MATH 20100 section 1.5

General Exponential function (graph)

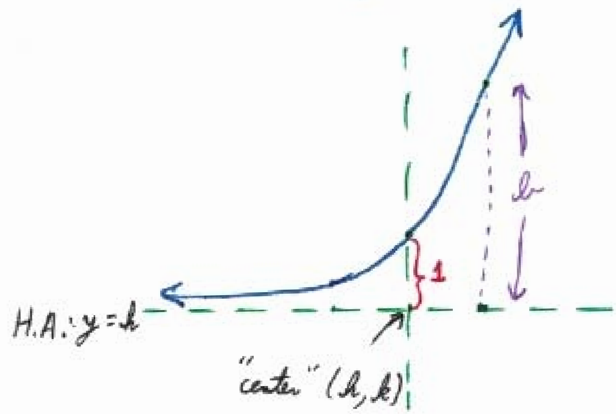
$$y = a\,b^{(x-h)} + k \quad \text{where } (b>0)$$

domain: $(-\infty, \infty)$

horizontal asymptote: $y = k$

H-shift: h

V-shift: k



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2) $y = 3^x = 3^{(x-0)} + 0$ $\qquad y = 8^x = 8^{(x-0)} + 0$

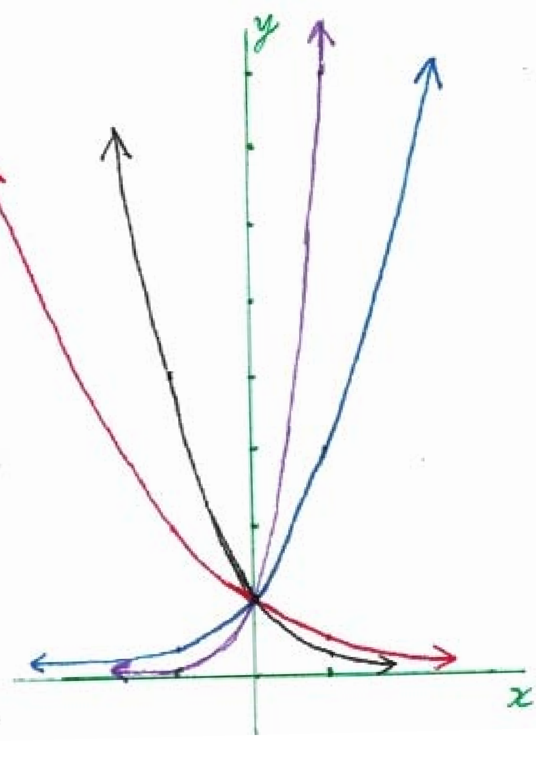
$y = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$ $\qquad y = \left(\frac{1}{4}\right)^x = 4^{-x}$

$= 2^{-(x-0)} + 0$ $\qquad = 4^{-(x-0)} + 0$

all functions have

H.A. : $y = 0$

and no shifts



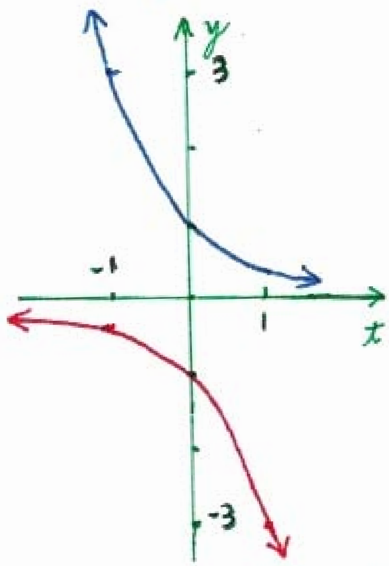
4) $y = 3^{-t} = 3^{-(t-0)} + 0$

$y = -3^{t} = -3^{(t-0)} + 0$

both functions have

H.A.: $y = 0$

and no shifts



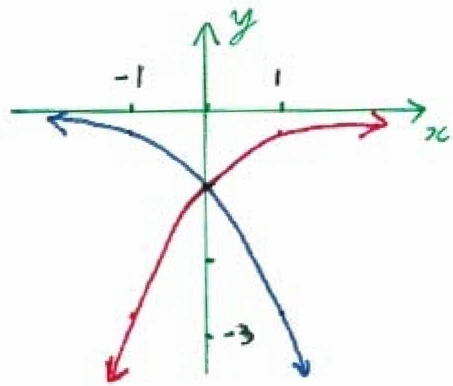
6) $y = -e^{x} = -e^{(x-0)} + 0$

$y = -e^{-x} = -e^{-(x-0)} + 0$

both functions have

H.A.: $y = 0$

and no shifts



8) $y = 3^{x} + 2 = 3^{(x-0)} + 2$

$y = 3^{-x} + 2 = 3^{-(x-0)} + 2$

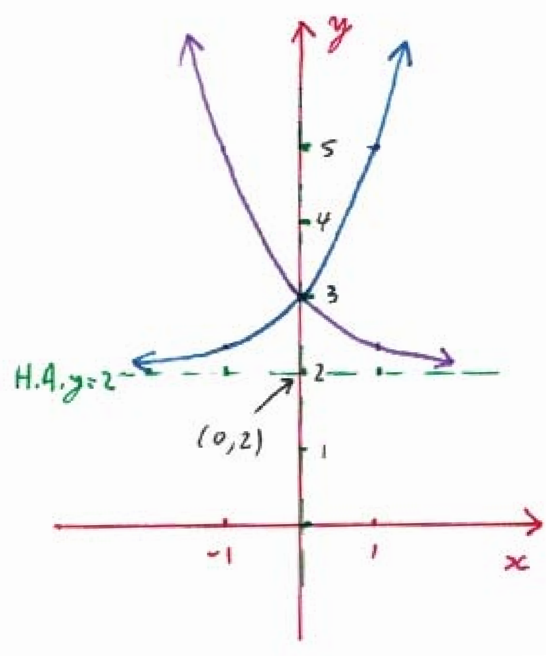
both functions have

H.A.: $y = 2$

H-shift: 0

V-shift: +2

"center": (0,2)



10) $y = -1 - e^{x} = -e^{(x-0)} + (-1)$

$y = -1 - e^{-x} = -e^{-(x-0)} + (-1)$

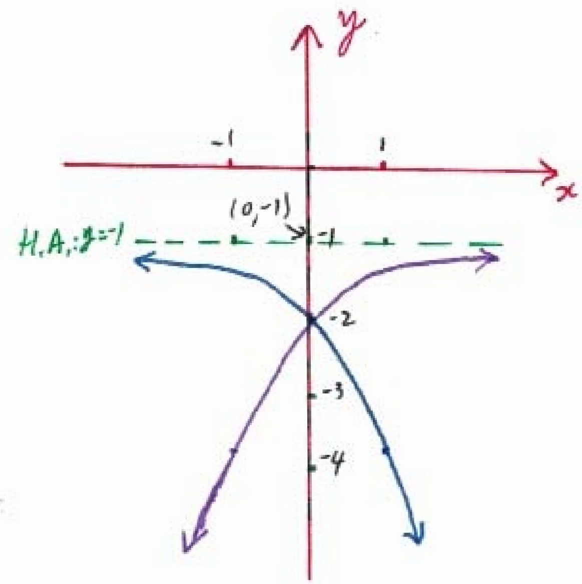
both functions have

H.A.: $y = -1$

H-shift: 0

V-shift: -1

"center": $(0,-1)$



12) $9^{\frac{1}{3}} \cdot 9^{\frac{1}{6}} = 9^{\left(\frac{1}{3}+\frac{1}{6}\right)} = 9^{\left(\frac{4}{12}+\frac{2}{12}\right)} = 9^{\frac{6}{12}} = 9^{\frac{1}{2}} = \sqrt{9} = 3$

14) $\dfrac{3^{\frac{5}{3}}}{3^{\frac{2}{3}}} = 3^{\left(\frac{5}{3}-\frac{2}{3}\right)} = 3^{\frac{3}{3}} = 3^{1} = 3$

16) $\left(13^{\sqrt{2}}\right)^{\frac{\sqrt{2}}{2}} = 13^{(\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)} = 13^{\frac{2}{2}} = 13^{1} = 13$

18) $(\sqrt{3})^{\frac{1}{2}} \cdot (\sqrt{12})^{\frac{1}{2}} = (\sqrt{3})^{\frac{1}{2}} \cdot (\sqrt{4}\sqrt{3})^{\frac{1}{2}} = (\sqrt{3})^{\frac{1}{2}}(2\sqrt{3})^{\frac{1}{2}}$

$= \{(\sqrt{3})(2\sqrt{3})\}^{\frac{1}{2}} = \{2(3)\}^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$

20) $\left(\dfrac{\sqrt{6}}{3}\right)^{2} = \left(\dfrac{\sqrt{6}}{3}\right)\left(\dfrac{\sqrt{6}}{3}\right) = \dfrac{6}{9} = \dfrac{2}{3}$

22) $g(t) = \cos(e^{-t})$

since domain of e^{-t} and $\cos\theta$ is $(-\infty, \infty)$, the domain of $g(t)$ is $\underline{\underline{(-\infty, \infty)}}$.

the range of e^{-t} is $(0, \infty)$ and $\cos\theta$ with domain $(-\infty, \infty)$ produces multiple full waves, the range of $g(t)$ is $\underline{\underline{[-1, 1]}}$

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24) $f(x) = \dfrac{3}{1-e^{2x}}$ since the numerator is a constant, we only need to check the restriction on denominator.

V.A.: $1-e^{2x}=0$
$1=e^{2x}$
$\Downarrow$
$\ln(1)=2x$
$0=2x$
$x=0$

domain: $\underline{\underline{(-\infty, 0) \cup (0, \infty)}}$

range: visualize the graph of e^{2x}.

we need to investigate the range for each pieces of the domain.

domain $(0, \infty)$ part: this is when $x>0$ and the value of e^{2x} is $1<e^{2x}<\infty$ and we are subtracting a value larger than 1 on denominator; so this part generates range of $(-\infty, 0)$.

domain $(-\infty, 0)$ part: this is when $x<0$ and the value of e^{2x} is $0<e^{2x}<1$ and here we are subtracting a value smaller than 1 on denominator; so this part generates range of $(3, \infty)$

range: $\underline{\underline{(-\infty, 0) \cup (3, \infty)}}$

34) let P_0 initial investment at time $t=0$ and t in years.
Since the investment is being compounded continuously, we can use the formula
$P(t) = P_0 e^{rt}$ where P_0 is initial principal
r is interest rate.

We need to find t in years so that $P(t) = 3P_0$ (triple the invested value).

$$r = 5.75\% = 0.0575$$

$$P(t) = 3P_0 \, , \quad t = ?$$

$$(3P_0) = P_0 e^{0.0575t}$$

$$3 = e^{0.0575t}$$

$$\Downarrow$$

$$\ln 3 = 0.0575t$$

$$\underline{\underline{t = \frac{\ln 3}{0.0575} \text{ years}}} \approx 19.10630067 \text{ years}$$

exact answer