MATH 20100 section 1.3

Pythagorean

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Longrightarrow 1 + \tan^2\theta = \sec^2\theta$$

Addition Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Double Angle Formulas

If we let $A = B = \theta$ and use the formula above then

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

using Pythagorean identity

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

Half Angle Formulas

"Reduction of Power" Formulas

solving for $\cos^2\theta$ and $\sin^2\theta$, we get

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

2) $s = r\theta \qquad r = 8,\ s = 10\pi \qquad \theta = ?$

$(10\pi) = (8)\theta$

$\theta = \frac{10\pi}{8} = \underline{\underline{\frac{5\pi}{4}\ rad.}} \qquad \theta = \frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right) = \underline{\underline{225^\circ}}$

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4) 1 m diameter $\Rightarrow$ 0.5 m radius $\Rightarrow r = 0.5\,m$

$s = 30\,cm = 0.3\,m \qquad \theta = ?$ [exact answer]

$(0.3\,m) = (0.5\,m)\theta$

$\theta = \frac{0.3}{0.5} = \underline{\underline{\frac{3}{5}\ rad}} \qquad \theta = \left(\frac{3}{5}\right)\left(\frac{180}{\pi}\right) = \underline{\underline{\left(\frac{108}{\pi}\right)^\circ}}$

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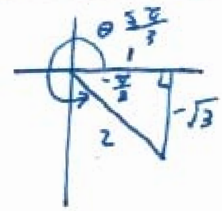
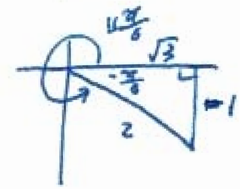
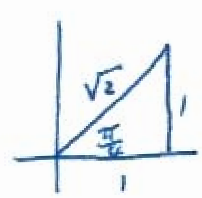
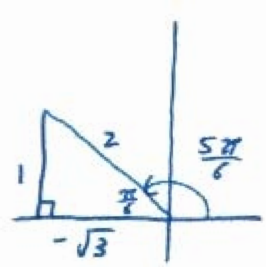
6)

θ	$\frac{-3\pi}{2} = \frac{\pi}{2}$	$\frac{-\pi}{3} = \frac{5\pi}{3}$	$\frac{-\pi}{6} = \frac{11\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{-\sqrt{3}}{2}$
$\tan\theta$	UND	$\frac{-\sqrt{3}}{1} = -\sqrt{3}$	$\frac{-1}{\sqrt{3}}$	$\frac{1}{1} = 1$	$\frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$
$\cot\theta$	0	$\frac{1}{-\sqrt{3}} = \frac{-1}{\sqrt{3}}$	$\frac{\sqrt{3}}{-1} = -\sqrt{3}$	$\frac{1}{1} = 1$	$\frac{-\sqrt{3}}{1} = -\sqrt{3}$
$\sec\theta$	UND	$\frac{2}{1} = 2$	$\frac{2}{\sqrt{3}}$	$\frac{\sqrt{2}}{1} = \sqrt{2}$	$\frac{2}{-\sqrt{3}} = \frac{-2}{\sqrt{3}}$
$\csc\theta$	1	$\frac{2}{-\sqrt{3}} = \frac{-2}{\sqrt{3}}$	$\frac{2}{-1} = -2$	$\frac{\sqrt{2}}{1} = \sqrt{2}$	$\frac{2}{1} = 2$

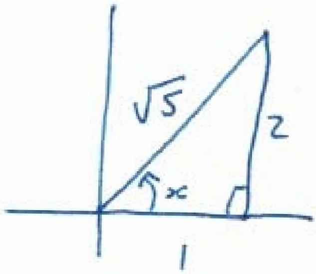








8) $\tan x = 2 = \frac{2}{1}$ $x \in [0, \frac{\pi}{2}] \Rightarrow 0 \le x \le \frac{\pi}{2}$ Q I



$\sin x = \frac{2}{\sqrt{5}}$ $\cos x = \frac{1}{\sqrt{5}}$

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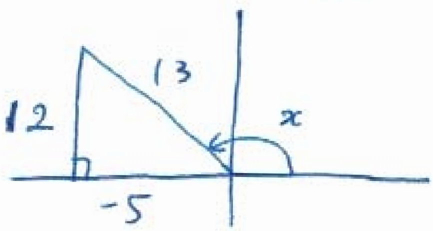
10) $\cos x = -\frac{5}{13} = \frac{-5}{13}$ $x \in [\frac{\pi}{2}, \pi] \Rightarrow \frac{\pi}{2} \le x \le \pi$ Q II



$\sin x = \frac{12}{13}$ $\tan x = \frac{12}{-5} = \frac{-12}{5}$

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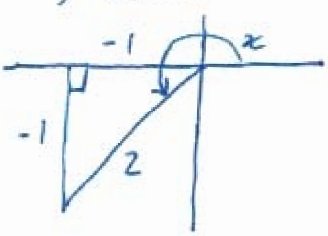
12) $\sin x = -\frac{1}{2} = \frac{-1}{2}$ $x \in [\pi, \frac{3\pi}{2}] \Rightarrow \pi \le x \le \frac{3\pi}{2}$ Q III



$\cos x = \frac{-1}{2}$ $\tan x = \frac{-1}{-1} = \frac{1}{1} = 1$

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14) $\sin\left(\frac{x}{2}\right) = \sin\frac{1}{2}(x)$

frequency: $k = \frac{1}{2}$

period: $\frac{2\pi}{k} = \frac{2\pi}{\left(\frac{1}{2}\right)} = \underline{\underline{4\pi}}$

16) $\cos\frac{\pi x}{2} = \cos\frac{\pi}{2}(x)$

frequency: $k = \frac{\pi}{2}$

period: $\frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2}} = \underline{\underline{4}}$

22) $\cos\left(x+\frac{2\pi}{3}\right)-2=\cos 1\left(x-\left(\frac{-2\pi}{3}\right)\right)-2$

frequency: $k=1$ period: $\frac{2\pi}{\cancel{k}}=\frac{2\pi}{1}=\underline{\underline{2\pi}}$

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24) $s=-\tan \pi t$

$s=-\tan \pi(t-(0))+(0)$

amplitude: $|A|=|-1|=1$

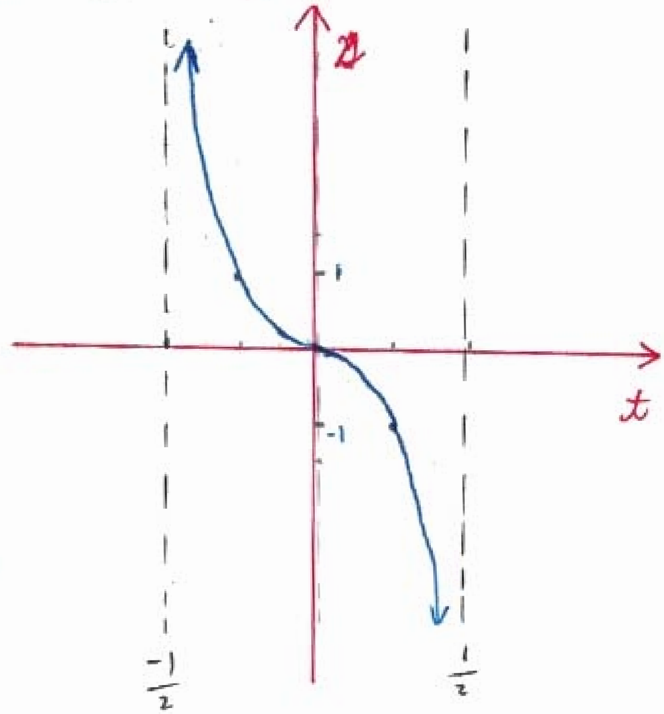
frequency: $k=\pi$

period: $\frac{\pi}{k}=\frac{\pi}{\pi}=\underline{\underline{1}}$

H-shift: 0

V-shift: 0

symmetry: $\underline{\underline{\text{origin}}}$



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26) $s=\csc\left(\frac{t}{2}\right)=1\csc\frac{1}{2}(t-(0))+(0)$

amplitude: $|A|=1$

frequency: $k=\frac{1}{2}$

period: $\frac{2\pi}{k}=\frac{2\pi}{\frac{1}{2}}=\underline{\underline{4\pi}}$

H-shift: 0

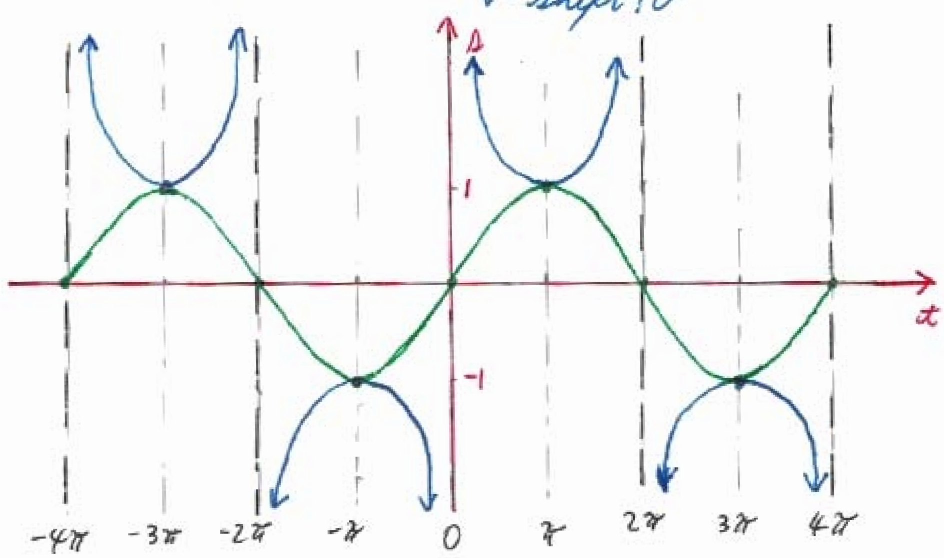
V-shift: 0

since $\cancel{\cos}\ \theta=\frac{1}{\sin\theta}$, we must find all information as if sketching sine function.

1st sketch the graph of $s=1\sin\frac{1}{2}(t-(0))+(0)$

then sketch $s=1\csc\frac{1}{2}(t-(0))+(0)$

symmetry: $\underline{\underline{\text{origin}}}$



32) $\cos\left(x+\frac{\pi}{2}\right) = (\cos x)\left(\cos\frac{\pi}{2}\right) - (\sin x)\left(\sin\frac{\pi}{2}\right)$

$= (\cos x)(0) - (\sin x)(1) = -\sin x$

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34) $\sin\left(x-\frac{\pi}{2}\right) = (\sin x)\left(\cos\frac{\pi}{2}\right) - (\cos x)\left(\sin\frac{\pi}{2}\right)$

$= (\sin x)(0) - (\cos x)(1) = -\cos x$

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36) $\sin(A-B) = \sin(A+(-B)) = (\sin A)(\cos(-B)) + (\cos A)(\sin(-B))$

$= (\sin A)(\cos B) + (\cos A)(-\sin B)$

$= \sin A\cos B - \cos A\sin B$

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38) $B = 2\pi$

$\sin(A+2\pi) = (\sin A)(\cos 2\pi) + (\cos A)(\sin 2\pi)$

$= (\sin A)(1) + (\cos A)(0) = \sin A$

$\cos(A+2\pi) = (\cos A)(\cos 2\pi) - (\sin A)(\sin 2\pi)$

$= (\cos A)(1) - (\sin A)(0) = \cos A$

this shows that the period of sine and cosine functions is 2π.

40) $\sin(2\pi - x) = (\sin 2\pi)(\cos x) - (\cos 2\pi)(\sin x)$

$= (0)(\cos x) - (1)(\sin x) = \underline{\underline{-\sin x}}$

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42) $\cos\left(\frac{3\pi}{2} + x\right) = \left(\cos\frac{3\pi}{2}\right)(\cos x) - \left(\sin\frac{3\pi}{2}\right)(\sin x)$

$= (0)(\cos x) - (-1)(\sin x) = \underline{\underline{\sin x}}$

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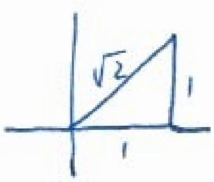
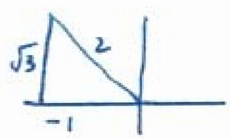
44) $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \left(\cos\frac{\pi}{4}\right)\left(\cos\frac{2\pi}{3}\right) - \left(\sin\frac{\pi}{4}\right)\left(\sin\frac{2\pi}{3}\right)$





$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{-1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \underline{\underline{\frac{-1-\sqrt{3}}{2\sqrt{2}}}}$

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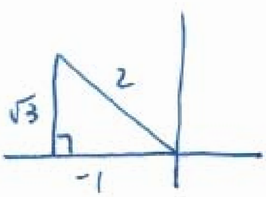
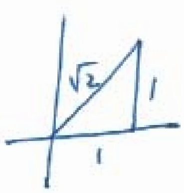
46) $\sin\frac{5\pi}{12} = \sin\left(\frac{8\pi}{12} - \frac{3\pi}{12}\right) = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$





$= \left(\sin\frac{2\pi}{3}\right)\left(\cos\frac{\pi}{4}\right) - \left(\cos\frac{2\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{-1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \underline{\underline{\frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}}}$

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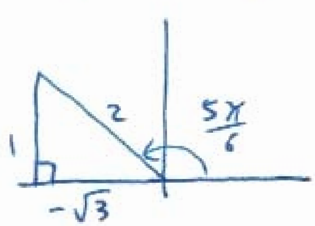
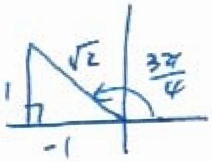
48) $\cos^2\frac{5\pi}{12} = \frac{1 + \cos\left(2\left(\frac{5\pi}{12}\right)\right)}{2} = \frac{1 + \cos\frac{5\pi}{6}}{2} = \frac{1 + \left(\frac{-\sqrt{3}}{2}\right)}{2}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} = \underline{\underline{\frac{2-\sqrt{3}}{4}}}$



50) $\sin^2 \frac{3\pi}{8} = \frac{1}{2}\left(1-\cos\left(2\left(\frac{3\pi}{8}\right)\right)\right) = \frac{1}{2}\left(1-\cos\frac{3\pi}{4}\right)$



$= \frac{1}{2}\left(1-\left(\frac{-1}{\sqrt{2}}\right)\right) = \frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) = \frac{\sqrt{2}+1}{2\sqrt{2}} = \underline{\underline{\frac{1+\sqrt{2}}{2\sqrt{2}}}}$

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52) $\sin^2\theta = \cos^2\theta$

$\sin^2\theta = (1-\sin^2\theta)$

$2\sin^2\theta - 1 = 0$

$(\sqrt{2}\sin\theta+1)(\sqrt{2}\sin\theta-1)=0$

$\sqrt{2}\sin\theta+1=0$		$\sqrt{2}\sin\theta-1=0$	
$\sin\theta = \frac{-1}{\sqrt{2}}$		$\sin\theta = \frac{1}{\sqrt{2}}$	
QIII	QIV	QI	QII
$\underline{\underline{\theta = \frac{5\pi}{4}}}$	$\underline{\underline{\theta = \frac{7\pi}{4}}}$	$\underline{\underline{\theta = \frac{\pi}{4}}}$	$\underline{\underline{\theta = \frac{3\pi}{4}}}$

54) $\cos 2\theta + \cos\theta = 0$

$(2\cos^2\theta - 1) + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(\cos\theta+1)(2\cos\theta-1)=0$

$\cos\theta+1=0$	$2\cos\theta-1=0$	
$\cos\theta=-1$	$\cos\theta=\frac{1}{2}$	
	QI	QIV
$\underline{\underline{\theta=\frac{3\pi}{2}}}$	$\underline{\underline{\theta=\frac{\pi}{3}}}$	$\underline{\underline{\theta=\frac{5\pi}{3}}}$

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56) $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$

dividing all terms by $\cos A\cos B$ →

$$= \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}}$$

$$= \frac{\tan A - \tan B}{1+\tan A\tan B}$$

58-a) $\cos(A-B) = \cos A \cos B + \sin A \sin B$, $\sin\theta = \cos(\frac{\pi}{2}-\theta)$

note $\sin\phi = \cos(\frac{\pi}{2}-\phi)$ and $\cos\phi = \sin(\frac{\pi}{2}-\phi)$

Let $\theta = (A+B)$ and substituting θ in $\sin\theta = \cos(\frac{\pi}{2}-\theta)$ we get

$\sin(A+B) = \cos \frac{\pi}{2}-(A+B) = \cos \frac{\pi}{2}-A-B = \cos\{(\frac{\pi}{2}-A)-B\}$

$= (\cos(\frac{\pi}{2}-A))(\cos B) + (\sin(\frac{\pi}{2}-A))(\sin B)$

now use the identities above written in black

$= (\sin A)(\cos B) + (\cos A)(\sin B)$

$= \sin A \cos B + \cos A \sin B$

58-b) $\cos(A+B) = \cos(A-(-B)) = (\cos A)(\cos(-B)) + (\sin A)(\sin(-B))$

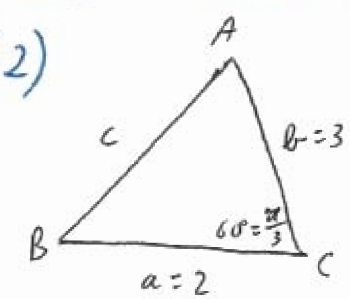
cosine function is even : $\cos(-\theta) = \cos\theta$

sine function is odd: $\sin(-\theta) = -\sin\theta$

$= (\cos A)(\cos B) + (\sin A)(-\sin B)$

$= \cos A \cos B - \sin A \sin B$

62)



using law of cosine $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (2)^2 + (3)^2 - 2(2)(3)\cos(\frac{\pi}{3}) = 4 + 9 - 12(\frac{1}{2}) = 13 - 6 = 7$

$c = \pm\sqrt{7}$ since distance is always positive, $c = +\sqrt{7}$

now using law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{3} = \frac{\sin\frac{\pi}{3}}{\sqrt{7}} \Rightarrow \sin B = \frac{3\sin\frac{\pi}{3}}{\sqrt{7}} = \frac{3\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{7}} = \underline{\underline{\frac{3\sqrt{3}}{2\sqrt{7}}}}$