MATH 20100 section 3.5

"Basic 3"

$$\frac{d}{dx}(\sin x) = \cos x\,(1) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x\,(1) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x\,(1) = \sec^2 x$$

"Other 3"

$$\frac{d}{dx}(\csc x) = -\csc x \cot x\,(1) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x\,(1) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x\,(1) = -\csc^2 x$$

---

2) $y = \frac{3}{x} + 5\sin x = 3x^{-1} + 5\sin x$

$$\frac{dy}{dx} = 3\left[-1x^{-2}\right] + 5\left[\cos x\,(1)\right] = \frac{-3}{x^2} + 5\cos x$$

---

4) $y = \sqrt{x}\,\sec x + 3 = (x^{\frac{1}{2}})(\sec x) + 3$

$$\frac{dy}{dx} = \left\{(x^{\frac{1}{2}})\left[\sec x \tan x\,(1)\right] + (\sec x)\left[\tfrac{1}{2}x^{-\frac{1}{2}}\right]\right\} + [0]$$

$$= \sqrt{x}\,\sec x \tan x + \frac{\sec x}{2\sqrt{x}}$$

$$= \frac{2x \sec x \tan x + \sec x}{2\sqrt{x}}$$

6) $y = x^2 \cot x - \frac{1}{x^2} = (x^2)(\cot x) - x^{-2}$

$$\frac{dy}{dx} = \{(x^2)[-\csc^2 x(1)] + (\cot x)[2x]\} - [-2x^{-3}]$$

$$= -x^2\csc^2 x + 2x\cot x + \frac{2}{x^3}$$

---

8) $g(x) = \frac{\cos x}{\sin^2 x} = \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) = (\csc x)(\cot x)$

$$\frac{dg}{dx} = (\csc x)[-\csc^2 x(1)] + (\cot x)[-\csc x \cot x(1)]$$

$$= -\csc x\,\{\csc^2 x + \cot^2 x\}$$

---

10) $y = (\sin x + \cos x)\sec x = (\sin x + \cos x)\left(\frac{1}{\cos x}\right)$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} = \tan x + 1$$

$$\frac{dy}{dx} = [\sec^2 x(1)] + [0] = \sec^2 x$$

$$\frac{dy}{dx} = ((\sin x + \cos x))[\sec x \tan x(1)] + (\sec x)\,[[\cos x(1)] + [-\sin x(1)]]$$

$$= (\sin x + \cos x)\left[\left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)\right] + [\cos x - \sin x]\left(\frac{1}{\cos x}\right)$$

$$= \frac{\sin^2 x + \sin x\cos x}{\cos^2 x} + \left(\frac{\cos x - \sin x}{\cos x}\right)\left(\frac{\cos x}{\cos x}\right)$$

$$= \frac{\sin^2 x + \sin x\cos x + \cos^2 x - \sin x\cos x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

12) $y = \dfrac{\cos x}{1+\sin x}$

$$\frac{dy}{dx} = \frac{(1+\sin x)[-\sin x(1)] - (\cos x)[\cos x(1)]}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \frac{-\sin x - (\cos^2 x + \sin^2 x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1-\sin x}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2} = \frac{-1}{1+\sin x}$$

---

14) $y = \dfrac{\cos x}{x} + \dfrac{x}{\cos x}$

$$\frac{dy}{dx} = \left\{\frac{(x)[-\sin x(1)] - (\cos x)[1]}{(x)^2}\right\} + \left\{\frac{(\cos x)[1] - (x)[-\sin x(1)]}{(\cos x)^2}\right\}$$

$$= \left\{\frac{-x\sin x - \cos x}{x^2}\right\} + \left\{\frac{\cos x + x\sin x}{\cos^2 x}\right\}$$

$$= \frac{\cos x + x\sin x}{\cos^2 x} - \frac{x\sin x + \cos x}{x^2}$$

16) $y = x^2\cos x - 2x\sin x - 2\cos x$

$$\frac{dy}{dx} = \left\{(x^2)[-\sin x(1)] + (\cos x)[2x]\right\} - \left\{(2x)[\cos x(1)] + (\sin x)[2]\right\} - 2[-\sin x(1)]$$

$$= \left\{-x^2\sin x + 2x\cos x\right\} - \left\{2x\cos x + 2\sin x\right\} + 2\sin x$$

$$= -x^2\sin x$$

---

18) $g(x) = (2-x)\tan^2 x$

for this, it is best to use the Chain Rule (sec. 3.6) on the derivative of $\tan^2 x$.

$$\frac{dg}{dx} = (2-x)\left[2\tan x(\sec^2 x(1))\right] + (\tan^2 x)[-1]$$

$$= \tan x\left\{2(2-x)\sec^2 x - \tan x\right\}$$

---

20) $s = t^2 - \sec t + 5e^t$

$$\frac{ds}{dt} = [2t] - [\sec t\tan t(1)] + 5[e^t(1)]$$

$$= 2t - \sec t\tan t + 5e^t$$

---

22) $s = \dfrac{\sin t}{1-\cos t}$

$$\frac{ds}{dt} = \frac{(1-\cos t)[\cos t(1)] - (\sin t)[-[-\sin t(1)]]}{(1-\cos t)^2} = \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}$$

$$= \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1-\cos t)^2} = \frac{\cos t - 1}{(1-\cos t)^2} = \frac{-(1-\cos t)}{(1-\cos t)^2} = \frac{-1}{1-\cos t}$$

$$= \frac{1}{\cos t - 1}$$

24) $r = \theta \sin\theta + \cos\theta$

$$\frac{dr}{d\theta} = \{(\theta)[\cos\theta(1)] + (\sin\theta)[1]\} + [-\sin\theta(1)]$$

$$= \theta\cos\theta + \sin\theta - \sin\theta = \theta\cos\theta$$

---

26) $r = (1+\sec\theta)\sin\theta = \sin\theta + \sin\theta\sec\theta = \sin\theta + (\sin\theta)\left(\frac{1}{\cos\theta}\right)$

$$= \sin\theta + \frac{\sin\theta}{\cos\theta} = \sin\theta + \tan\theta$$

$$\frac{dr}{d\theta} = [\cos\theta(1)] + [\sec^2\theta(1)] = \cos\theta + \sec^2\theta$$

---

28) $p = (1+\csc q)\cos q = \cos q + \frac{\cos q}{\sin q} = \cos q + \cot q$

$$\frac{dp}{dq} = [-\sin q(1)] + [-\csc^2 q(1)] = -\sin q - \csc^2 q$$

---

30) $p = \dfrac{\tan q}{1+\tan q}$

$$\frac{dp}{dq} = \frac{(1+\tan q)[\sec^2 q(1)] - (\tan q)[\sec^2 q(1)]}{(1+\tan q)^2}$$

$$= \frac{\sec^2 q + \sec^2 q\tan q - \sec^2 q\tan q}{(1+\tan q)^2}$$

$$= \frac{\sec^2 q}{(1+\tan q)^2}$$

32) $p = \dfrac{3q + \tan q}{q \sec q} = \dfrac{3q + \tan q}{\{q \sec q\}}$

$$\frac{dp}{dq} = \frac{(\{q \sec q\})[3 + \sec^2 q(1)] - (3q + \tan q)[\{(q)[\sec q \tan q(1)] + [1](\sec q)\}]}{(\{q \sec q\})^2}$$

$$= \frac{3q \sec q + q \sec^3 q - (3q^2 \sec q \tan q + 3q \sec q + q \sec q \tan^2 q + \sec q \tan q)}{(q \sec q)^2}$$

$$= \frac{q \sec^3 q - 3q^2 \sec q \tan q - q \sec q \tan q - \sec q \tan q}{(q \sec q)^2}$$

34-a) $y = -2 \sin x$

$$\frac{dy}{dx} = -2[\cos x(1)]$$
$$= -2 \cos x$$

$$\frac{d^2y}{dx^2} = -2[-\sin x(1)]$$
$$= 2 \sin x$$

$$\frac{d^3y}{dx^3} = 2[\cos x(1)]$$
$$= 2 \cos x$$

$$\frac{d^4y}{dx^4} = 2[-\sin x(1)]$$
$$= -2 \sin x$$

34-b) $y = 9 \cos x$

$$\frac{dy}{dx} = 9[-\sin x(1)]$$
$$= -9 \sin x$$

$$\frac{d^2y}{dx^2} = -9[\cos x(1)]$$
$$= -9 \cos x$$

$$\frac{d^3y}{dx^3} = -9[-\sin x(1)]$$
$$= 9 \sin x$$

$$\frac{d^4y}{dx^4} = 9[\cos x(1)]$$
$$= 9 \cos x$$

40) $y = 2x + \sin x$

horizontal tangent line has slope $m(x) = \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 2[1] + [\cos x(1)] = 2 + \cos x$$

$$0 = m(x) = \frac{dy}{dx} = 2 + \cos x$$

$0 = 2 + \cos x$

$-2 = \cos x$

no real number solution

Since $\cos x = -2$ has no solutions, the tangent line is never horizontal for $y = 2x + \sin x$

---

42) $y = x + 2\cos x$

$$\frac{dy}{dx} = [1] + 2[-\sin x(1)] = 1 - 2\sin x$$

$$0 = m(x) = \frac{dy}{dx} = 1 - 2\sin x$$

$0 = 1 - 2\sin x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$

When $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ the tangent line is horizontal for $y = x + 2\cos x$

50) $\lim_{x\to\frac{-\pi}{6}} \sqrt{1+\cos(\pi\csc x)} = \sqrt{1+\cos(\pi\csc(\frac{-\pi}{6}))}$

$= \sqrt{1+\cos(\pi(-2))} = \sqrt{1+\cos(-2\pi)} = \sqrt{1+(1)} = \sqrt{2}$

52) $\lim_{\theta\to\frac{\pi}{4}} \frac{\tan\theta - 1}{\theta - \frac{\pi}{4}} = \lim_{\theta\to\frac{\pi}{4}} \frac{\frac{d}{d\theta}(\tan\theta - 1)}{\frac{d}{d\theta}(\theta - \frac{\pi}{4})} = \lim_{\theta\to\frac{\pi}{4}} \frac{[\sec^2\theta(1)]}{[1]}$

$= \lim_{\theta\to\frac{\pi}{4}} \sec^2\theta = \sec^2(\frac{\pi}{4}) = (\sqrt{2})^2 = 2$

54) $\lim_{x\to 0} \sin\left(\frac{\pi + \tan x}{\tan x - 2\sec x}\right) = \sin\left(\frac{\pi + \tan(0)}{\tan(0) - 2\sec(0)}\right)$

$= \sin\left(\frac{\pi + (0)}{(0) - 2(1)}\right) = \sin\left(\frac{-\pi}{2}\right) = -1$

56) $\lim_{\theta\to 0} \cos\left(\frac{\pi\theta}{\sin\theta}\right) = \lim_{\theta\to 0} \cos\left(\frac{\pi}{\frac{\sin\theta}{\theta}}\right) = \cos\left(\frac{\pi}{\lim_{\theta\to 0}\frac{\sin\theta}{\theta}}\right)$

$= \cos\left(\frac{\pi}{(1)}\right) = \cos(\pi) = -1$

60) $g(x) = \begin{cases} x + b & , x < 0 \\ \cos x & , x \geq 0 \text{ or } 0 \leq x \end{cases}$

continuous at $x = 0$?

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} (x + b) = (0^-) + b = b$

60) continued

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}(\cos x) = \cos(0^+) = 1$$

$$b = \lim_{x\to 0^-} g(x) = \lim_{x\to 0} g(x) = \lim_{x\to 0^+} g(x) = 1$$

$$b = 1$$

if we let $b=1$, $g(x)$ is continuous at $x=0$.

differentiable at $x=0$?

$$\lim_{x\to 0^-} g'(x) = \lim_{x\to 0^-} \frac{d}{dx}(x+b) = \lim_{x\to 0^-}[1] = 1$$

$$\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} \frac{d}{dx}(\cos x) = \lim_{x\to 0^+}[-\sin x(1)] = -\sin(0^+) = 0$$

since $1 = \lim_{x\to 0^-} g'(x) \neq \lim_{x\to 0^+} g'(x) = 0$,

$g(x)$ is not differentiable at $x=0$ for any value of b.