Def. The derivative of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Alternate Formula for the Derivative

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

Notation of Derivatives
(the ones written in blue color will be the ones I'll be using in the examples and class lectures.)

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = D(f)(x) = D_x f(x)$$

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \left.\frac{df}{dx}\right|_{x=a} = \left.\frac{d}{dx}f(x)\right|_{x=a}$$

Theorem 1: Differentiability Implies Continuity
If f has a derivative at $x=c$, then f is continuous at $x=c$.

2) $F(x) = (x-1)^2 + 1 = (x^2 - 2x + 1) + 1 = (x^2 - 2x + 2)$

$F(x+h) = ((x+h)-1)^2 + 1 = (x+h-1)^2 + 1 = (x^2 + 2xh + h^2 - 2x - 2h + 1) + 1$

$= (x^2 + 2xh + h^2 - 2x - 2h + 2)$

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{(x^2 + 2xh + h^2 - 2x - 2h + 2) - (x^2 - 2x + 2)}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 2h}{h} = \lim_{h \to 0} \frac{h(2x + h - 2)}{h} = \lim_{h \to 0} (2x + h - 2)$$

$$= 2x + (0) - 2 = 2x - 2$$

$F'(-1) = 2(-1) + 2 = -2 + 2 = 0$ $\quad F'(0) = 2(0) + 2 = 2$

$F'(2) = 2(2) + 2 = 4 + 2 = 6$

---

4) $k(z) = \frac{1-z}{2z} = \left(\frac{1-z}{2z}\right)$

$k(z+h) = \frac{1-(z+h)}{2(z+h)} = \left(\frac{1-z-h}{2z+2h}\right)$

$$k'(x) = \lim_{h \to 0} \frac{k(z+h) - k(z)}{h} = \lim_{h \to 0} \left(\frac{\left(\frac{1-z-h}{2z+2h}\right) - \left(\frac{1-z}{2z}\right)}{\frac{h}{1}} \right) \left(\frac{\frac{2z(2z+2h)}{1}}{\frac{2z(2z+2h)}{1}} \right)$$

$$= \lim_{h \to 0} \frac{(1-z-h)(2z) - (1-z)(2z+2h)}{h(2z)(2z+2h)} = \lim_{h \to 0} \frac{(2z - 2z^2 - 2zh) - (2z + 2h - 2z^2 - 2z\ldots}{h(2z)(2z+2h)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(2z)(2z+2h)} = \lim_{h \to 0} \frac{-1}{2z(2z+2h)} = \frac{-1}{2z(2z+2(0))} = \frac{-1}{2z(2z)} = \frac{-1}{4z^2}$$

$k'(-1) = \frac{-1}{4(-1)^2} = \frac{-1}{4}$ $\quad k'(1) = \frac{-1}{4(1)^2} = \frac{-1}{4}$ $\quad k'(\sqrt{2}) = \frac{-1}{4(\sqrt{2})^2} = \frac{-1}{4(2)} = \frac{-1}{8}$

6) $r(s) = \sqrt{2s+1}$ $\qquad$ $r(s+h) = \sqrt{2(s+h)+1} = \sqrt{2s+2h+1}$

$$r'(s) = \lim_{h\to 0} \frac{r(s+h) - r(s)}{h} = \lim_{h\to 0} \left(\frac{\sqrt{2s+2h+1} - \sqrt{2s+1}}{h}\right)\left(\frac{\sqrt{2s+2h+1} + \sqrt{2s+1}}{\sqrt{2s+2h+1} + \sqrt{2s+1}}\right)$$

$$= \lim_{h\to 0} \frac{(2s+2h+1) - (2s+1)}{h(\sqrt{2s+2h+1} + \sqrt{2s+1})} = \lim_{h\to 0} \frac{2h}{h(\sqrt{2s+2h+1} + \sqrt{2s+1})}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2s+2h+1} + \sqrt{2s+1}} = \frac{2}{\sqrt{2s+2(0)+1} + \sqrt{2s+1}} = \frac{2}{\sqrt{2s+1} + \sqrt{2s+1}}$$

$$= \frac{2}{2\sqrt{2s+1}} = \frac{1}{\sqrt{2s+1}}$$

$$r'(0) = \frac{1}{\sqrt{2(0)+1}} = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1 \qquad r'(1) = \frac{1}{\sqrt{2(1)+1}} = \frac{1}{\sqrt{2+1}} = \frac{1}{\sqrt{3}}$$

$$r'\left(\tfrac{1}{2}\right) = \frac{1}{\sqrt{2(\frac{1}{2})+1}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

---

8) $r = s^3 - 2s^2 + 3 \therefore r(s) = (s^3 - 2s^2 + 3)$

$$r(s+h) = (s+h)^3 - 2(s+h)^2 + 3 = (s^3 + 3s^2h + 3sh^2 + h^3) - 2(s^2 + 2sh + h^2) + 3$$

$$= (s^3 + 3s^2h + 3sh^2 + h^3 - 2s^2 - 4sh - 2h^2 + 3)$$

$$\frac{dr}{ds} = \lim_{h\to 0} \frac{r(s+h) - r(s)}{h} = \lim_{h\to 0} \frac{(s^3 + 3s^2h + 3sh^2 + h^3 - 2s^2 - 4sh - 2h^2 + 3) - (s^3 - 2s^2 + 3)}{h}$$

$$= \lim_{h\to 0} \frac{3s^2h + 3sh^2 + h^3 - 4sh - 2h^2}{h} = \lim_{h\to 0} \frac{h(3s^2 + 3sh + h^2 - 4s - 2h)}{h}$$

$$= \lim_{h\to 0} (3s^2 + 3sh + h^2 - 4s - 2h) = 3s^2 + 3s(0) + (0)^2 - 4s - 2(0) = 3s^2 - 4s$$

10) $v = t - \frac{1}{t}$: $v(t) = \left(t - \frac{1}{t}\right)$

$$v(t+h) = (t+h) - \frac{1}{(t+h)} = \left(t + h - \frac{1}{t+h}\right)$$

$$\frac{dv}{dt} = \lim_{h\to 0} \frac{v(t+h) - v(t)}{h} = \lim_{h\to 0} \frac{\left(t+h-\frac{1}{t+h}\right) - \left(t - \frac{1}{t}\right)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\frac{h}{1} - \frac{1}{t+h} + \frac{1}{t}}{\frac{h}{1}}\right)\left(\frac{\frac{t(t+h)}{1}}{\frac{t(t+h)}{1}}\right) = \lim_{h\to 0} \frac{ht(t+h) - 1(t) + 1(t+h)}{ht(t+h)}$$

$$= \lim_{h\to 0} \frac{t^2h + th^2 - t + t + h}{ht(t+h)} = \lim_{h\to 0} \frac{t^2h + th^2 + h}{ht(t+h)}$$

$$= \lim_{h\to 0} \frac{h(t^2 + th + 1)}{ht(t+h)} = \lim_{h\to 0} \frac{t^2 + th + 1}{t(t+h)} = \frac{t^2 + t(0) + 1}{t(t+(0))} = \frac{t^2+1}{t^2} = 1 + \frac{1}{t^2}$$

---

12) $z = \frac{1}{\sqrt{w^2-1}}$: $z(w) = \left(\frac{1}{\sqrt{w^2-1}}\right)$ $z(w+h) = \left(\frac{1}{\sqrt{(w+h)^2-1}}\right)$

$$\frac{dz}{dw} = \lim_{h\to 0} \frac{z(w+h) - z(w)}{h} = \lim_{h\to 0} \left(\frac{\left(\frac{1}{\sqrt{(w+h)^2-1}}\right) - \left(\frac{1}{\sqrt{w^2-1}}\right)}{\frac{h}{1}}\right)\left(\frac{\frac{\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}}{1}}{\frac{\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}}{1}}\right)$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{w^2-1} - \sqrt{(w+h)^2-1}}{h\left(\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\right)}\right)\left(\frac{\sqrt{w^2-1} + \sqrt{(w+h)^2-1}}{\sqrt{w^2-1} + \sqrt{(w+h)^2-1}}\right)$$

$$= \lim_{h\to 0} \frac{(w^2-1) - ((w+h)^2-1)}{h\left(\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\right)\left(\sqrt{w^2-1} + \sqrt{(w+h)^2-1}\right)}$$

$$= \lim_{h\to 0} \frac{(w^2-1) - (w^2 + 2wh + h^2 - 1)}{h\left(\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\right)\left(\sqrt{w^2-1} + \sqrt{(w+h)^2-1}\right)} = \lim_{h\to 0} \frac{-2wh - h^2}{h\left(\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\right)\left(\sqrt{w^2-1} + \sqrt{(w+h)^2-1}\right)}$$

$$= \lim_{h\to 0} \frac{h(-2w-h)}{h\left(\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\right)\left(\sqrt{w^2-1} + \sqrt{(w+h)^2-1}\right)} = \lim_{h\to 0} \frac{-2w-h}{\sqrt{w^2-1}\,\sqrt{(w+h)^2-1}\left(\sqrt{w^2-1} + \sqrt{(w+h)^2-1}\right)}$$

12) continued

$$\frac{dz}{dw} = \frac{-2w-(0)}{\sqrt{w^2-1}\,\sqrt{(w+(0))^2-1}\,\left(\sqrt{w^2-1}+\sqrt{(w+(0))^2-1}\right)}$$

$$= \frac{-2w}{\sqrt{w^2-1}\,\sqrt{w^2-1}\,\left(\sqrt{w^2-1}+\sqrt{w^2-1}\right)} = \frac{-2w}{\sqrt{w^2-1}\,\sqrt{w^2-1}\,\left(2\sqrt{w^2-1}\right)}$$

$$= \frac{-w}{\left(\sqrt{w^2-1}\right)^3} = \frac{-w}{(w^2-1)^{3/2}}$$

---

14) $k(x) = \left(\frac{1}{2+x}\right)$, $x = 2$ $\quad k(x+h) = \frac{1}{2+(x+h)} = \left(\frac{1}{2+x+h}\right)$

$$k'(x) = \lim_{h\to 0} \frac{k(x+h)-k(x)}{h} = \lim_{h\to 0} \left(\frac{\left(\frac{1}{2+x+h}\right)-\left(\frac{1}{2+x}\right)}{\frac{h}{1}}\right)\left(\frac{\frac{(2+x)(2+x+h)}{1}}{\frac{(2+x)(2+x+h)}{1}}\right)$$

$$\frac{dk}{dx} = \lim_{h\to 0} \frac{(2+x)-(2+x+h)}{h(2+x)(2+x+h)} = \lim_{h\to 0} \frac{-h}{h(2+x)(2+x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{(2+x)(2+x+h)} = \frac{-1}{(2+x)(2+x+(0))} = \frac{-1}{(2+x)(2+x)}$$

$$= \frac{-1}{(2+x)^2}$$

$$k'(2) = \left.\frac{dk}{dx}\right|_{x=2} = \frac{-1}{(2+(2))^2} = \frac{-1}{(4)^2} = \frac{-1}{16}$$

16) $y = \dfrac{x+3}{1-x}$, $x = -2$ $\qquad y(x) = \left(\dfrac{x+3}{1-x}\right)$

$$y(x+h) = \frac{(x+h)+3}{1-(x+h)} = \left(\frac{x+h+3}{1-x-h}\right)$$

$$y'(x) = \lim_{h\to 0}\frac{y(x+h)-y(x)}{h} = \lim_{h\to 0}\left(\frac{\left(\frac{x+h+3}{1-x-h}\right)-\left(\frac{x+3}{1-x}\right)}{\frac{h}{1}}\right)\left(\frac{\frac{(1-x)(1-x-h)}{1}}{\frac{(1-x)(1-x-h)}{1}}\right)$$

$$= \frac{dy}{dx} = \lim_{h\to 0}\frac{(1-x)(x+h+3)-(x+3)(1-x-h)}{h(1-x)(1-x-h)}$$

$$= \lim_{h\to 0}\frac{(x+h+3-x^2-xh-3x)-(x-x^2-xh+3-3x-3h)}{h(1-x)(1-x-h)}$$

$$= \lim_{h\to 0}\frac{4h}{h(1-x)(1-x-h)} = \lim_{h\to 0}\frac{4}{(1-x)(1-x-h)} = \frac{4}{(1-x)(1-x-(0))}$$

$$= \frac{4}{(1-x)(1-x)} = \frac{4}{(1-x)^2}$$

$$y'(-2) = \left.\frac{dy}{dx}\right|_{x=-2} = \frac{4}{(1-(-2))^2} = \frac{4}{(3)^2} = \frac{4}{9}$$

18) $w = g(z) = \left(1 + \sqrt{4-z}\right)$, $(z, w) = (3, 2)$

$$g(z+h) = 1 + \sqrt{4-(z+h)} = \left(1 + \sqrt{4-z-h}\right)$$

$$\frac{dg}{dz} = g'(z) = \lim_{h\to 0} \frac{g(z+h) - g(z)}{h} = \lim_{h\to 0} \frac{\left(1+\sqrt{4-z-h}\right) - \left(1+\sqrt{4-z}\right)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{4-z-h} - \sqrt{4-z}}{h}\right)\left(\frac{\sqrt{4-z-h} + \sqrt{4-z}}{\sqrt{4-z-h} + \sqrt{4-z}}\right)$$

$$= \lim_{h\to 0} \frac{(4-z-h)-(4-z)}{h\left(\sqrt{4-z-h} + \sqrt{4-z}\right)} = \lim_{h\to 0} \frac{-h}{h\left(\sqrt{4-z-h} + \sqrt{4-z}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{4-z-h} + \sqrt{4-z}} = \frac{-1}{\sqrt{4-z-(0)} + \sqrt{4-z}} = \frac{-1}{\sqrt{4-z} + \sqrt{4-z}} = \frac{-1}{2\sqrt{4-z}}$$

$$m = \left.\frac{dg}{dz}\right|_{z=3} = g'(3) = \frac{-1}{2\sqrt{4-(3)}} = \frac{-1}{2\sqrt{1}} = \frac{-1}{2}$$

$$w - (2) = \frac{-1}{2}\left(z - (3)\right)$$

$$w - 2 = \frac{-1}{2}(z-3)$$

$$w - 2 = \frac{-1}{2}z + \frac{3}{2}$$

$$w = \frac{-1}{2}z + \frac{3}{2} + 2$$

$$w = \frac{-1}{2}z + \frac{7}{2}$$

20) $y = 1 - \frac{1}{x}$, $\left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = ?$ $\quad y(x) = \left(1 - \frac{1}{x}\right)$

$$y(x+h) = 1 - \frac{1}{(x+h)} = \left(1 - \frac{1}{x+h}\right)$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} = \lim_{h\to 0} \frac{\left(1 - \frac{1}{x+h}\right) - \left(1 - \frac{1}{x}\right)}{h}$$

$$= \lim_{h\to 0} \left(\frac{\frac{1}{x} - \frac{1}{x+h}}{\frac{h}{1}}\right)\left(\frac{\frac{x(x+h)}{1}}{\frac{x(x+h)}{1}}\right) = \lim_{h\to 0} \frac{(x+h) - (x)}{h\,x\,(x+h)}$$

$$= \lim_{h\to 0} \frac{h}{h\,x(x+h)} = \lim_{h\to 0} \frac{1}{x(x+h)} = \frac{1}{x(x+(0))} = \frac{1}{x(x)} = \frac{1}{x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=\sqrt{3}} = \frac{1}{(\sqrt{3})^2} = \frac{1}{3}$$

---

22) $w = z + \sqrt{z}$, $\left.\frac{dw}{dz}\right|_{z=4} = ?$ $\quad w(z) = (z + \sqrt{z})$

$$w(z+h) = (z+h) + \sqrt{(z+h)} = \left(z + h + \sqrt{z+h}\right)$$

$$\frac{dw}{dz} = \lim_{h\to 0} \frac{w(z+h) - w(z)}{h} = \lim_{h\to 0} \frac{\left(z + h + \sqrt{z+h}\right) - \left(z + \sqrt{z}\right)}{h}$$

$$= \lim_{h\to 0} \frac{h + \sqrt{z+h} - \sqrt{z}}{h} = \lim_{h\to 0} \left(\frac{h}{h} + \frac{\sqrt{z+h} - \sqrt{z}}{h}\right)$$

22) continued

$\vdots$

$$\frac{dw}{dz} = \lim_{h\to 0}\left(1 + \left(\frac{\sqrt{z+h} - \sqrt{z}}{h}\right)\left(\frac{\sqrt{z+h} + \sqrt{z}}{\sqrt{z+h} + \sqrt{z}}\right)\right)$$

$$= \lim_{h\to 0}\left(1 + \frac{(z+h)-(z)}{h(\sqrt{z+h} + \sqrt{z})}\right) = \lim_{h\to 0}\left(1 + \frac{h}{h(\sqrt{z+h} + \sqrt{z})}\right)$$

$$= \lim_{h\to 0}\left(1 + \frac{1}{\sqrt{z+h} + \sqrt{z}}\right) = 1 + \frac{1}{\sqrt{z+(0)} + \sqrt{z}} = 1 + \frac{1}{\sqrt{z}+\sqrt{z}} = 1 + \frac{1}{2\sqrt{z}}$$

$$\left.\frac{dw}{dz}\right|_{z=4} = 1 + \frac{1}{2\sqrt{(4)}} = 1 + \frac{1}{2(2)} = 1 + \frac{1}{4} = \frac{5}{4}$$

---

24) $f(x) = (x^2 - 3x + 4) \qquad f(z) = (z^2 - 3z + 4)$

$$f'(x) = \lim_{z\to x}\frac{f(z) - f(x)}{z - x} = \lim_{z\to x}\frac{(z^2 - 3z + 4) - (x^2 - 3x + 4)}{z - x}$$

$$= \lim_{z\to x}\frac{z^2 - 3z - x^2 + 3x}{z - x} = \lim_{z\to x}\frac{z^2 - x^2 - 3z + 3x}{z - x}$$

$$= \lim_{z\to x}\frac{(z+x)(z-x) - 3(z-x)}{z - x} = \lim_{z\to x}\frac{(z-x)\{(z+x) - 3\}}{z - x}$$

$$= \lim_{z\to x}\{z + x - 3\} = (x) + x - 3 = 2x - 3$$

26) $g(x) = (1+\sqrt{x})$ $\qquad$ $g(z) = (1+\sqrt{z})$

$$g'(x) = \lim_{z\to x} \frac{g(z)-g(x)}{z-x} = \lim_{z\to x} \frac{(1+\sqrt{z})-(1+\sqrt{x})}{z-x}$$

$$= \lim_{z\to x} \left(\frac{\sqrt{z}-\sqrt{x}}{z-x}\right)\left(\frac{\sqrt{z}+\sqrt{x}}{\sqrt{z}+\sqrt{x}}\right) = \lim_{z\to x} \frac{z-x}{(z-x)(\sqrt{z}+\sqrt{x})}$$

$$= \lim_{z\to x} \frac{1}{\sqrt{z}+\sqrt{x}} = \frac{1}{\sqrt{(x)}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

---

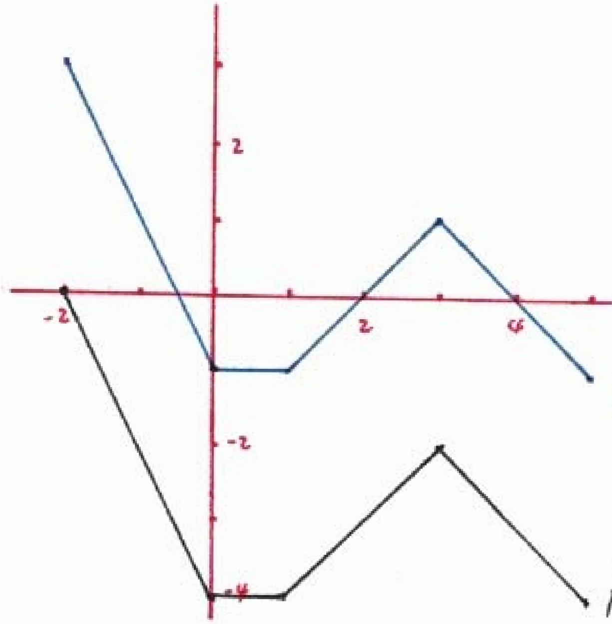
28) Note that the slope of the tangent line is never negative (also at $x=0$, the slope of the tangent line is 0). For $(-\infty, 0)$, $f_2'(x)$ is positive but decreasing as x approaches 0. Then at $x=0$, $\left.\frac{df_2}{dx}\right|_{x=0} = 0$. For $(0,\infty)$, $f_2'(x)$ is positive and increasing without bound. So $\frac{df_2}{dx}$ graph is (a).

---

30) the graph of $y = f_4(x)$ is a graph of a symmetric polynomial of 4th degree (most likely). Also when we take a derivative of a polynomial, the derivative function is a polynomial function that is one degree lower. The graph (c) is a 3rd graph of a 3rd degree symmetric polynomial.

So $\frac{df_4}{dx}$ graph is (c).

32)



42) $g(x) = \begin{cases} x^{\frac{2}{3}}, & x \geq 0 \\ x^{1/3}, & x < 0 \end{cases}$

for $x < 0$ (left-hand derivative)

$g(0) = 0 \qquad g(0+h) = (0+h)^{\frac{1}{3}} = h^{\frac{1}{3}}$

$$\lim_{h \to 0^-} \frac{g(0+h)-g(0)}{h} = \lim_{h \to 0^-} \frac{(h^{\frac{1}{3}})-(0)}{h} = \lim_{h \to 0^-} \frac{1}{h^{2/3}} = \lim_{h \to 0^-} \frac{1}{(\sqrt[3]{h})^2}$$

$$= \frac{1}{(\sqrt[3]{(0^-)})^2} = \frac{1}{(0^-)^2} = \frac{1}{0^+} = +\infty$$

for $x \geq 0$ (right-hand derivative)

$g(0) = 0 \qquad g(0+h) = (0+h)^{\frac{2}{3}} = h^{\frac{2}{3}}$

$$\lim_{h \to 0^+} \frac{g(0+h)-g(0)}{h} = \lim_{h \to 0^+} \frac{(h^{\frac{2}{3}})-(0)}{h} = \lim_{h \to 0^+} \frac{1}{h^{1/3}} = \lim_{h \to 0^+} \frac{1}{\sqrt[3]{h}}$$

$$= \frac{1}{\sqrt[3]{(0^+)}} = \frac{1}{0^+} = +\infty$$

42) continued

Since $\lim_{h\to0^-}\dfrac{g(0+h)-g(0)}{h}=\lim_{h\to0^+}\dfrac{g(0+h)-g(0)}{h}=+\infty$, $g'(0)$ D.N.E.

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44) $g(x)=\begin{cases}2x-x^3-1 & , x\ge0\\ x-\frac{1}{x+1} & , x<0\end{cases}$

for $x<0$ (left-hand derivative)

$g(0)=-1 \qquad g(0+h)=(0+h)-\dfrac{1}{(0+h)+1}=h-\dfrac{1}{h+1}$

$$\lim_{h\to0^-}\frac{g(0+h)-g(0)}{h}=\lim_{h\to0^-}\frac{\left(h-\frac{1}{h+1}\right)-(-1)}{h}=\lim_{h\to0}\left(\frac{\frac{h}{1}-\frac{1}{h+1}+\frac{1}{1}}{\frac{h}{1}}\right)\left(\frac{\frac{h+1}{1}}{\frac{h+1}{1}}\right)$$

$$=\lim_{h\to0^-}\frac{h(h+1)-1+(h+1)}{h(h+1)}=\lim_{h\to0^-}\frac{h^2+h-1+h-1}{h(h+1)}=\lim_{h\to0^-}\frac{h^2+2h}{h(h+1)}$$

$$=\lim_{h\to0^-}\frac{h(h+2)}{h(h+1)}=\lim_{h\to0^-}\frac{h+2}{h+1}=\frac{(0^-)+2}{(0^-)+1}=\frac{2}{1}=2$$

for $x\ge0$ (right-hand derivative)

$g(0)=-1 \qquad g(0+h)=2(0+h)-(0+h)^3-1=\ 2h-h^3-1$

$$\lim_{h\to0^+}\frac{g(0+h)-g(0)}{h}=\lim_{h\to0^+}\frac{(2h-h^3-1)-(-1)}{h}=\lim_{h\to0^+}\frac{2h-h^3}{h}$$

$$=\lim_{h\to0^+}\frac{h(2-h^2)}{h}=\lim_{h\to0^+}(2-h^2)=2-(0^+)^2=2$$

Since $\lim_{h\to0^-}\dfrac{g(0+h)-g(0)}{h}=2$ and $\lim_{h\to0^+}\dfrac{g(0+h)-g(0)}{h}=2$

$$\lim_{h\to0}\frac{g(0+h)-g(0)}{h}=2$$

44) continued

now we need to check if $\lim_{x\to 0} g(x) = g(0)$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}\left(x - \frac{1}{x+1}\right) = (0^-) - \frac{1}{(0^-)+1} = \frac{-1}{1} = -1$$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}\left(2x - x^3 - 1\right) = 2(0^+) - (0^+)^3 - 1 = -1$$

$$g(0) = 2(0) - (0)^3 - 1 = -1$$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = \lim_{x\to 0} g(x) = g(0) = -1$$

we can conclude that $g(x)$ is continuous at $x=0$.

With $\lim_{h\to 0} \frac{g(0+h)-g(0)}{h} = 2$, we can state

that the derivative $g'(0) = 2$.