \frac{\pi}{2} = \frac{-1}{2}x + \frac{\pi}{8}$

42) $x^2 \cos^2 y - \sin y = 0$, $(0, \pi)$

$$\left\{ (x^2)\left[2\cos y \left(-\sin y \frac{dy}{dx}\right)\right] + (\cos^2 y)[2x] \right\} - \left[\cos y \frac{dy}{dx}\right] = 0$$

$$-2x^2 \sin y \cos y \frac{dy}{dx} + 2x\cos^2 y - \cos y \frac{dy}{dx} = 0$$

$$2x\cos^2 y = 2x^2 \sin y \cos y \frac{dy}{dx} + \cos y \frac{dy}{dx}$$

$$2x\cos^2 y = \frac{dy}{dx}\left(2x^2 \sin y \cos y + \cos y\right)$$

$$\frac{2x\cos^2 y}{2x^2 \sin y \cos y + \cos y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos y\,(2x\cos y)}{\cos y\,(2x^2 \sin y + 1)} = \frac{2x\cos y}{2x^2 \sin y + 1}$$

a) $m = \left.\frac{dy}{dx}\right|_{x=0,\, y=\pi} = \frac{2(0)\cos(\pi)}{2(0)^2 \sin(\pi) + 1} = \frac{2(0)(-1)}{2(0)^2(0)+1} = \frac{0}{1} = 0$

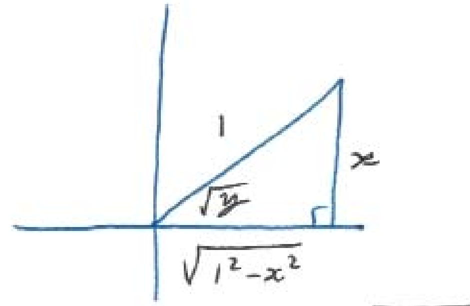
$$y - (\pi) = 0\,(x - (0))$$

$$y - \pi = 0$$

$y = \pi$ {horizontal line}

b) $x = 0$ {vertical line}

58-a) $y = (\sin^{-1} x)^2$



$$\sqrt{y} = \sin^{-1} x$$

$$\Downarrow$$

$$\sin(\sqrt{y}) = x = \frac{x}{1}$$

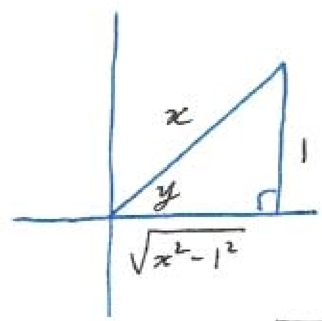
$$\cos(\sqrt{y}) = \frac{\sqrt{1^2 - x^2}}{1}$$

$$\left[\cos(\sqrt{y})\left(\tfrac{1}{2} y^{-1/2} \frac{dy}{dx}\right)\right] = [1]$$

$$\frac{\cos(\sqrt{y})}{2\sqrt{y}} \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{2\sqrt{y}}{\cos\sqrt{y}} = \frac{2\sqrt{(\sin^{-1} x)^2}}{\left(\frac{\sqrt{1^2 - x^2}}{1}\right)} = \frac{2 \sin^{-1} x}{\sqrt{1 - x^2}}$$

58-b) $y = \sin^{-1}\left(\frac{1}{x}\right)$



$$\Downarrow$$

$$\sin y = \frac{1}{x} = x^{-1}$$

$$\left[\cos y \frac{dy}{dx}\right] = \left[-1 x^{-2}\right]$$

$$\cos y = \frac{\sqrt{x^2 - 1^2}}{x}$$

$$\cos y \frac{dy}{dx} = \frac{-1}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2 \cos y} = \frac{-1}{x^2\left(\frac{\sqrt{x^2 - 1^2}}{x}\right)} = \frac{-1}{x\sqrt{x^2 - 1}}$$

We will be using these procedures shown on these pages to avoid memorizing 6 formulas in section 3.9.