Def. The instantaneous rate of change of f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x)}{h},$$

provided the limit exists.

Def. Velocity (instantaneous velocity) is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}.$$

Def. Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Def. Acceleration is the derivative of velocity with respect to time. If a body's position at time is $s = f(t)$, then the body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Jerk is the derivative of acceleration with respect to time:

$$j(t) = \frac{da}{dt} = \frac{d^2v}{dt^2} = \frac{d^3s}{dt^3}$$

2) $s = 6t - t^2$, $0 \le t \le 6$: $s(t) = 6t - t^2$

a) $s(0) = 6(0) - (0)^2 = 0$ $\quad s(6) = 6(6) - (6)^2 = 36 - 36 = 0$

displacement: $\Delta s = s(6) - s(0) = (0) - (0) = 0\ m$

$$v_{ave} = \frac{s(6)-s(0)}{(6)-(0)} = \frac{(0)-(0)}{6} = \frac{0}{6} = 0\ m/sec$$

b) $v(t) = \frac{ds}{dt} = 6 - 2t$ $\quad |v(0)| = |6-2(0)| = |6| = 6\ m/sec$

$|v(6)| = |6-2(6)| = |6-12| = |-6| = 6\ m/sec$

$a(t) = \frac{dv}{dt} = -2$ $\quad a(0) = -2\ m/sec^2$ $\quad a(6) = -2\ m/sec^2$

c) $v(t) = 0$, $t = ?$

$0 = v(t) = 6 - 2t$

$0 = 6 - 2t$

$2t = 6$

$t = 3\ sec$

when $t = 3$ sec, the body changes direction

$v(t) > 0$ when $0 < t < 3$ and

$v(t) < 0$ when $3 < t < 6$

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4) $s = \frac{t^4}{4} - t^3 + t^2$, $0 \le t \le 3$: $s(t) = \frac{1}{4}t^4 - t^3 + t^2$

a) $s(0) = \frac{1}{4}(0)^4 - (0)^3 + (0)^2 = 0\ m$

$s(3) = \frac{1}{4}(3)^4 - (3)^3 + (3)^2 = \frac{1}{4}(81) - (27) + (9) = \frac{81}{4} - 18 = \frac{81}{4} - \frac{72}{4} = \frac{9}{4}\ m$

$\Delta s = s(3) - s(0) = \left(\frac{9}{4}\right) - (0) = \frac{9}{4}\ m$

$$v_{ave} = \frac{s(3)-s(0)}{(3)-(0)} = \frac{\left(\frac{9}{4}\right)-(0)}{(3)-(0)} = \frac{\frac{9}{4}}{3} = \frac{3}{4}\ m/sec$$

4) continued

b) $v(t) = \frac{ds}{dt} = t^3 - 3t^2 + 2t \quad |v(0)| = |(0)^3 - 3(0)^2 + 2(0)| = |0| = 0$ m/sec

$|v(3)| = |(3)^3 - 3(3)^2 + 2(3)| = |27 - 27 + 6| = |6| = 6$ m/sec

$a(t) = \frac{dv}{dt} = 3t^2 - 6t + 2 \quad a(0) = 3(0)^2 - 6(0) + 2 = 2$ m/sec^2

$a(3) = 3(3)^2 - 6(3) + 2 = 27 - 18 + 2 = 11$ m/sec^2

c) $v(t) = 0$, $t = ?$

$0 = v(t) = t^3 - 3t^2 + 2t$

$0 = t^3 - 3t^2 + 2t$

$0 = t(t^2 - 3t + 2)$

$0 = t(t-1)(t-2)$

$t=0$ discard (starting time)	$t-1=0$ $t=1$	$t-2=0$ $t=2$

the body changes direction at $t = 1$ sec and $t = 2$ sec.

	$(0,1)$	$(1,2)$	$(2,3)$
t	POS	POS	POS
$(t-1)$	neg	POS	POS
$(t-2)$	neg	neg	POS
$t(t-1)(t-2)$	POS	neg	POS

$v(t) > 0$ when $0 < t < 1$ and $2 < t < 3$

$v(t) < 0$ when $1 < t < 2$

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6) $s = \frac{25}{t+5}$, $-4 \le t \le 0$; $s(t) = \frac{25}{t+5}$

a) $s(-4) = \frac{25}{(-4)+5} = \frac{25}{1} = 25$ m $\quad s(0) = \frac{25}{(0)+5} = \frac{25}{5} = 5$ m

$\Delta s = s(0) - s(-4) = (5) - (25) = -20$ m

$v_{ave} = \frac{s(0) - s(-4)}{(0) - (-4)} = \frac{(5) - (25)}{(0) - (-4)} = \frac{-20}{4} = -5$ m/sec

6) continued

b) $v(t)=\frac{ds}{dt}=\frac{(t+5)[0]-(25)[1]}{(t+5)^2}=\frac{-25}{(t+5)^2}=\frac{-25}{t^2+10t+25}$

$|v(-4)|=\left|\frac{-25}{((-4)+5)^2}\right|=\left|\frac{-25}{(1)^2}\right|=\left|\frac{-25}{1}\right|=|-25|=25 \text{ m/sec}$

$|v(0)|=\left|\frac{-25}{((0)+5)^2}\right|=\left|\frac{-25}{(5)^2}\right|=\left|\frac{-25}{25}\right|=|-1|=1 \text{ m/sec}$

$a(t)=\frac{dv}{dt}=\frac{(t^2+10t+25)[0]-(-25)[2t+10]}{(t^2+10t+25)^2}=\frac{25(2t+10)}{((t+5)^2)^2}$

$=\frac{25(2)(t+5)}{(t+5)^4}=\frac{50}{(t+5)^3}$

$a(-4)=\frac{50}{((-4)+5)^3}=\frac{50}{(1)^3}=\frac{50}{1}=50 \text{ m/sec}^2$

$a(0)=\frac{50}{((0)+5)^3}=\frac{50}{(5)^3}=\frac{2}{5} \text{ m/sec}^2$

c) $v(t)=0,\ t=?$

$0=v(t)=\frac{-25}{(t+5)^2}$

no solution

since $v(t)\neq 0$, the body does not change direction on $-4\le t\le 0$

$v(t)<0$ when $-4\le t\le 0$

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8) $v(t)=t^2-4t+3,\ t\ge 0$

$a(t)=\frac{dv}{dt}=2t-4$

8) continued

a) $v(t)=0$, $t=?$

$0=v(t)=t^2-4t+3$

$0=t^2-4t+3$

$0=(t-1)(t-3)$

$t-1=0 \quad | \quad t-3=0$

$t=1 \quad | \quad t=3$

$t=1$: $a(1)=2(1)-4=-2\ m/sec^2$

$t=3$: $a(3)=2(3)-4=6-4=2\ m/sec^2$

b)

	$[0,1)$	$(1,3)$	$(3,\infty)$
$(t-1)$	neg	POS	POS
$(t-3)$	neg	neg	POS
$(t-1)(t-3)$	POS	neg	POS

moving forward:
$v(t)>0$ when $0\le t<1$
and $3<t$

moving backward:
$v(t)<0$ when $1<t<3$

c) $a(t)=0$, $t=?$

$0=a(t)=2t-4$

$0=2t-4$

$4=2t$

$2=t$

	$[0,2)$	$(2,\infty)$
$2t-4$	neg	POS

velocity increasing:
$a(t)>0$ when $2<t$

velocity decreasing
$a(t)<0$ when $0\le t<2$

12) moon

$s(t) = 832t - 2.6t^2$

$s(t) = 0$, $t = ?$

$0 = s(t) = 832t - 2.6t^2$

$0 = 832t - 2.6t^2$

$0 = 2t(416 - 1.3t)$

$2t = 0$	$416 - 1.3t = 0$
$t = 0$	$416 = 1.3t$
discard (starting time)	$t = \frac{416}{1.3} = \frac{(32)(13)}{1.3}$
	$= \frac{32}{0.1} = 320$ sec

The bullet will be aloft on the moon for 320 sec.

maximum height:

$v(t) = \frac{ds}{dt} = 832 - 5.2t$

$v(t) = 0$, $t = ?$

$0 = v(t) = 832 - 5.2t$

$0 = 832 - 5.2t$

$5.2t = 832$

$t = \frac{832}{5.2} = \frac{4(208)}{4(1.3)} = \frac{(16)(13)}{1.3} = \frac{16}{0.1} = 160$ sec

$s(160) = 832(160) - 2.6(160)^2$

$= 160\{832 - 2.6(160)\} = 160\{832 - 416\}$

$= 160\{416\} = 66\,560$ ft

max height: $s(160) = 66\,560$ ft

Earth

$s = 832t - 16t^2$

$s(t) = 0$, $t = ?$

$0 = s(t) = 832t - 16t^2$

$0 = 832t - 16t^2$

$0 = 16t(52 - t)$

$16t = 0$	$52 - t = 0$
$t = 0$	$52 = t$
discard (starting time)	$t = 52$ sec

The bullet will be aloft on Earth for 52 sec.

maximum height:

$v(t) = \frac{ds}{dt} = 832 - 32t$

$v(t) = 0$, $t = ?$

$0 = v(t) = 832 - 32t$

$0 = 832 - 32t$

$32t = 832$

$t = \frac{832}{32} = \frac{16(52)}{16(2)} = \frac{52}{2} = 26$ sec

$s(26) = 832(26) - 16(26)^2$

$= 26\{832 - 16(26)\} = 26\{832 - 416\}$

$= 26\{416\} = 10\,816$ ft

max height: $s(26) = 10816$ ft

16-a) particle P is:

moving left: $2 < t < 3$ and $5 < t < 6$

moving right: $0 < t < 1$

standing still: $1 < t < 2$ and $3 < t < 5$

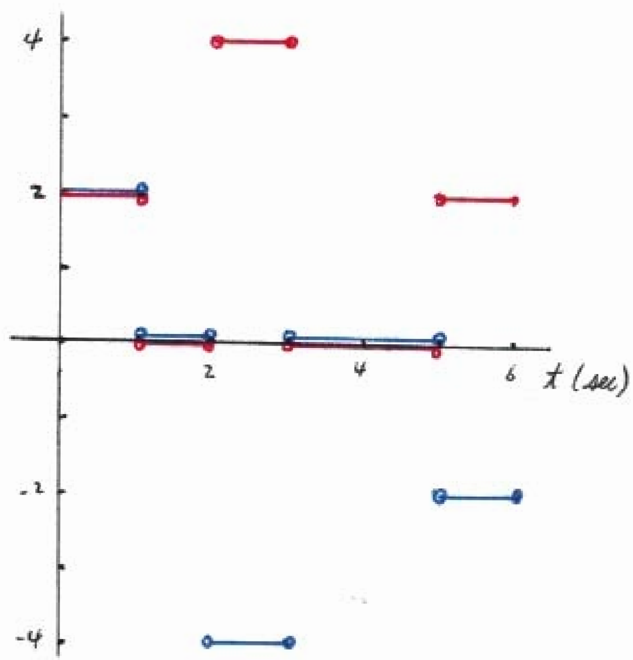
16-b) $0 < t < 1$: $v_{ave} = \frac{s(1)-s(0)}{(1)-(0)} = \frac{(2)-(0)}{(1)-(0)} = \frac{2}{1} = 2$ cm/sec

$1 < t < 2$: $v_{ave} = \frac{s(2)-s(1)}{(2)-(1)} = \frac{(2)-(2)}{(2)-(1)} = \frac{0}{1} = 0$ cm/sec

$2 < t < 3$: $v_{ave} = \frac{s(3)-s(2)}{(3)-(2)} = \frac{(-2)-(2)}{(3)-(2)} = \frac{-4}{1} = -4$ cm/sec

$3 < t < 5$: $v_{ave} = \frac{s(5)-s(3)}{(5)-(3)} = \frac{(-2)-(-2)}{(5)-(3)} = \frac{0}{2} = 0$ cm/sec

$5 < t < 6$: $v_{ave} = \frac{s(6)-s(5)}{(6)-(5)} = \frac{(-4)-(-2)}{(6)-(5)} = \frac{-2}{1} = -2$ cm/sec



blue: velocity
$v(t)$ {cm/sec}

red: speed
$|v(t)|$ {cm/sec}

18) a) forward: $(v(t)>0)$ $\quad 0 \le t < 1$ and $5 < t < 7$

backward: $(v(t)<0)$ $\quad 1 < t < 5$

speeds up: (0 to high $|v(t)|$) $\quad 1 < t < 2$ and $5 < t < 6$

slows down: (high $|v(t)|$ to 0) $\quad 0 \le t \le 1$ and $6 < t < 7$

b) acceleration positive: (low $v(t)$ to high $v(t)$)

$3 < t < 6$

acceleration negative: (high $v(t)$ to low $v(t)$)

$0 \le t < 2$ and $6 < t < 7$

acceleration zero: ($v(t)$ constant)

$2 < t < 3$ and $7 < t < 9$

c) greatest speed: (high $|v(t)|$)

$t = 0$ and $2 < t < 3$

d) particle stand still: ($v(t) = 0$)

$7 \le t \le 9$

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26) $S = \frac{1}{60}\sqrt{wh}$, $h = 180\text{cm}$ $\quad S(w) = \frac{1}{60}\sqrt{w(180)} = \frac{3}{60}\sqrt{20w} = \frac{\sqrt{20}}{20} w^{\frac{1}{2}}$

$$\frac{dS}{dw} = \frac{\sqrt{20}}{20}\left[\frac{1}{2} w^{-\frac{1}{2}}\right] = \frac{1}{\sqrt{20}}\left[\frac{1}{2\sqrt{w}}\right] = \frac{1}{4\sqrt{5}}\left[\frac{1}{2\sqrt{w}}\right] = \frac{1}{(8\sqrt{5})\sqrt{w}}$$

S increases more rapidly at lower weights where the derivative $\frac{dS}{dw}$ is greater.