MATH 20100 section 3.3

Derivative of a Constant Function

If f has the constant value $f(x) = c$, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Power Rule (General Version)

If n is any real number, then

$$\frac{d}{dx}(x^n) = n x^{n-1},$$

for all x where the powers x^n and x^{n-1} are defined.

Derivative Constant Multiple Rule

If u is a differentiable function of x, and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

Derivative Sum (Difference) Rule

If u and v are differentiable functions of x, then their sum (difference) $u \pm v$ is differentiable at every point where u and v are both differentiable. At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

the most general version of this derivative is

$$\frac{d}{dx}(e^x) = e^x(1) \qquad \{\text{note: } e^x \text{ is a function}\}$$

which involves chain rule. We will see chain rule in section 3.6.

Derivative Product Rule

If u and v are differentiable at x, then so is their product uv, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left(f(x)\right)\left[\frac{dg}{dx}\right] + \left(g(x)\right)\left[\frac{df}{dx}\right]$$

Derivative Quotient Rule

If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient $\frac{u}{v}$ is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\left(g(x)\right)\left[\frac{df}{dx}\right] - \left(f(x)\right)\left[\frac{dg}{dx}\right]}{\left(g(x)\right)^2}$$

We need to use the Product and Quotient Rules exactly the order defined above because in vector calculus if not used as described above, we will not get the correct result.

Second- and Higher-Order Derivatives

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y'' = D^2(f)(x) = D_x^2 f(x)$$

{2nd derivative}

$$f^{(5)}(x) = \frac{d^5y}{dx^5} = \frac{d}{dx}\left(\frac{d^4y}{dx^4}\right) = \frac{dy^{(4)}}{dx} = y^{(5)} = D^5(f)(x) = D_x^5 f(x)$$

{5th derivative}

$$f^{(n)}(x) = \frac{d^ny}{dx^n} = \frac{d}{dx}\left(\frac{d^{(n-1)}y}{dx^{(n-1)}}\right) = \frac{dy^{(n-1)}}{dx} = y^n = D^n(f)(x) = D_x^n f(x)$$

{n th derivative}

2) $y = x^2 + x + 8$

$$\frac{dy}{dx} = [2x] + [1] + [0] = 2x + 1$$

$$\frac{d^2y}{dx^2} = 2[1] + [0] = 2$$

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4) $w = 3z^7 - 7z^3 + 21z^2$

$$\frac{dw}{dz} = 3[7z^6] - 7[3z^2] + 21[2z] = 21z^6 - 21z^2 + 42z$$

$$\frac{d^2w}{dz^2} = 21[6z^5] - 21[2z] + 42[1] = 126z^5 - 42z + 42$$

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6) $y = \frac{x^3}{3} + \frac{x^2}{2} + e^{-x} = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{e^x}$

$$\frac{dy}{dx} = \frac{1}{3}[3x^2] + \frac{1}{2}[2x] + [e^{-x}(-1)] = x^2 + x - e^{-x}$$

$$= x^2 + x - \frac{1}{e^x}$$

$$\frac{d^2y}{dx^2} = [2x] + [1] - [e^{-x}(-1)] = 2x + 1 + e^{-x} = 2x + 1 + \frac{1}{e^x}$$

8) $s = -2t^{-1} + \frac{4}{t^2} = -2t^{-1} + 4t^{-2}$

$$\frac{ds}{dt} = -2[-1t^{-2}] + 4[-2t^{-3}] = 2t^{-2} - 8t^{-3} = \frac{2}{t^2} - \frac{8}{t^3}$$

$$\frac{d^2s}{dt^2} = 2[-2t^{-3}] - 8[-3t^{-4}] = -4t^{-3} + 24t^{-4} = \frac{-4}{t^3} + \frac{24}{t^4}$$

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10) $y = 4 - 2x - x^{-3} = 4 - 2x - \frac{1}{x^3}$

$$\frac{dy}{dx} = [0] - 2[1] - [-3x^{-4}] = -2 + 6x^{-4} = -2 + \frac{6}{x^4}$$

$$\frac{d^2y}{dx^2} = [0] + 6[-4x^{-5}] = -24x^{-5} = \frac{-24}{x^5}$$

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12) $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4} = 12\theta^{-1} - 4\theta^{-3} + \theta^{-4}$

$$\frac{dr}{d\theta} = 12[-1\theta^{-2}] - 4[-3\theta^{-4}] + [-4\theta^{-5}] = -12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}$$

$$= \frac{-12}{\theta^2} + \frac{12}{\theta^4} - \frac{4}{\theta^5}$$

$$\frac{d^2r}{d\theta^2} = -12[-2\theta^{-3}] + 12[-4\theta^{-5}] - 4[-5\theta^{-6}] = 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$$

$$= \frac{24}{\theta^3} - \frac{48}{\theta^5} + \frac{20}{\theta^6}$$

14) $y = (2x+3)(5x^2-4)$

a) $\frac{dy}{dx} = (2x+3)[10x] + (5x^2-4)[2] = (20x^2+30x) + (10x^2-8)$

$= 30x^2 + 30x - 8$

b) $y = 10x^3 - 8x + 15x^2 - 12 = 10x^3 + 15x^2 - 8x - 12$

$\frac{dy}{dx} = 30x^2 + 30x - 8$

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16) $y = (1+x^2)(x^{3/4} - x^{-3})$

a) $\frac{dy}{dx} = (1+x^2)[\frac{3}{4}x^{-1/4} + 3x^{-4}] + (x^{3/4} - x^{-3})[2x]$

$= (\frac{3}{4}x^{-1/4} + 3x^{-4} + \frac{3}{4}x^{7/4} + 3x^{-2}) + (2x^{7/4} - 2x^{-2})$

$= \frac{3}{4}x^{-1/4} + 3x^{-4} + \frac{11}{4}x^{7/4} + x^{-2}$

$= \frac{3}{4(\sqrt[4]{x})} + \frac{3}{x^4} + \frac{11}{4}(\sqrt[4]{x})^7 + \frac{1}{x^2}$

b) $y = x^{3/4} - x^{-3} + x^{\frac{11}{4}} - x^{-1}$

$\frac{dy}{dx} = \frac{3}{4}x^{-1/4} + 3x^{-4} + \frac{11}{4}x^{7/4} + x^{-2}$

$= \frac{3}{4(\sqrt[4]{x})} + \frac{3}{x^4} + \frac{11}{4}(\sqrt[4]{x})^7 + \frac{1}{x^2}$

18) $z = \dfrac{4-3x}{3x^2+x}$

$$\frac{dz}{dx} = \frac{(3x^2+x)[-3]-(4-3x)[6x+1]}{(3x^2+x)^2} = \frac{(-9x^2-3x)-(-18x^2+21x+4)}{(3x^2+x)^2}$$

$$= \frac{9x^2-24x-4}{(3x^2+x)^2}$$

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20) $f(t) = \dfrac{t^2-1}{t^2+t-2}$ {without simplifying $f(t)$}

$$\frac{df}{dt} = \frac{(t^2+t-2)[2t]-(t^2-1)[2t+1]}{(t^2+t-2)^2} = \frac{(2t^3+2t^2-4t)-(2t^3+t^2-2t-1)}{(t^2+t-2)^2}$$

$$= \frac{t^2-2t+1}{(t^2+t-2)^2} = \frac{(t-1)^2}{((t+2)(t-1))^2} = \frac{1}{(t+2)^2}$$

$f(t) = \dfrac{t^2-1}{t^2+t-2} = \dfrac{(t+1)(t-1)}{(t+2)(t-1)} = \dfrac{t+1}{t+2}$ when $t \neq 1$

$$\frac{df}{dt} = \frac{(t+2)[1]-(t+1)[1]}{(t+2)^2} = \frac{(t+2)-(t+1)}{(t+2)^2}$$

$$= \frac{1}{(t+2)^2}$$

22) $w = (2x-7)^{-1}(x+5) = \frac{x+5}{2x-7}$

$$\frac{dw}{dx} = \frac{(2x-7)[1]-(x+5)[2]}{(2x-7)^2} = \frac{(2x-7)-(2x+10)}{(2x-7)^2} = \frac{-17}{(2x-7)^2}$$

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24) $u = \frac{5x+1}{2\sqrt{x}} = \frac{5x+1}{2x^{1/2}}$

$$\frac{du}{dx} = \frac{(2x^{1/2})[5]-(5x+1)[x^{-1/2}]}{(2x^{1/2})^2} = \frac{(10x^{1/2})-(5x^{1/2}+x^{-1/2})}{(2x^{1/2})^2}$$

$$= \frac{5x^{1/2}-x^{-1/2}}{(2x^{1/2})^2} = \left(\frac{\frac{5\sqrt{x}}{1}-\frac{1}{\sqrt{x}}}{\frac{(2\sqrt{x})^2}{1}}\right)\left(\frac{\frac{\sqrt{x}}{1}}{\frac{\sqrt{x}}{1}}\right) = \frac{5x-1}{4(\sqrt{x})^3}$$

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26) $r = 2\left(\frac{1}{\sqrt{\theta}}+\sqrt{\theta}\right) = 2\left(\frac{1}{\theta^{1/2}}+\theta^{1/2}\right) = 2\theta^{-1/2}+2\theta^{1/2}$

$$\frac{dr}{d\theta} = 2\left[\frac{-1}{2}\theta^{-3/2}\right]+2\left[\frac{1}{2}\theta^{-1/2}\right] = -\theta^{-3/2}+\theta^{-1/2} = \frac{-1}{\theta^{3/2}}+\frac{1}{\theta^{1/2}}$$

$$= \frac{1}{\sqrt{\theta}} - \frac{1}{(\sqrt{\theta})^3}$$

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28) $y = \frac{(x+1)(x+2)}{(x-1)(x-2)} = \frac{x^2+3x+2}{x^2-3x+2}$

$$\frac{dy}{dx} = \frac{(x^2-3x+2)[2x+3]-(x^2+3x+2)[2x-3]}{(x^2-3x+2)^2}$$

$$= \frac{(2x^3-6x^2+4x+3x^2-9x+6)-(2x^3+6x^2+4x-3x^2-9x-6)}{(x^2-3x+2)^2}$$

28) continued

$$\frac{dy}{dx} = \frac{(2x^3-3x^2-5x+6)-(2x^3+3x^2-5x-6)}{(x^2-3x+2)^2}$$

$$= \frac{-6x^2+12}{(x^2-3x+2)^2} = \frac{12-6x^2}{(x^2-3x+2)^2} = \frac{6(2-x^2)}{(x^2-3x+2)^2} = \frac{6(2-x^2)}{((x-1)(x-2))^2}$$

$$= \frac{6(2-x^2)}{(x-1)^2(x-2)^2}$$

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30) $y = \dfrac{x^2+3e^x}{2e^x-x}$

$$\frac{dy}{dx} = \frac{(2e^x-x)[2x+3e^x(1)]-(x^2+3e^x)[2e^x(1)-1]}{(2e^x-x)^2}$$

$$= \frac{(4xe^x+6e^{2x}-2x^2-3xe^x)-(2x^2e^x-x^2+6e^{2x}-3e^x)}{(2e^x-x)^2}$$

$$= \frac{(xe^x+6e^{2x}-2x^2)-(2x^2e^x-x^2+6e^{2x}-3e^x)}{(2e^x-x)^2}$$

$$= \frac{xe^x-x^2-2x^2e^x+3e^x}{(2e^x-x)^2}$$

32) $w = r e^{-r} = (r)(e^{-r})$ note: e^x is a function

$$\frac{dw}{dr} = (r)[e^{-r}(-1)] + (e^{-r})[1] = -re^{-r} + e^{-r} = e^{-r} - re^{-r}$$
$$= e^{-r}(1-r) = \frac{1-r}{e^r}$$

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34) $y = x^{-3/5} + \pi^{3/2}$ {here π is a constant}

$$\frac{dy}{dx} = \left[\frac{-3}{5} x^{\frac{-8}{5}}\right] + [0] = \frac{-3}{5} x^{\frac{-8}{5}} = \frac{-3}{5\,x^{8/5}} = \frac{-3}{5(\sqrt[5]{x})^8}$$

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36) $w = \frac{1}{z^{1.4}} + \frac{\pi}{\sqrt{z}} = z^{-1.4} + \pi z^{-1/2}$

$$\frac{dw}{dz} = [-1.4 z^{-2.4}] + \pi\left[\frac{-1}{2} z^{-3/2}\right] = -1.4 z^{-2.4} - \frac{\pi}{2} z^{-3/2}$$
$$= \frac{-1.4}{z^{2.4}} - \frac{\pi}{2z^{3/2}} = \frac{-1.4}{z^{2.4}} - \frac{\pi}{2(\sqrt{z})^3}$$

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38) $y = \sqrt[3]{x^{9.6}} + 2e^{1/3} = x^{\left(\frac{9.6}{3}\right)} + 2e^{\frac{1}{3}}$

$y = x^{3.2} + 2e^{\frac{1}{3}}$ {note: $e^{\frac{1}{3}}$ is a constant}

$$\frac{dy}{dx} = [3.2 x^{2.2}] + [0] = 3.2 x^{2.2}$$

40) $r = e^{\theta}\left(\frac{1}{\theta^{2}} + \theta^{-\pi/2}\right) = (e^{\theta})(\theta^{-2} + \theta^{-\pi/2})$

$$\frac{dr}{d\theta} = (e^{\theta})\left[-2\theta^{-3} - \frac{\pi}{2}\theta^{\left(-\frac{\pi}{2}-1\right)}\right] + \left(e^{-2} + \theta^{-\frac{\pi}{2}}\right)\left[e^{\theta}(1)\right]$$

$$= e^{\theta}\left\{(1)\left[-2\theta^{-3} - \frac{\pi}{2}\theta^{-\frac{\pi}{2}-1}\right] + [1]\left(\theta^{-2} + \theta^{-\frac{\pi}{2}}\right)\right\}$$

$$= e^{\theta}\left\{\frac{-2}{\theta^{3}} - \frac{\pi}{2\theta^{\frac{\pi}{2}+1}} + \frac{1}{\theta^{2}} + \frac{1}{\theta^{\frac{\pi}{2}}}\right\} = e^{\theta}\left\{\frac{-2}{\theta^{3}} - \frac{\pi}{2(e^{\frac{\pi}{2}})(\theta^{1})} + \frac{1}{\theta^{2}} + \frac{1}{\theta^{\frac{\pi}{2}}}\right\}$$

$$= e^{\theta}\left\{\frac{-2}{\theta^{3}} - \frac{\pi}{2(\theta^{\frac{\pi}{2}})(\theta^{1})} + \frac{\theta}{\theta^{3}} + \frac{2\theta}{2(\theta^{\frac{\pi}{2}})(\theta^{1})}\right\} = e^{\theta}\left\{\frac{\theta-2}{\theta^{3}} + \frac{2\theta-\pi}{2(\theta^{\frac{\pi}{2}})(\theta^{1})}\right\}$$

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46) $s = \frac{t^{2}+5t-1}{t^{2}} = \frac{t^{2}}{t^{2}} + \frac{5t}{t^{2}} - \frac{1}{t^{2}} = 1 + \frac{5}{t} - \frac{1}{t^{2}} = 1 + 5t^{-1} - t^{-2}$

$$\frac{ds}{dt} = [0] + 5[-1t^{-2}] - [-2t^{-3}] = -5t^{-2} + 2t^{-3} = \frac{-5}{t^{2}} + \frac{2}{t^{3}} = \frac{2-5t}{t^{3}}$$

$$\frac{d^{2}s}{dt^{2}} = -5[-2t^{-3}] + 2[-3t^{-4}] = 10t^{-3} - 6t^{-4} = \frac{10}{t^{3}} - \frac{6}{t^{4}} = \frac{10t-6}{t^{4}}$$

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48) $u = \frac{(x^{2}+x)(x^{2}-x+1)}{x^{4}} = \frac{x^{4}-x^{3}+x^{2}+x^{3}-x^{2}+x}{x^{4}} = \frac{x^{4}+x}{x^{4}}$

$$= \frac{x^{4}}{x^{4}} + \frac{x}{x^{4}} = 1 + \frac{1}{x^{3}} = 1 + x^{-3}$$

$$\frac{du}{dx} = [0] - [-3x^{-4}] = 3x^{-4} = \frac{3}{x^{4}} \qquad \frac{d^{2}u}{dx^{2}} = 3[-4x^{-5}] = -12x^{-5} = \frac{-12}{x^{5}}$$

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50) $p = \frac{q^{2}+3}{(q-1)^{3}+(q+1)^{3}} = \frac{q^{2}+3}{(q^{3}-3q^{2}+3q-1)+(q^{3}+3q^{2}+3q+1)} = \frac{q^{2}+3}{2q^{3}+6q}$

$$= \frac{q^{2}+3}{2q(q+3)} = \frac{1}{2q} = \frac{1}{2}q^{-1}$$

$$\frac{dp}{dq} = \frac{1}{2}[-1q^{-2}] = \frac{-1}{2}q^{-2} = \frac{-1}{2q^{2}} \qquad \frac{d^{2}p}{dq^{2}} = \frac{-1}{2}[-2q^{-3}] = q^{-3} = \frac{1}{q^{3}}$$

52) $w = e^z(z-1)(z^2+1) = (e^z)(z^3+z-z^2-1) = (e^z)(z^3-z^2+z-1)$

$$\frac{dw}{dz} = (e^z)[3z^2-2z+1] + (z^3-z^2+z-1)[e^z(1)]$$

$$= e^z\{(1)[3z^2-2z+1] + [1](z^3-z^2+z-1)\}$$

$$= e^z\{z^3+2z^2-z\}$$

$$\frac{d^2w}{dz^2} = (e^z)[3z^2+4z-1] + (z^3+2z^2-z)[e^z(1)]$$

$$= e^z\{(1)[3z^2+4z-1] + [1](z^3+2z^2-z)\}$$

$$= e^z\{z^3+5z^2+3z-1\}$$

56-a) $y = x^3 - 3x - 2$ $\qquad \frac{dy}{dx} = 3x^2 - 3$

tangent line being horizontal implies that

$m(x) = 0$ so $3x^2 - 3 = \frac{dy}{dx} = m(x) = 0$

$$3x^2 - 3 = 0$$

$$3(x^2-1) = 0$$

$$3(x+1)(x-1) = 0$$

$x+1=0$	$x-1=0$
$x=-1$	$x=1$

when $x=-1$:

$y = (-1)^3 - 3(-1) - 2 = 0$

point: $(-1, 0)$

tangent line: $y-(0) = 0(x-(-1))$

$y - 0 = 0$

$y = 0$

and the $\perp$ line is $x = -1$

56-a) continued

when $x=1$:

$y=(1)^3-3(1)-2=-4$

point: $(1,-4)$

tangent line: $y-(-4)=0(x-(1))$

$y+4=0$

$y=-4$

and the $\perp$ line is $x=1$

56-b) Since $\frac{dy}{dx}=3x^2-3$ which is a parabola opening upwards, the smallest value will occur when $x=0$ and $\frac{dy}{dx}=-3$. So $m(x)=-3$ is the smallest slope. This occurs at $x=0$.

when $x=0$: $y=(0)^3-3(0)-2=-2$ point: $(0,-2)$

tangent line: $y-(-2)=-3(x-(0))$

$y+2=-3x$

$y=-3x-2$

and the $\perp$ line; $m_2=\frac{-1}{m_1}=\frac{-1}{-3}=\frac{1}{3}$

$y-(-2)=\frac{1}{3}(x-(0))$

$y+2=\frac{1}{3}x$

$y=\frac{1}{3}x-2$

60) $y = x^2 + ax + b$ $\quad$ $y = cx - x^2$ $\quad$ common tangent line at $(1,0)$

since $y = cx - x^2$ has only 1 unknown constant c, we start here

we have a tangent line at $(1,0)$ which indicates that

$y = cx - x^2$ passes the point $(1,0)$.

so $(0) = C(1) - (1)^2$

$0 = C - 1$

$1 = C$ $\quad$ and the actual curve is $y = (1)x - x^2$

$y = x - x^2$

also $\frac{dy}{dx} = 1 - 2x$

the slope of the tangent line is $m = \frac{dy}{dx}\Big|_{x=1} = 1 - 2(1) = -1$

having a common tangent line also means that the slope of the tangent line of $y = x^2 + ax + b$ is -1.

so $\frac{dy}{dx} = 2x + a$ $\quad$ $-1 = m = \frac{dy}{dx}\Big|_{x=1} = 2(1) + a$

$-1 = 2 + a$

$-3 = a$

so our equation is now $y = x^2 - 3x + b$.

this curve also passes $(1,0)$ and

$(0) = (1)^2 - 3(1) + b$

$0 = 1 - 3 + b$

$0 = -2 + b$

$2 = b$

$a = -3$, $b = 2$, $c = 1$

also

$y = x^2 - 3x + 2$ and $y = x - x^2$

66) $f(2)=3$, $f'(2)=-1$, $g(2)=-4$, $g'(2)=1$

$\perp$ to $F(x)=\dfrac{f(x)+3}{x-g(x)}$ at $x=2$

$$F(2)=\frac{f(2)+3}{(2)-g(2)}=\frac{(3)+3}{2-(-4)}=\frac{6}{6}=1 \quad \text{point: } (2,1)$$

$$\frac{dF}{dx}=\frac{(x-g(x))[f'(x)]-(f(x)+3)[1-g'(x)]}{(x-g(x))^2}$$

$$m_1=\left.\frac{dF}{dx}\right|_{x=2}=\frac{((2)-g(2))[f'(2)]-(f(2)+3)[1-g'(2)]}{((2)-g(2))^2}$$

$$=\frac{(2-(-4))[-1]-((3)+3)[1-(1)]}{(2-(-4))^2}$$

$$=\frac{(6)[-1]-(6)[0]}{(6)^2}=\frac{-6}{(6)^2}=\frac{-1}{6}$$

$\perp$ line:

$$m_2=\frac{-1}{m_1}=\frac{-1}{\left(\frac{-1}{6}\right)}=6$$

$$y-(1)=6(x-(2))$$

$$y-1=6x-12$$

$$y=6x-11$$

70) $\lim_{x\to -1} \frac{x^{2/9}-1}{x+1} = \lim_{x\to -1} \frac{(\sqrt[9]{x})^2-1}{x+1} = \lim_{x\to -1} \frac{\frac{d}{dx}(x^{2/9}-1)}{\frac{d}{dx}(x+1)}$

$= \lim_{x\to -1} \frac{[\frac{2}{9}x^{-7/9}]}{[1]} = \lim_{x\to -1} \frac{2}{9}x^{7/9} = \lim_{x\to -1} \frac{2}{9(\sqrt[9]{x})^7}$

$= \left.\frac{2}{9(\sqrt[9]{x})^7}\right|_{x=-1} = \frac{2}{9(\sqrt[9]{(-1)})^7} = \frac{2}{9(-1)^7} = \frac{2}{9(-1)} = \frac{-2}{9}$

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72) $f(x) = \begin{cases} ax+b, & x > -1 \text{ or } -1 < x \\ bx^2-3, & x \le -1 \end{cases}$; $\frac{df}{dx} = \begin{cases} a, & -1 < x \\ 2bx, & x \le -1 \end{cases}$

f is differentiable at $x=-1$

$\lim_{x\to -1^+} \frac{df}{dx} = \lim_{x\to -1^+} a = a$ and $\lim_{x\to -1^-} \frac{df}{dx} = \lim_{x\to -1^-} 2bx = 2b(-1) = -2b$

$a = \lim_{x\to -1^+} \frac{df}{dx} = \lim_{x\to -1^-} \frac{df}{dx} = -2b$

$a = -2b$

f is continuous at $x=-1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (ax+b) = a(-1)+b = -a+b$ and

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (bx^2-3) = b(-1)^2-3 = b-3$

$-a+b = \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x) = b-3$

$-a+b = b-3 \Rightarrow \begin{matrix} -a=-3 \\ a=3 \end{matrix}$ and $\begin{matrix} 3=-2b \\ -\frac{3}{2}=b \end{matrix}$

78)

$$\frac{d}{dx}\left(x^{-m}\right) = \frac{d}{dx}\left(\frac{1}{x^m}\right)$$

$$= \frac{(x^m)[0] - (1)\left[m x^{(m-1)}\right]}{(x^m)^2}$$

$$= \frac{-m x^{(m-1)}}{x^{2m}}$$

$$= -m x^{\{(m-1) - 2m\}}$$

$$= -m x^{-m-1}$$